# Numerical treatment of physical processes

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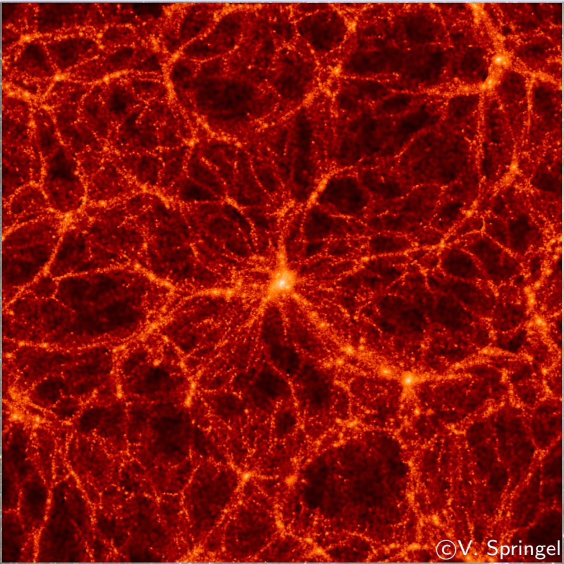
September 7th, 2018

H. Lesch, K. Dolag,

Many collaborators

# Outline

# What have we learned so far?

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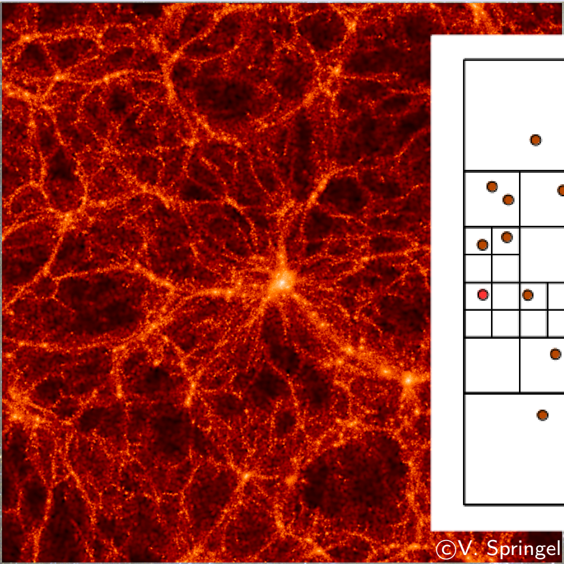


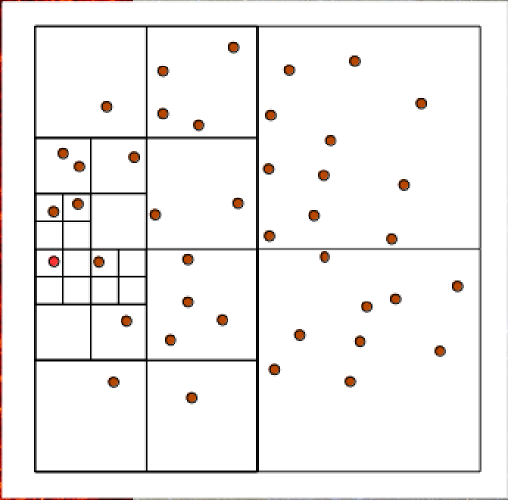

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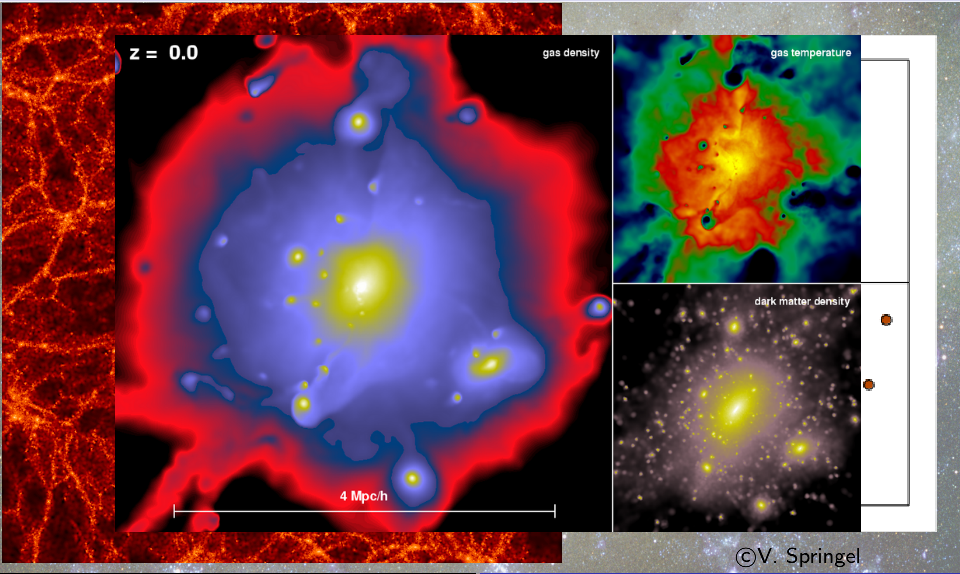

