Probing Weak Lensing Cosmology with Scattering Transform

Sijin Chen

Universitäts-Sternwarte München

With Stella Seitz, Laurence Gong

Master Science Day, April 9th, 2024

Scattering Transform

operation:

$$I' = |I \star \psi^{j,l}|$$

Scattering transform=

wavelet convolution

+ modulus

+ mean

Scattering Transform

Wavelet Convolution :

operation:

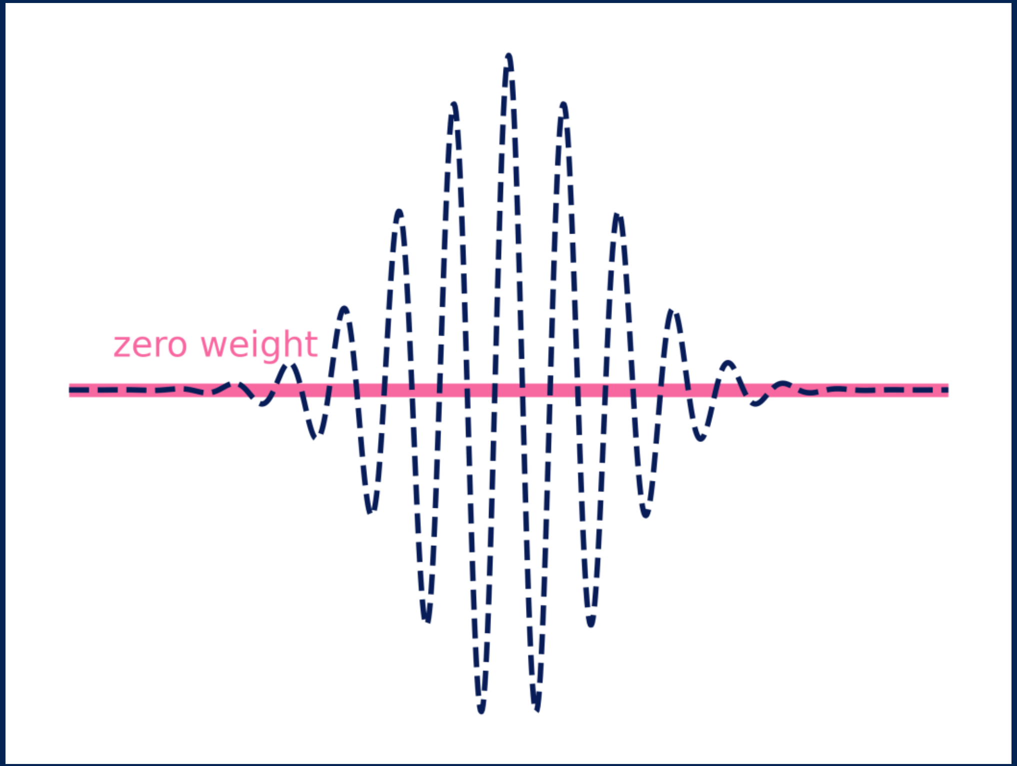
$$I' = |I \star \psi^{j,l}|$$

Scattering transform=

wavelet convolution

+ modulus

+ mean



in real space :

$$G(x) = \frac{1}{\sqrt{|\Sigma|}} e^{-\boldsymbol{x}^T \Sigma^{-1} \boldsymbol{x}/2} e^{i \boldsymbol{k}_0 \cdot \boldsymbol{x}}$$

- Σ : the covariance matrix describing the size and shape of the Gaussian envelope
- $\boldsymbol{k}_0$: the frequency of the modulated oscillation

Scattering Transform

Wavelet Convolution :

operation:

$$I' = |I \star \psi^{j,l}|$$

$J = 8, j = 0,1 \dots, 7$

$L = 4, l = 0,1,2,3$

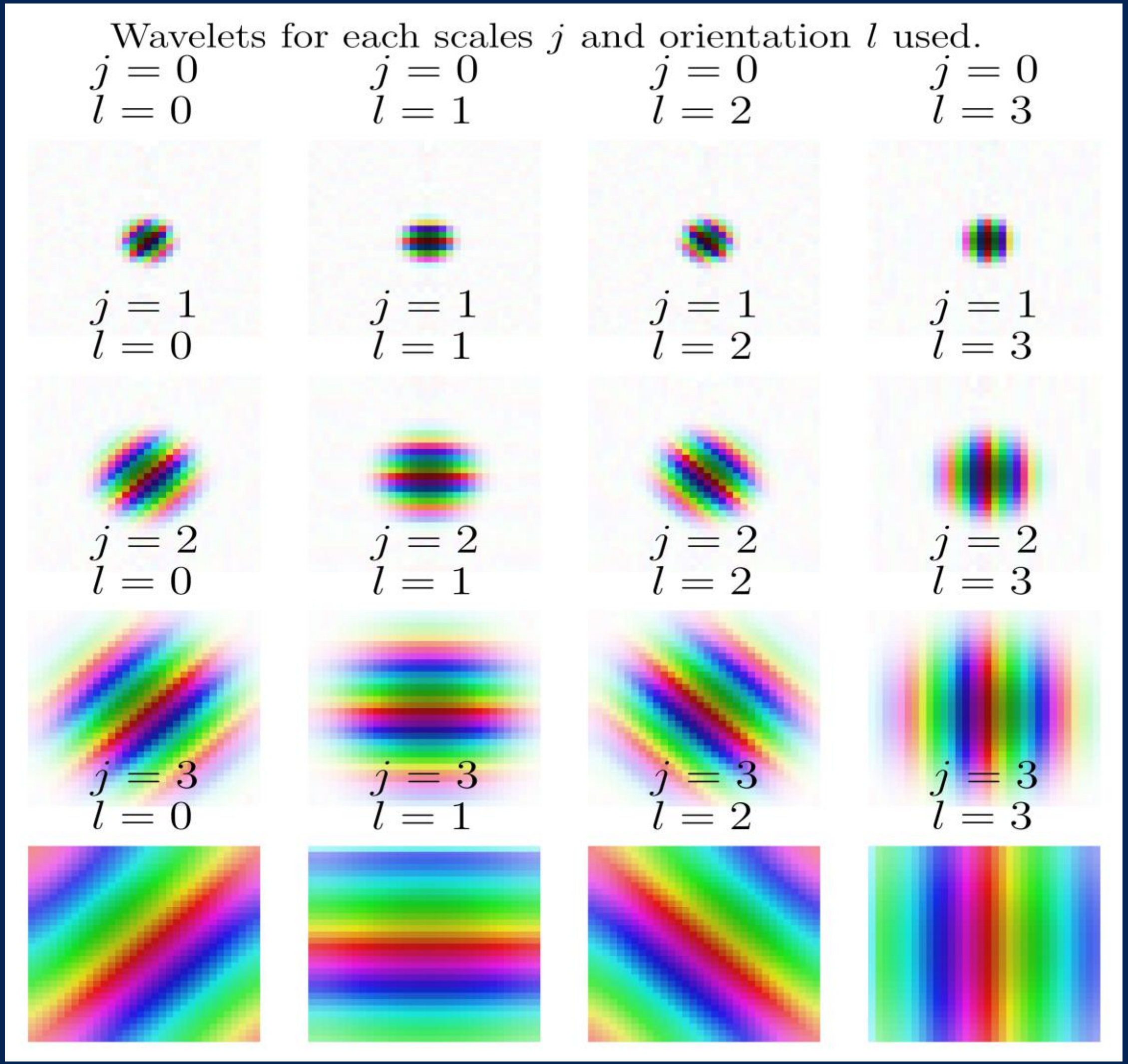
Scattering transform=

wavelet convolution

+ modulus

+ mean

- j : size (logarithmic spacing)
- l : orientation



Kymatio

Scattering Transform

Input field I_0

Coefficients: $S_0 \equiv < I_0 >$

operation:

$$I' = |I \star \psi^{j,l}|$$

Fields: $I_1 \equiv |I_0 \star \psi_1|$

Coefficients: $S_1 \equiv < I_1 >$

Scattering transform=

wavelet convolution

+ modulus

+ mean

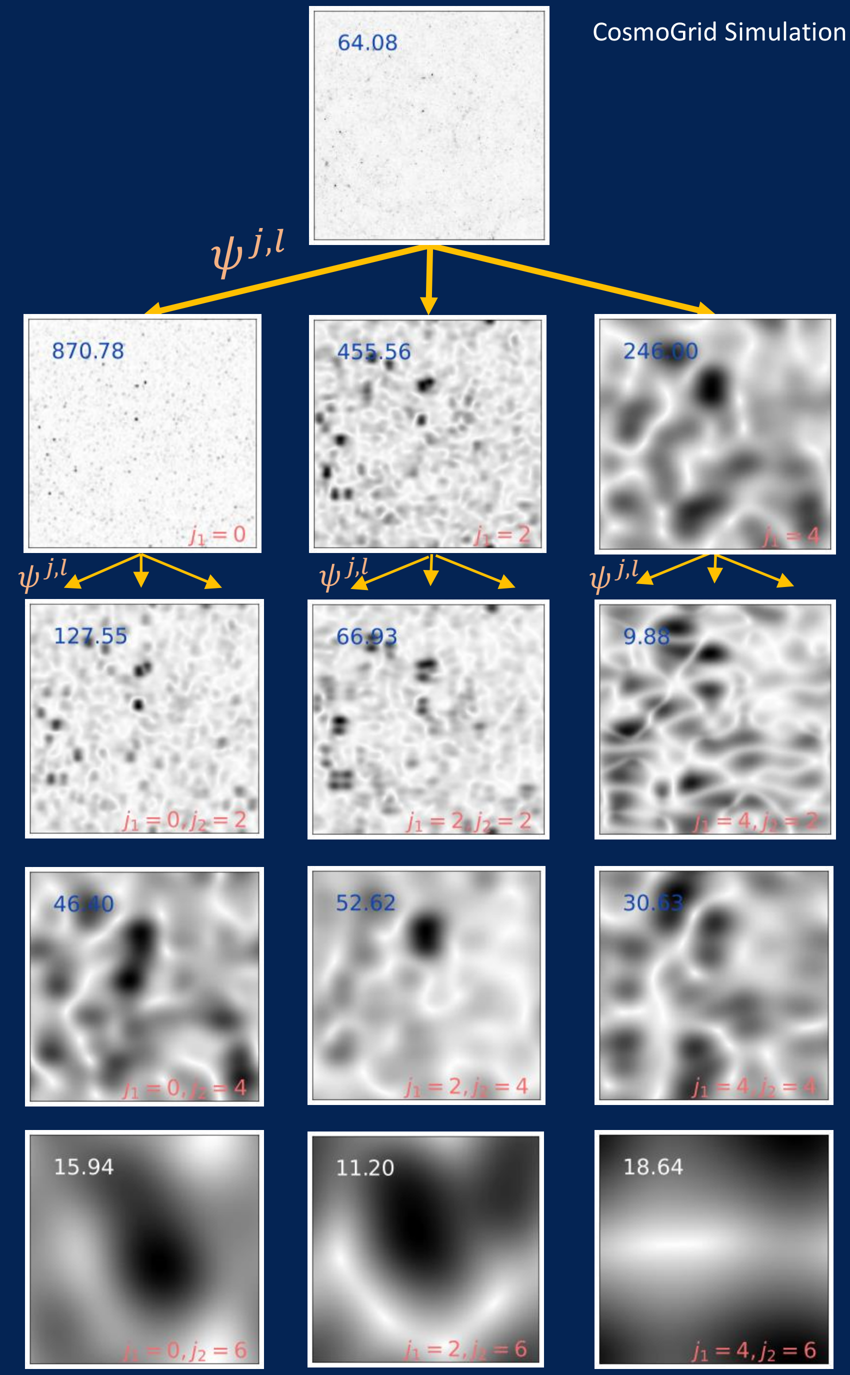
Fields $I_2 \equiv ||I_0 \star \psi_1| \star \psi_2|$

Coefficients: $S_2 \equiv < I_2 >$

(all convolved fields shown has orientation index $l_1 = 1$, $l_2 = 1$)