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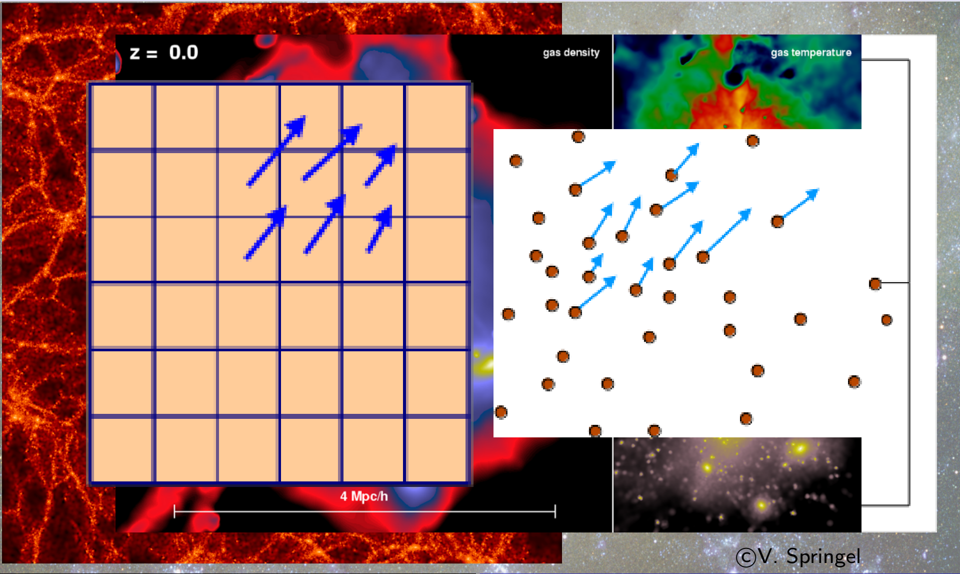
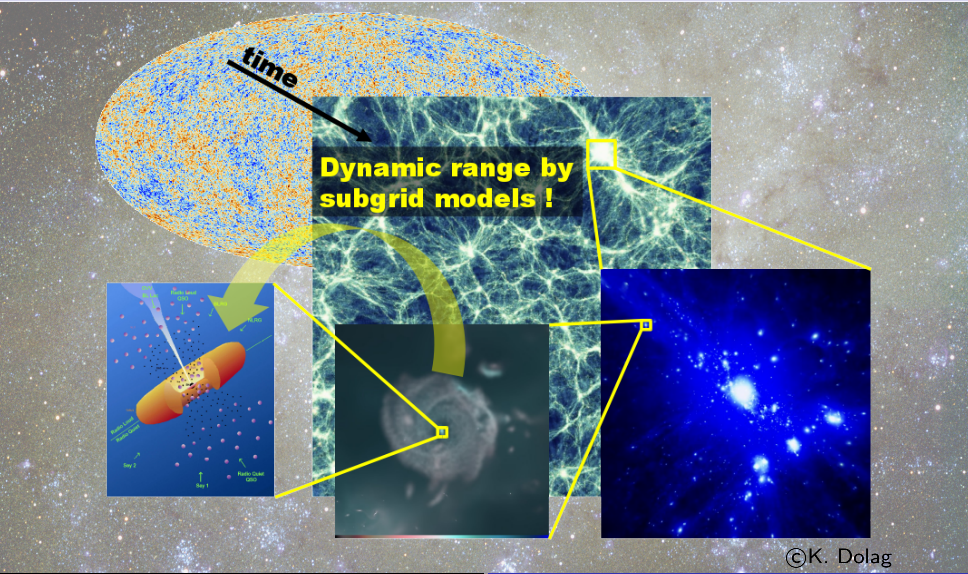


# Dynamic Range



# Which goals can we not reach yet?

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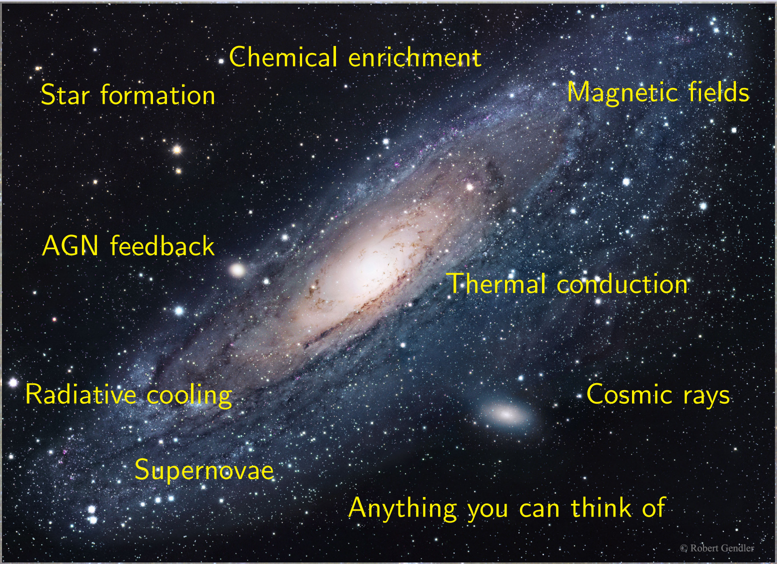
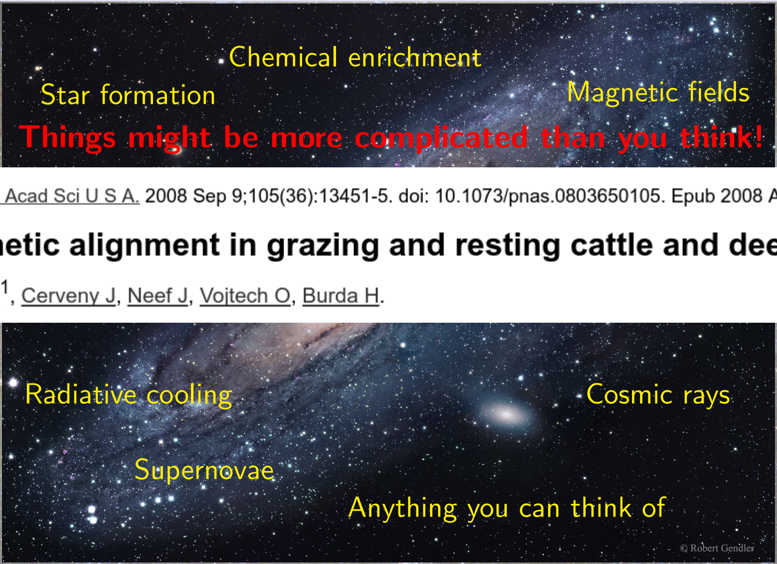


# Which goals can we not reach yet?

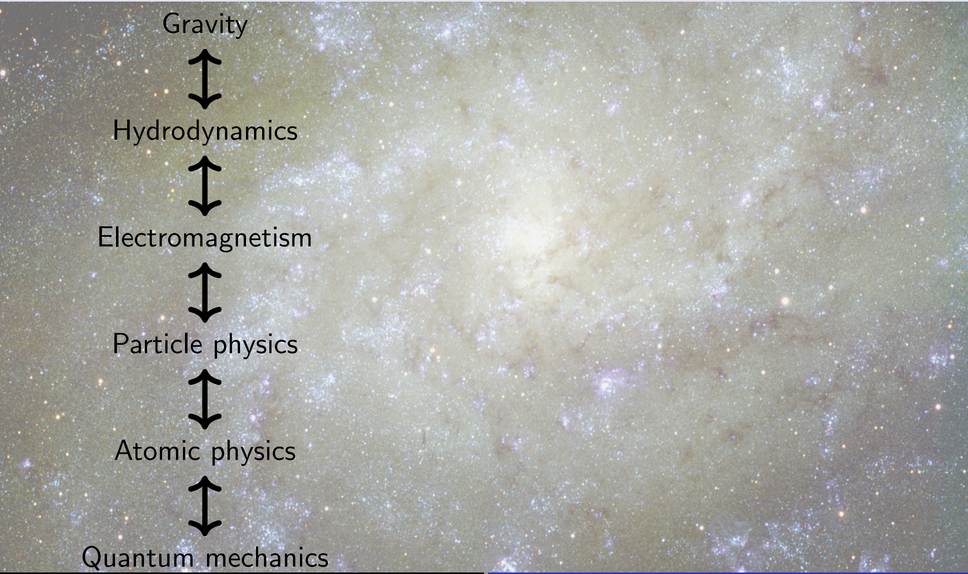



Proc Natl Acad Sci U S A. 2008 Sep 9;105(36):13451-5. doi: 10.1073/pnas.0803650105. Epub 2008 Aug 25.

**Magnetic alignment in grazing and resting cattle and deer.**

Begall S[1], Cerveny J, Neef J, Vojtech O, Burda H.

# But how?



Gravity

↕

Hydrodynamics

↕

Electromagnetism

↕

Particle physics

↕

Atomic physics

↕

Quantum mechanics

# But how?

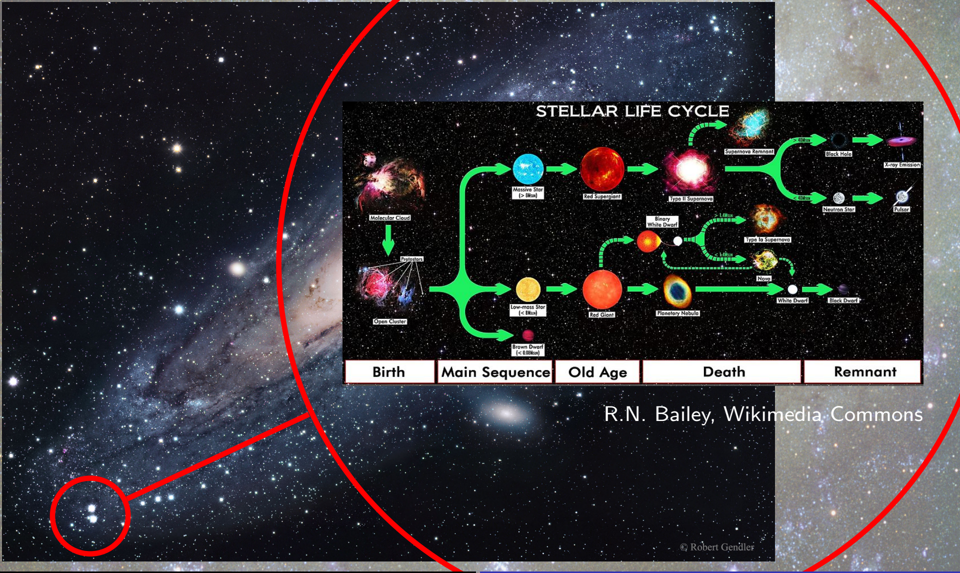
Gravity
↕
Hydrodynamics
↕
Electromagnetism
↕
Particle physics
↕
Atomic physics
↕
Quantum mechanics

Huge magnitude of scales to consider!

$\Rightarrow$ **Subgrid models**

# Subgrid modelling

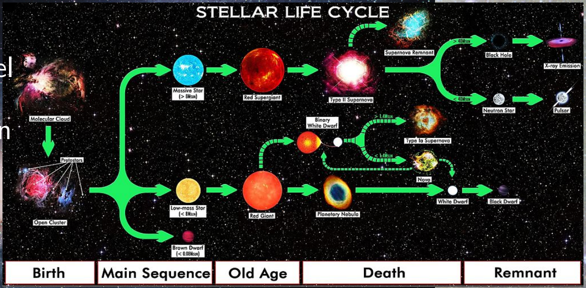
# Subgrid modelling




R.N. Bailey, Wikimedia Commons

# Subgrid modelling

- Input: Simulation state
- Statistical approach
- Below particle / cell level
- Output: Model evolution

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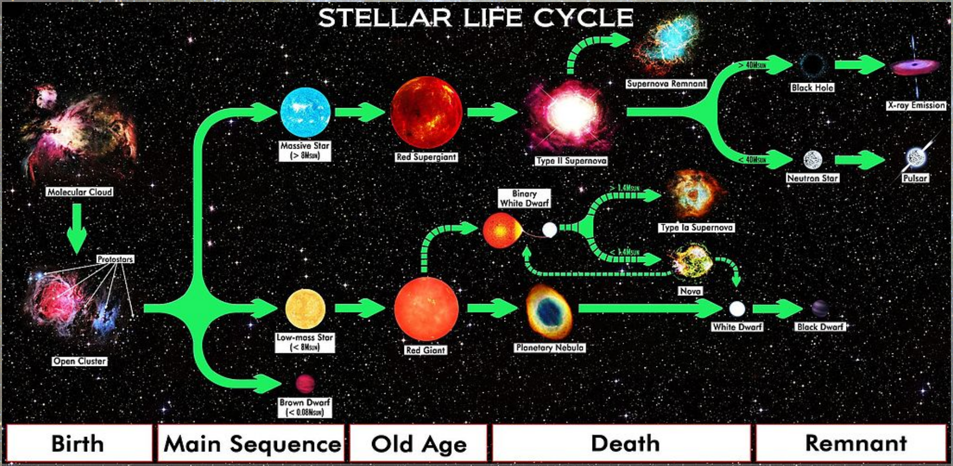
# Outline

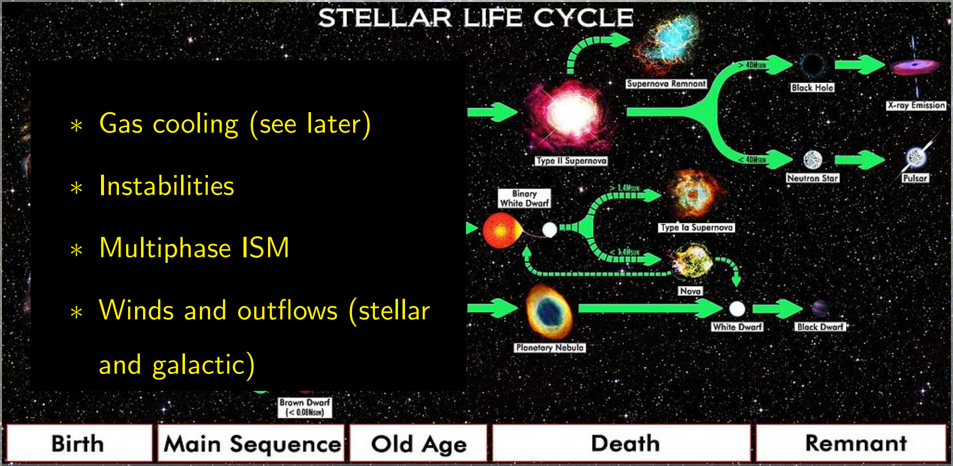
# Star formation & evolution




R.N. Bailey, Wikimedia Commons

# Star formation & evolution

- Gas cooling (see later)
- Instabilities
- Multiphase ISM
- Winds and outflows (stellar and galactic)