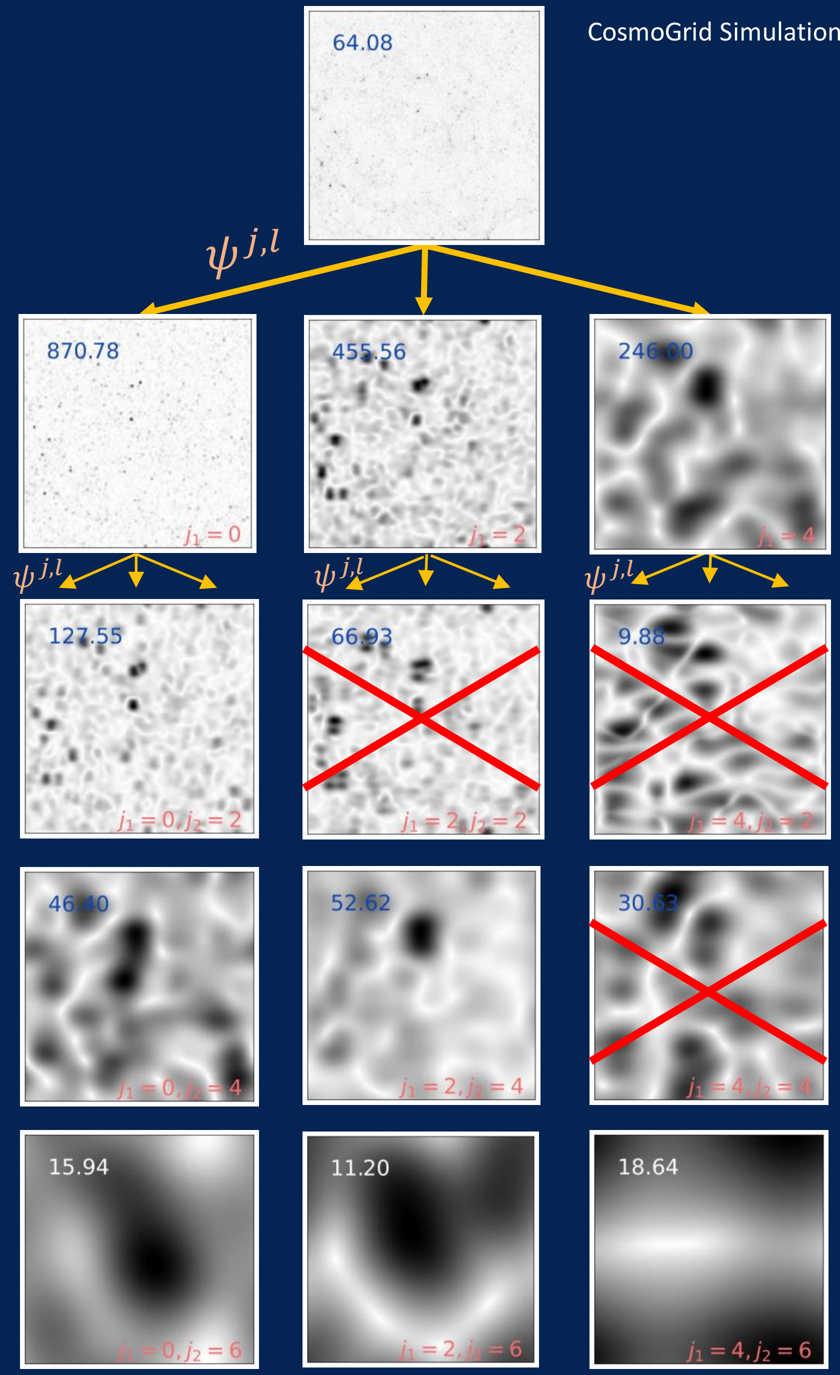
(All numbers shown are 10^6 times the real coefficients)

CosmoGrid Simulation Data



Scattering Transform

CosmoGrid Simulation Data



operation:

$$I' = |I \star \psi^{j,l}|$$

Scattering transform=

wavelet convolution

+ modulus

+ mean

When doing second order convolution, choose filter ψ^{j_2,l_2} with $j_2 > j_1$, since structures of particular size, say j_2, do not have any meaningful