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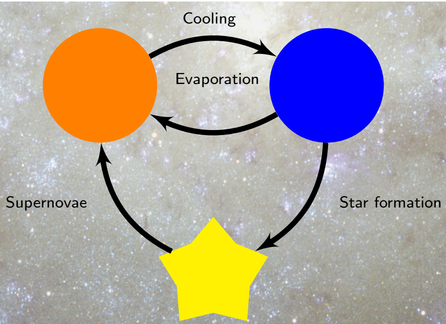
# Springel & Hernquist 2002

# Springel & Hernquist 2002


Stellar density

SN mass fraction

Star formation $$\frac{d\rho_\star}{dt} = (1-\beta)\,\frac{\rho_c}{t_\star}$$

SF timescale

Hot density

Cloud evaporation $$\left.\frac{d\rho_h}{dt}\right|_{\rm evap} = A\beta\,\frac{\rho_c}{t_\star}$$

Cloud growth $$\left.\frac{d\rho_c}{dt}\right|_{\rm TI} = -\left.\frac{d\rho_h}{dt}\right|_{\rm TI} = \frac{\Lambda_{\rm net}(\rho_h, u_h)}{u_h - u_c}$$

Cold density

Radiative losses

# The full equations of S&H 2002

Thermal energy budget:

supernova 'temperature' ~ $10^8$ K

$$\frac{\mathrm{d}}{\mathrm{d}t}(\rho_h u_h + \rho_c u_c) = -\Lambda_{\rm net}(\rho_h, u_h) + \beta \frac{\rho_c}{t_\star} u_{\rm SN} - (1-\beta)\frac{\rho_c}{t_\star} u_c,$$

Total energy | Cooling | Feedback | Loss to stars

cold clouds:

$$\frac{\mathrm{d}}{\mathrm{d}t}(\rho_c u_c) = -\frac{\rho_c}{t_\star} u_c - A\beta \frac{\rho_c}{t_\star} u_c + \frac{(1-f)u_c}{u_h - u_c}\Lambda_{\rm net}$$

$$f = \begin{cases} 1 & \text{normal cooling} \\ 0 & \text{thermal instability} \end{cases}$$

hot phase:

$$\frac{\mathrm{d}}{\mathrm{d}t}(\rho_h u_h) = \beta \frac{\rho_c}{t_\star}(u_{\rm SN} + u_c) + A\beta \frac{\rho_c}{t_\star} u_c - \frac{u_h - f u_c}{u_h - u_c}\Lambda_{\rm net}$$

Mass transfer budget:

cold clouds:

$$\frac{\mathrm{d}\rho_c}{\mathrm{d}t} = -\frac{\rho_c}{t_\star} - A\beta \frac{\rho_c}{t_\star} + \frac{(1-f)}{u_h - u_c}\Lambda_{\rm net}$$

Star formation | Evaporation | Cloud Growth

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Supernovas

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Star formation — Evaporati

McKee & Ostriker 1977

Parameter

hot phase:

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Temperature evolution:

hot phase:

$$\rho_h\frac{\mathrm{d}u_h}{\mathrm{d}t} = [u_{\mathrm{SN}} - (A+1)(u_h - u_c)]\,\beta\frac{\rho_c}{t_\star} - f\Lambda_{\mathrm{net}}$$

cold clouds: temperature assumed to be constant at ~ $10^4$ K

equilibirum temperature for star formation + thermal instability →

$$u_h = \frac{u_{\mathrm{SN}}}{A+1} + u_c$$

# The full equations of S&H 2002

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Temperature e

More complex modelling:

See e.g. Tornatore et al. 2003, 2007

hot phase:

$$\rho_h \frac{\mathrm{d}}{\mathrm{d}t} \ldots [u_{\rm SN} \ldots (A+1)(u_h \ldots u_c)] \ldots t_\star - f\Lambda_{\rm net}$$

cold clouds: temperature assumed to be constant at ~ $10^4$ K

equilibirum temperature for star formation
+ thermal instability

$$u_h = \frac{u_{\rm SN}}{A+1} + u_c$$

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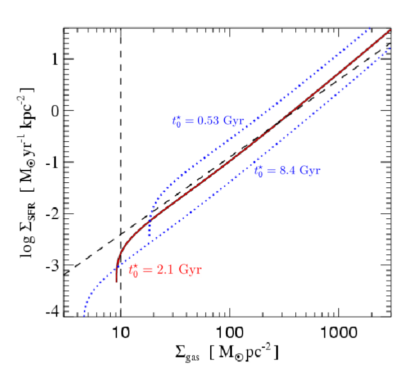
# Callibration of SF model: Kennicutt-Schmidt relation

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## Kennicutt 1998

$$\Sigma_{SFR} = (2.5 \pm 0.7) \cdot 10^{-4} \left( \frac{\Sigma_{gas}}{M_\odot pc^{-2}} \right)^{1.4 \pm 0.15} \frac{M_\odot}{yr\ \ kpc^2}$$

# Callibration of SF model: More ingredients

**Life-time:** Maeder & Meynet 1989; Padovani & Matteucci 1993

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**IMF:** Salpeter; Kroupa; Chabrier; Arimoto & Yoshi

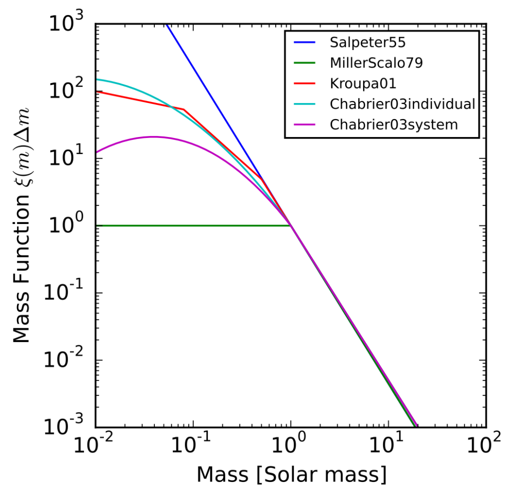
**Stellar yields:** AGB (Groenewegen; Karakas), SNIa (Thielemann), SNII (Woosly & Weaver; Romano; Kobayashi; . . . )

# Callibration of SF model: More ingredients

**Life-time:** ni & Mat-

**IMF:** rimoto &

**Stellar yiel** s), SNIa Weaver;

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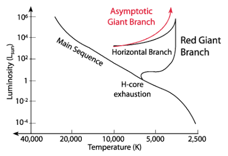
# Feedback processes

**Different origin**

AGB stars

Supernovae type Ia

Supernovae type II

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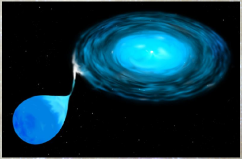

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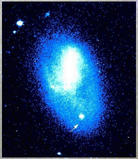

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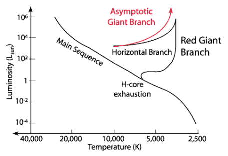
# Feedback processes

**Different origin**

- AGB stars
- Supernovae type Ia
- Supernovae type II

**Different form**

- Thermal heating
- Kinetic wind

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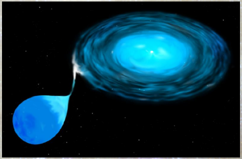

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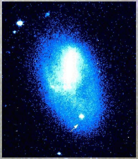

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# Exemplary galactic wind model

* Observations by Martin 1998, 1999: $\dot{M}_W = \eta \dot{M}_\star$
* Relate to SN energy: $\frac{1}{2}\dot{M}_W v_W^2 = \chi \epsilon_{SN} \dot{M}_\star$
* Typically $\eta \sim 2$, $\chi \sim 0.25$, $v_W \sim 250 km/s$

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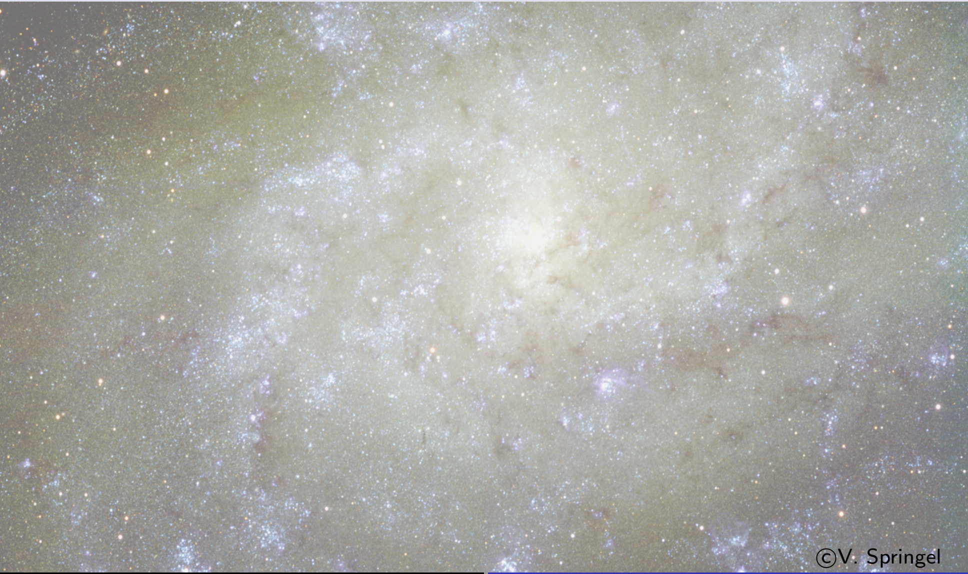
- Typically $\eta \sim 2$, $\chi \sim 0.25$, $v_W \sim 250 km/s$
- Star formation quenching by strong winds
- Popular approach to produce thin disks in simulations
- Recent observations hint to weaker feedback (Genzel et al. in prep.)

$\Rightarrow$ Deposit energy and metals in the halo

# SN driven galactic fountain

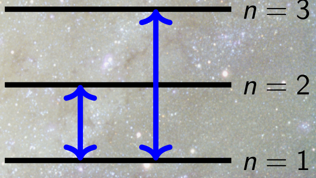


# Outline

# Basic process and assumptions

* Sampled gas contains multiple particle species: H, He (& higher?)
* Spontaneous & driven (de-) excitation

⇒ Photons



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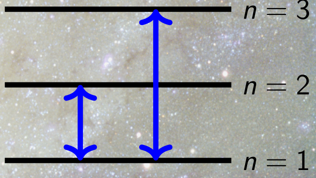
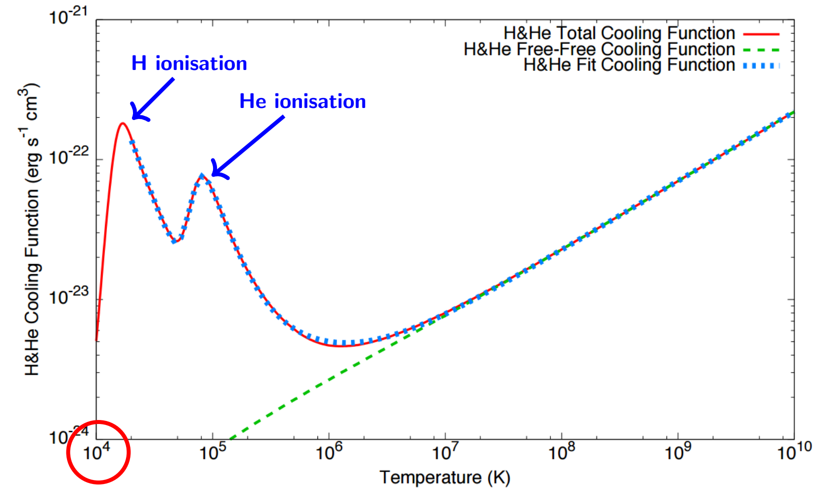