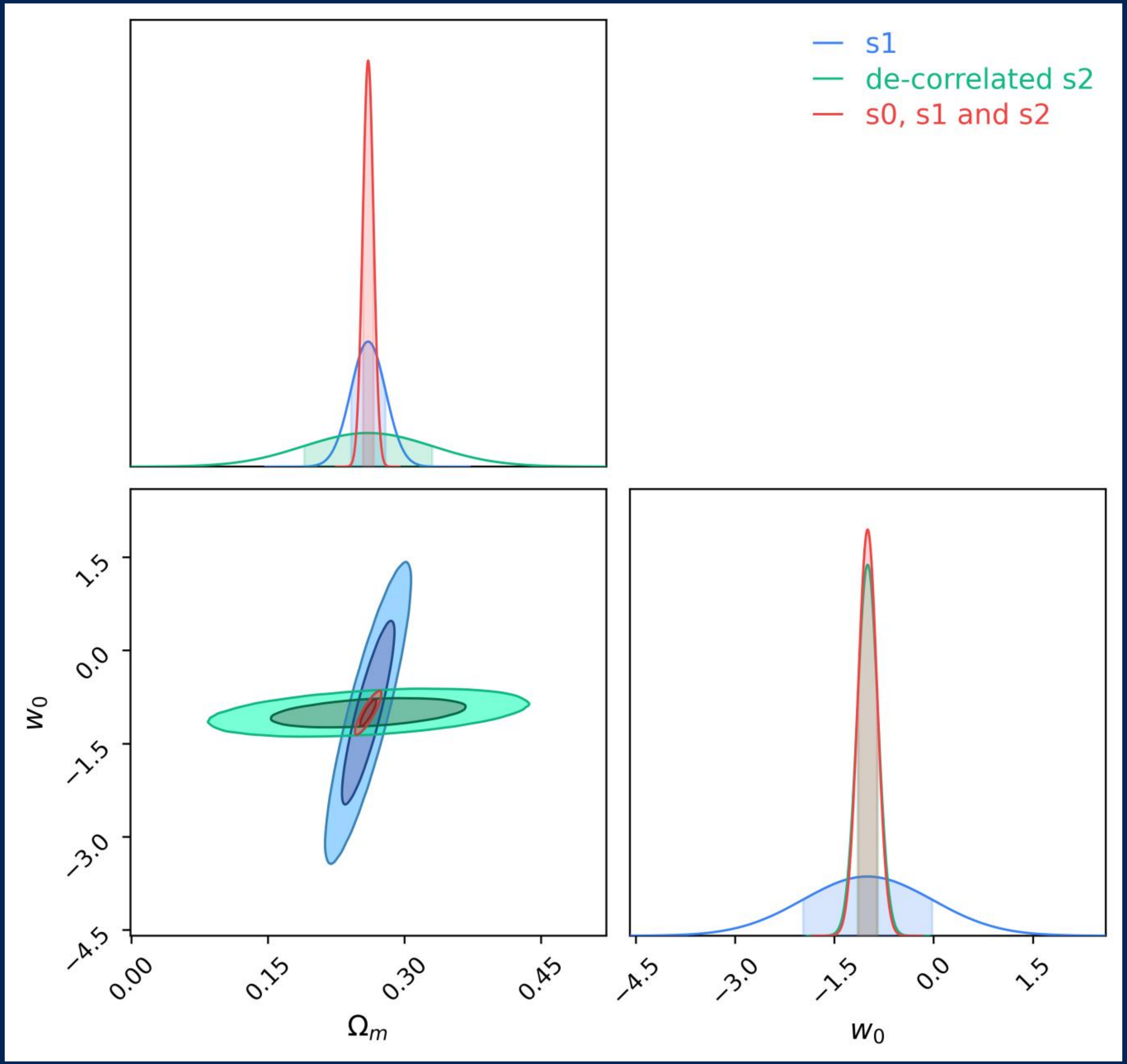
Fisher Forecast

$$s_1 \equiv \langle S_1^{j_1,l_1} \rangle_{l_1}$$

$$s_2 \equiv \langle S_2^{j_1,l_1,j_2,l_2} \rangle_{l_1,l_2}$$

De-correlated 2nd –order coefficients:
s_2/s_1

with a noiseless 20x20 deg^2 weak lensing map



Future Work

- Emulation
  - Emulate the scattering coefficients using 2500 cosmology from CosmoGridV1 simulation set
  - A fast emulator based on Neural Network (CosmoPower, Spurio-Mancini et al 2021)

- MCMC
  - Covariance estimated from simulations at fiducial cosmology
  - Sample the posterior with the fast emulator in multi-dimensional space
  - Obtaining cosmological parameter constraints with scattering transform

Scattering Transform

One operation:

$$I' = |I \star \psi^{j,l}|$$

Scattering transform=

wavelet convolution

+ modulus

+ mean

(Morlet) wavelets:

in real space :

$$G(x) = \frac{1}{\sqrt{|\Sigma|}} e^{-\boldsymbol{x}^T \Sigma^{-1} \boldsymbol{x}/2} e^{i\boldsymbol{k}_0 \cdot \boldsymbol{x}}$$

- Σ : the covariance matrix describing the size and shape of the Gaussian envelope
- $\boldsymbol{k}_0$: the frequency of the modulated oscillation

in Fourier space :

$$\tilde{G}(\boldsymbol{k}) = -\boldsymbol{e}^{-(\boldsymbol{k}-\boldsymbol{k}_0)^T \boldsymbol{\Sigma} (\boldsymbol{k}-\boldsymbol{k}_0)/2}$$

Scattering Transform

(Morlet) wavelets:

One operation:

$$I' = |I \star \psi^{j,l}|$$

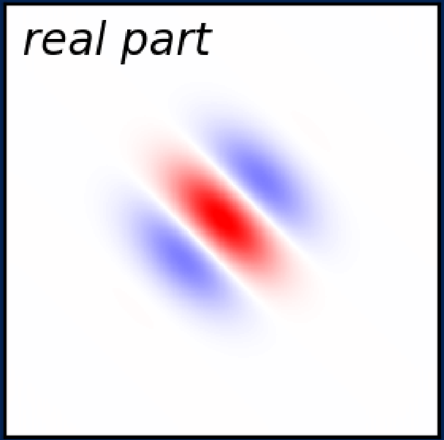
Scattering transform=

wavelet convolution

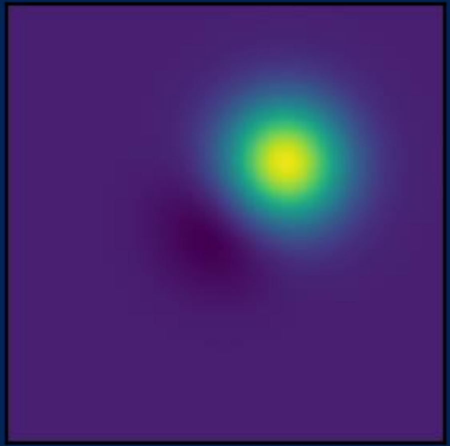
+ modulus

+ mean

Real space



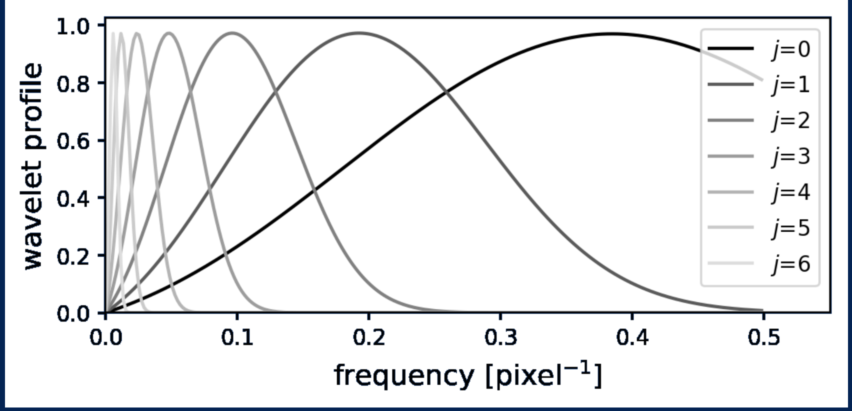
Fourier space



S. Cheng et al

Scattering Transform

(Morlet) wavelets:



S.Cheng et al

One operation:

$$I' = |I \star \psi^{j,l}|$$

$J = 8, j = 0,1 \ldots, 7$

$L = 4, l = 0,1,2,3$

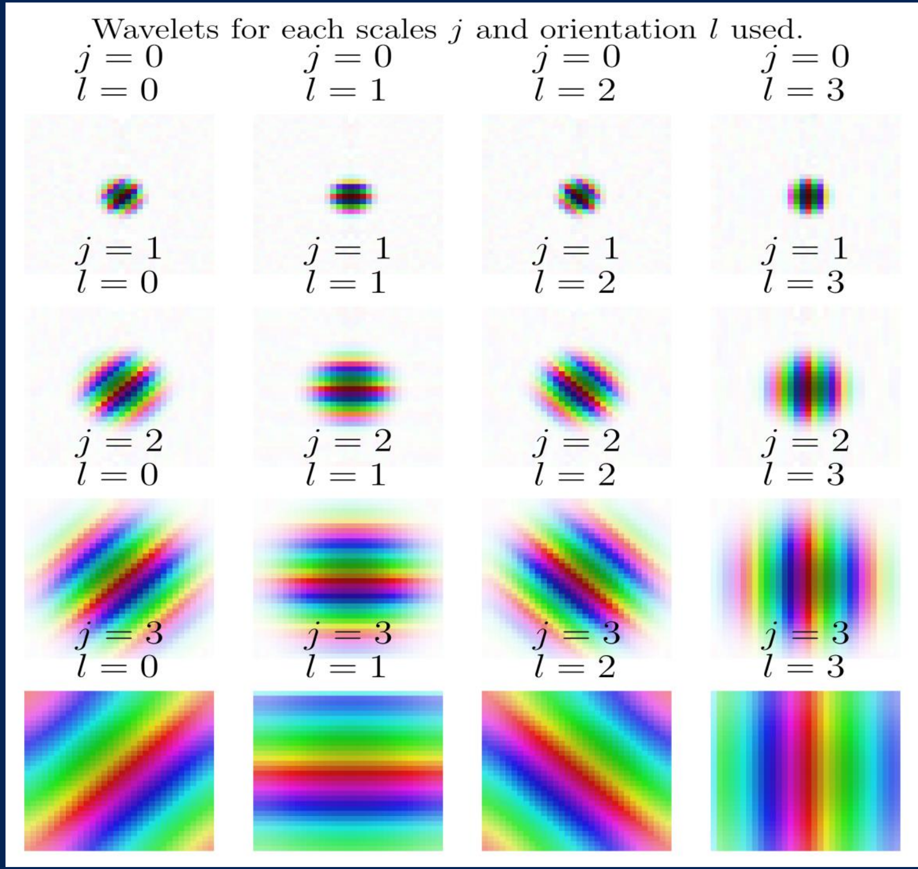
Scattering transform=

wavelet convolution

+ modulus

+ mean

- j : scales (logarithmic spacing)
- l : directions



Kymatio

Scattering Transform

Modulus: convert selected fluctuations into their local strength

One operation:

$$I' = |I \star \psi^{j,l}|$$

Scattering transform=

wavelet convolution

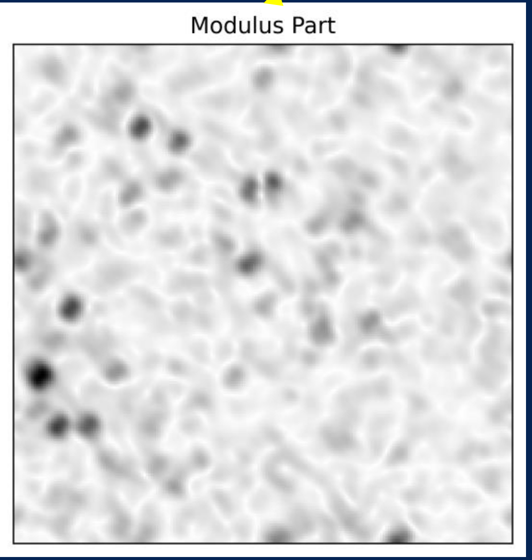
+ modulus

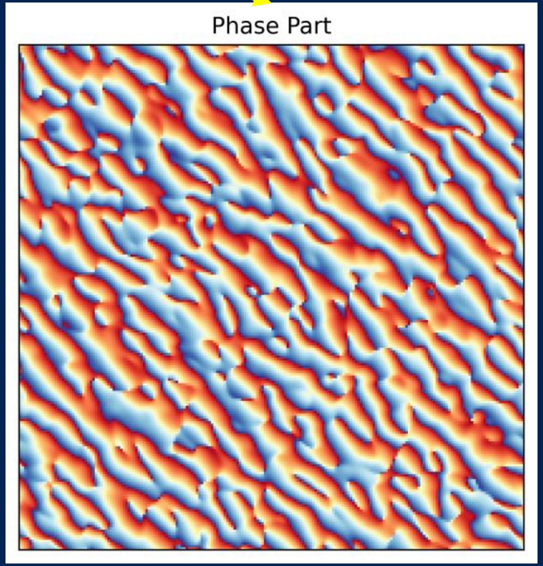
+ mean

What is phase part

$\psi^{j,l}$

After convolution





Scattering Transform

Mean: spatial average of the field

One operation:

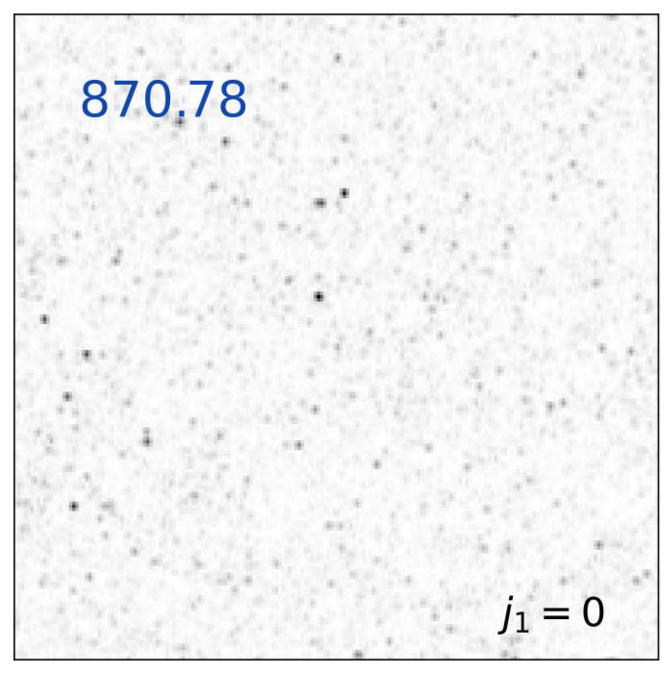
$$I' = |I \star \psi^{j,l}|$$

Scattering transform=

wavelet convolution

+ modulus

+ mean



$$S_0 \equiv < I_0 >$$

$$S_1^{j_1,l_1} \equiv < I_1^{j_1,l_1} > = < \left| I_0 \star \psi^{j_1,l_1} \right| >$$

$$S_2^{j_1,l_1,j_2,l_2} \equiv < I_2^{j_1,l_1,j_2,l_2} > = < \left| \left| I_0 \star \psi^{j_1,l_1} \right| \star \psi^{j_2,l_2} \right| >$$

Scattering Transform

- When using isotropic fields, the scattering coefficients S_n could be further reduced by taking the average over all the orientation indices.