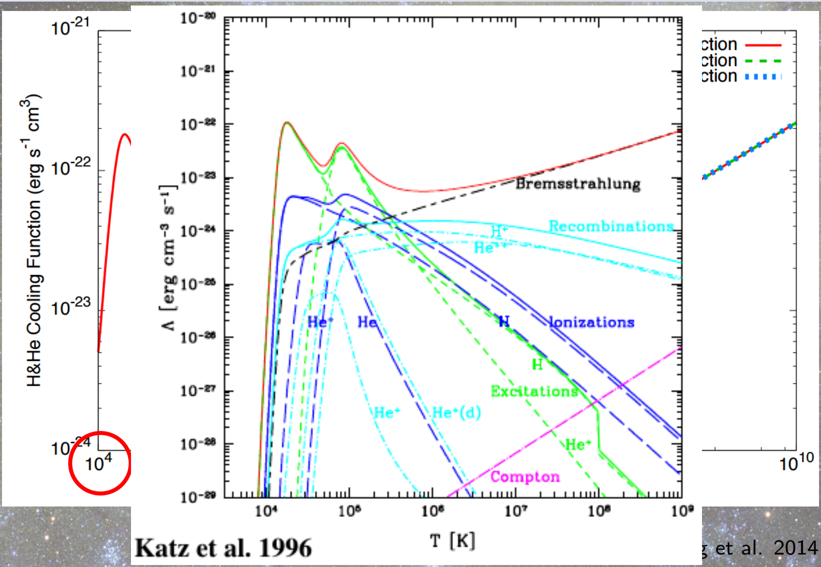

* Following photon field (radiative transfer) very expensive

⇒ Ionization equilibrium

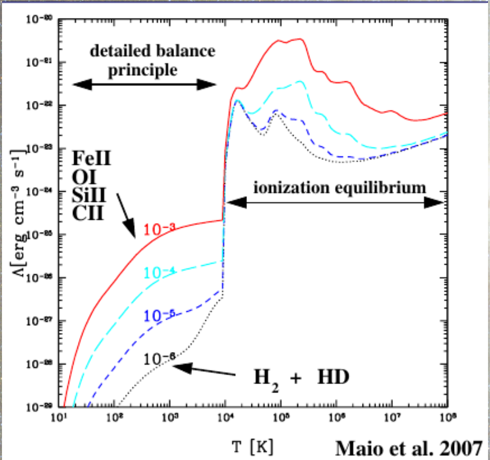
# H & He Cooling function



# H & He Cooling function

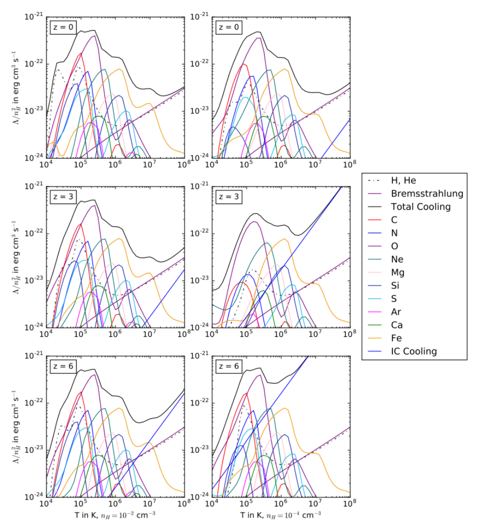


# Adding heavier elements




- ∗ Cool below $10^4 K$
- ∗ Produced in stars
- ∗ Distributed by supernovae
- ∗ Metal diffusion (transport processes, see later)

# Cooling function with redshift

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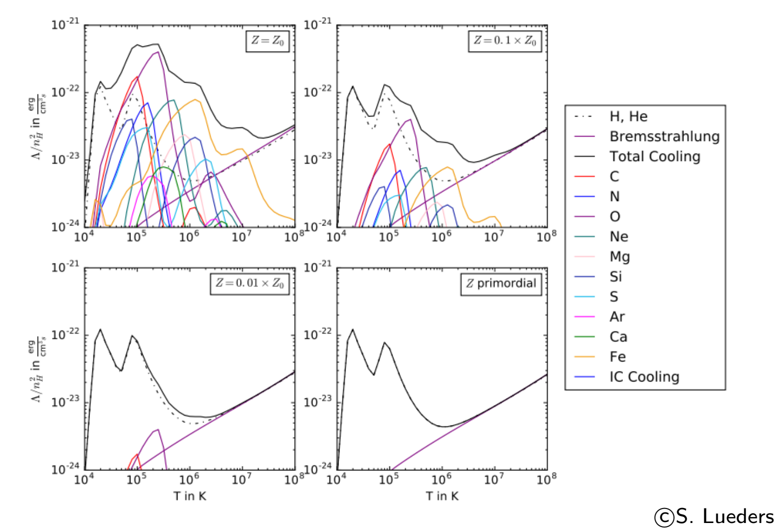
# Cooling function with metallicity

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# History of Gadget

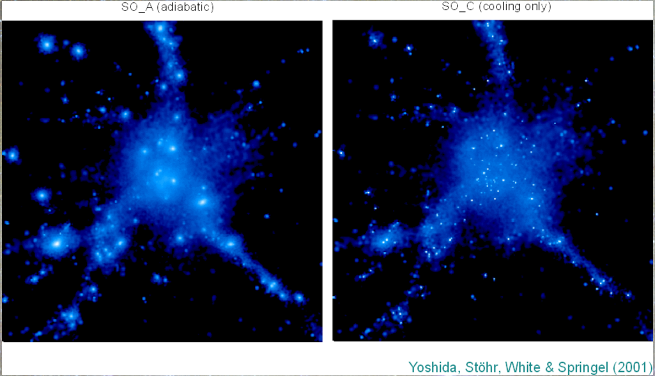
* Katz et al. 1996: Basic modelling
* Springel & Hernquist 2002++: Primordial composition, ionization equilibrium, UV background (e.g. Haardt & Madau)
* Yoshida et al. 2003++: $\mathrm{H}_2$ cooling at low $T$
* Scannapieco et al. 2005++: Multiphase model, metals
* Tornatore et al. 2004:++: Complex stellar evolution and metal model
* Maio et al. 2007: $\mathrm{H}_2$, HD and metals at low $T$
* Schaye et al. 2009++: Metals using detailed Cloudy tables
* Murante et al. 2010: Dynamical sub scale model Muppi

# Excursion: Cloudy → cooling tables

* See Ferland er al. 2013 and https://www.nublado.org/
* Open source!
* Spherical symmetric model
* Central ionisation source, UV background
* Depends on $\rho$, $T$, $z$, $Z_s$

⇒ Fraction per species

# Effects of cooling in cluster simulations



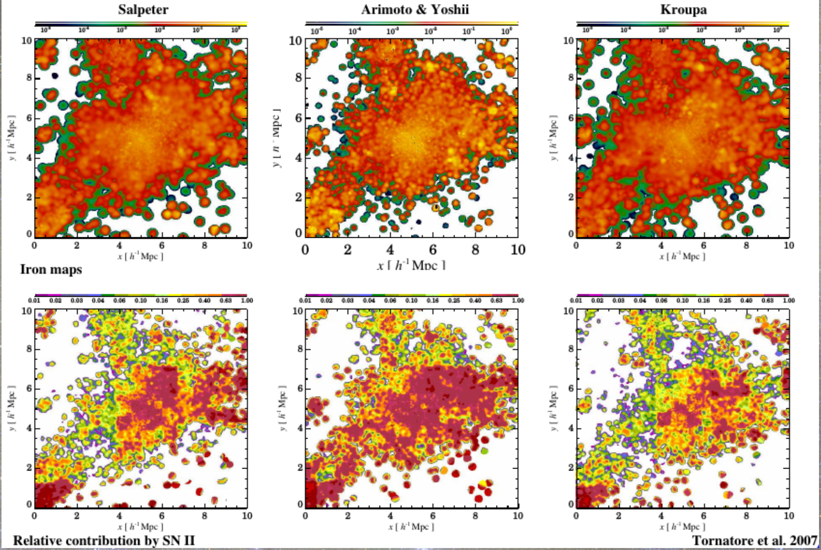

- ∗ Gravitational collapse
- ∗ Cooling
- ∗ Collapse due to energy loss

- ∗ Cooling flow
- ∗ Entropy loss

# Chemical Enrichment (Tornatore et al. 2004/2007)

* Model rate of SNIa
* Adopt stellar lifetime function $\tau(m)$
* Adopt metal yields $p_{Z_i}(m, Z)$
* Fix IMF for number stars / mass bin
* Follow evolution equations for SNIa, SNII, AGB stars along with metal production
* Let feedback enrich surrounding medium with H, He, Fe, O, C, Mg, S

# Chemical Enrichment (Tornatore et al. 2004/2007)



# Code Comparisons: Feedback models

Mon. Not. R. Astron. Soc. **423,** 1726–1749 (2012) doi:10.1111/j.1365-2966.2012.20993.

## The Aquila comparison project: the effects of feedback and numerical methods on simulations of galaxy formation

C. Scannapieco,[1★] M. Wadepuhl,[2] O. H. Parry,[3,4] J. F. Navarro,[5] A. Jenkins,[3] V. Springel,[6,7] R. Teyssier,[8,9] E. Carlson,[10] H. M. P. Couchman,[11] R. A. Crain,[12,13] C. Dalla Vecchia,[14] C. S. Frenk,[3] C. Kobayashi,[15,16] P. Monaco,[17,18] G. Murante,[17,19] T. Okamoto,[20] T. Quinn,[10] J. Schaye,[13] G. S. Stinson,[21] T. Theuns,[3,22] J. Wadsley,[11] S. D. M. White[2] and R. Woods[11]

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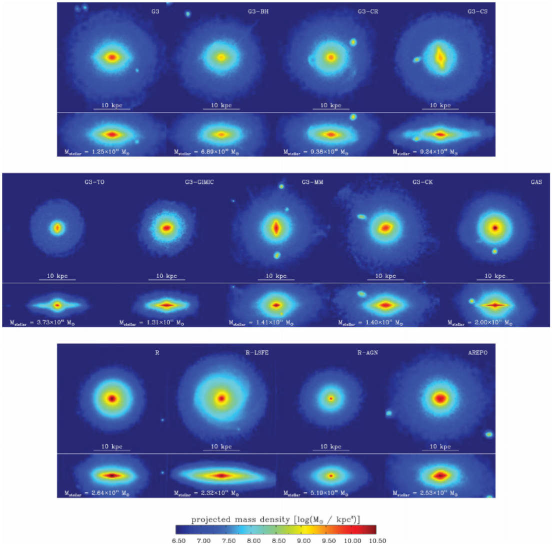

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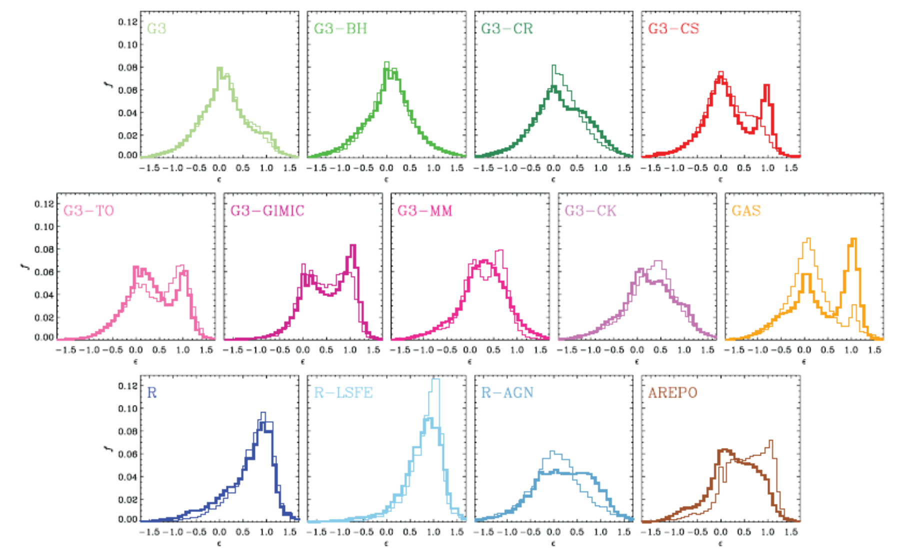
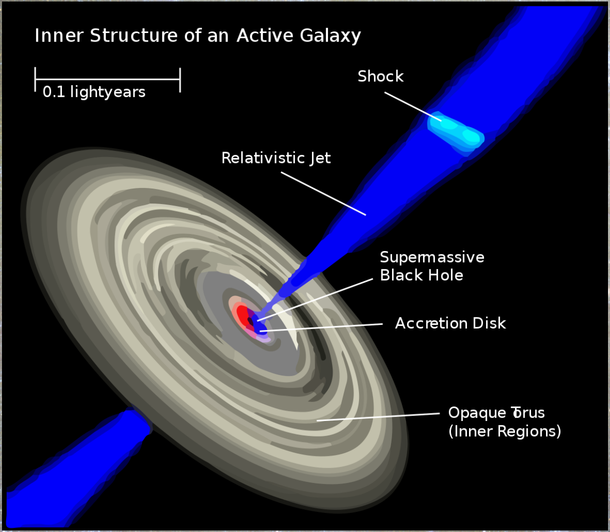
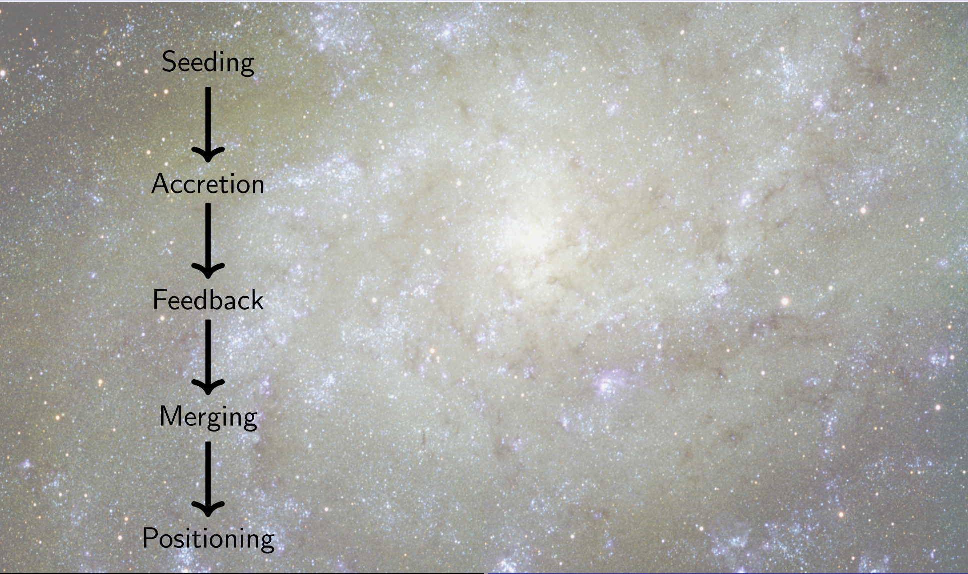