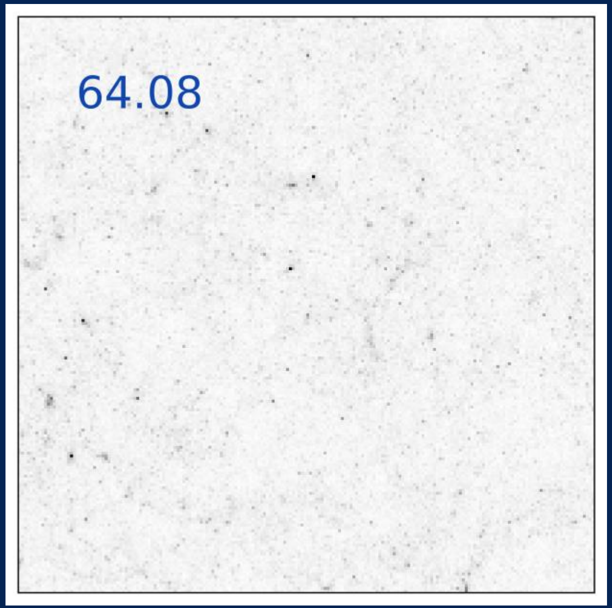
$$s_0 \equiv S_0$$

$$s_1(j_1) \equiv \langle S_1^{j_1,l_1} \rangle_{l_1}$$

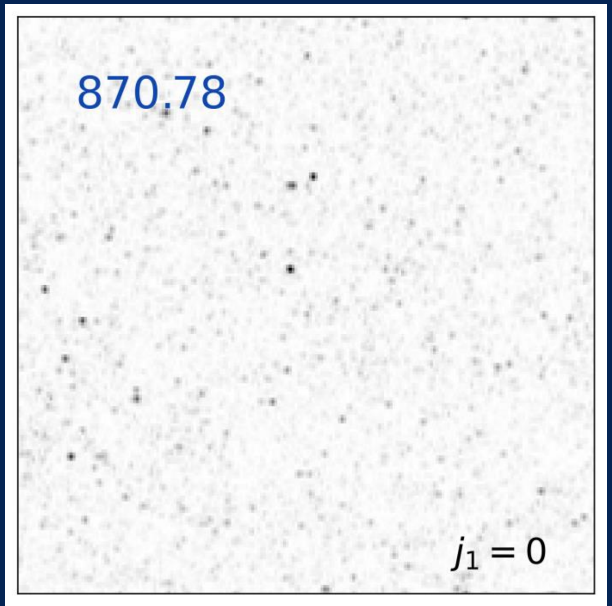
$$s_2(j_1, j_2) \equiv \langle S_2^{j_1,l_1,j_2,l_2} \rangle_{l_1,l_2}$$

Scattering Transform

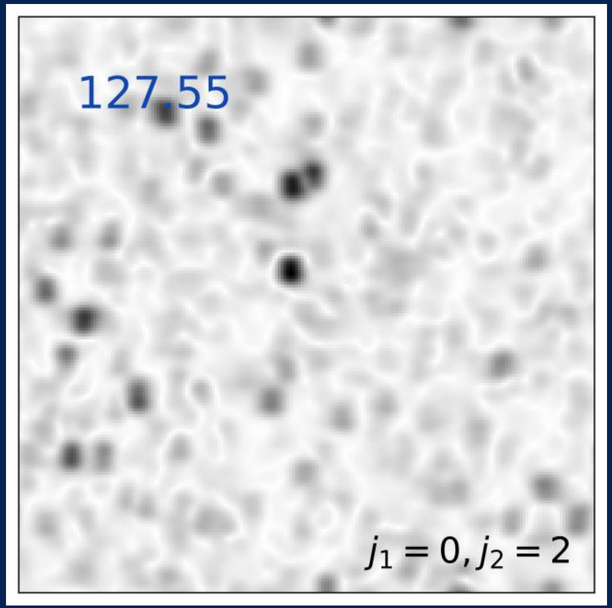
input field



1^{st} -order



2^{nd} -order



(All the numbers here are 10^6 times the real coefficients)

1^{st} order coefficients: the clustering of the particles

2^{nd} order coefficients: the clustering of the structures

fluctuation level

Gnomonic Projection