# Outline

# AGN Basics


Inner Structure of an Active Galaxy
0.1 lightyears
Shock
Relativistic Jet
Supermassive
Black Hole
Accretion Disk
Opaque Torus
(Inner Regions)

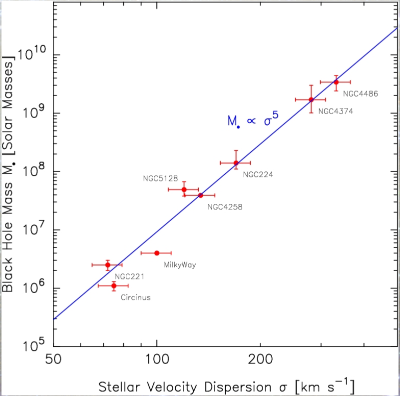
# Aspects of AGN models



# Aspects of AGN models

Seeding
↓
Accretion
↓
Feedback
↓
Merging
↓
Positioning

* Seeding in galaxies with $M_\star > 2.3 \cdot 10^{10} M_\odot$
* Constant seed mass $\sim 10^5 M_\odot$
* $M - \sigma$ relation seeding

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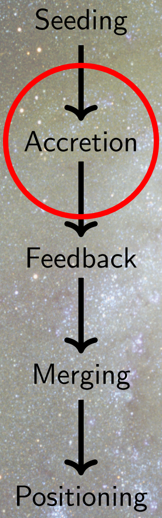
# Aspects of AGN models




* $\alpha$-Bondi (Springel et al. 2005):

Spherical $\dot{M}_B = \alpha \cdot 4\pi R_B^2 \rho c_s \approx \alpha \cdot \frac{4\pi G^2 M_\bullet^2 \rho}{(c_s + v^2)^{3/2}}$

$\dot{M}_\bullet = \min\left(\dot{M}_B, \dot{M}_{Eddington}\right)$

Eddington limit: $p_{rad}$ stops infall

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* $\beta$-Bondi (Booth & Schaye 2009)

$$\alpha = \begin{cases} 1 & \text{for } n_h < n_h^C \\ \left(\frac{n_h}{n_h^C}\right)^\beta & \text{else} \end{cases}$$

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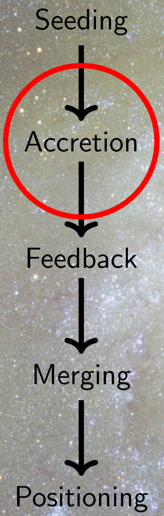


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- Cold / Hot (Steinborn et al. 2015)

  $\dot{M}_\bullet = \min\left(\dot{M}_{B,\mathrm{hot}} + \dot{M}_{B,\mathrm{cold}}, \dot{M}_{Eddington}\right)$

  $\alpha_{\mathrm{hot}} = 10,\ \alpha_{\mathrm{cold}} = 100$

# Aspects of AGN models


* Thermal (Springel et al. 2005):

$$\dot{E}_{\mathrm{feed}} = \epsilon_f \cdot L_r = \epsilon_f \cdot \epsilon_r \dot{M}_\bullet c^2$$

with $\epsilon_r \sim 0.1$, $\epsilon_f \sim 0.05$ to fix $M - \sigma$

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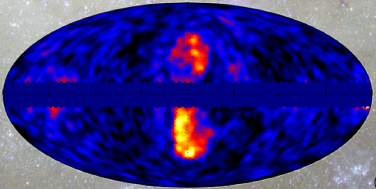
* Bubbles (Sijacki et al. 2007):

  Radio mode (thermal) $\underset{z \to 0}{\longrightarrow}$ quasar mode (kinetic bubble injection)

  Fermi bubble?



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  Radio mode (thermal) $\underset{z \to 0}{\longrightarrow}$ quasar mode

  (kinetic bubble injection)

* Mass dependent (Steinborn et al. 2015):

  Mechanical and radiative as thermal due to resolution

  $\epsilon_0 = \eta \frac{P_0 / L_{\mathrm{Edd}}}{\dot{M}_\bullet / \dot{M}_{\mathrm{Edd}}}, \quad \epsilon_r = \eta \frac{L / L_{\mathrm{Edd}}}{\dot{M}_\bullet / \dot{M}_{\mathrm{Edd}}}$

# Aspects of AGN models

Seeding
↓
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Positioning

* Based on distance:

  $|\vec{r}_1 - \vec{r}_2| < h$

* Based on velocity:

  $|\vec{v}_1 - \vec{v}_2| < c_s$

* Instant merging:

  2 particles become 1

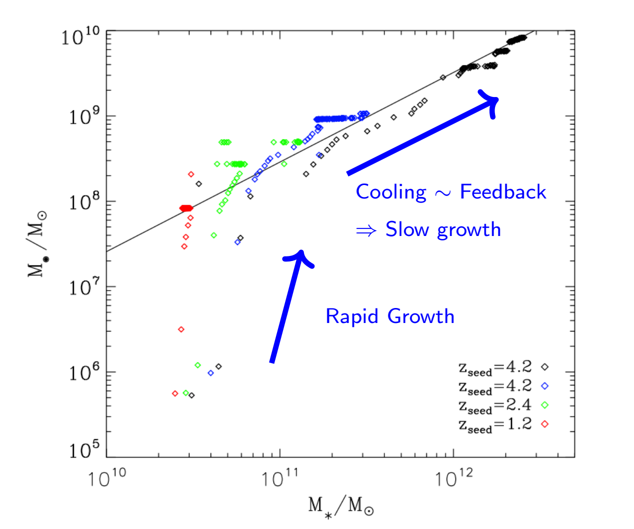
* ...

# Aspects of AGN models

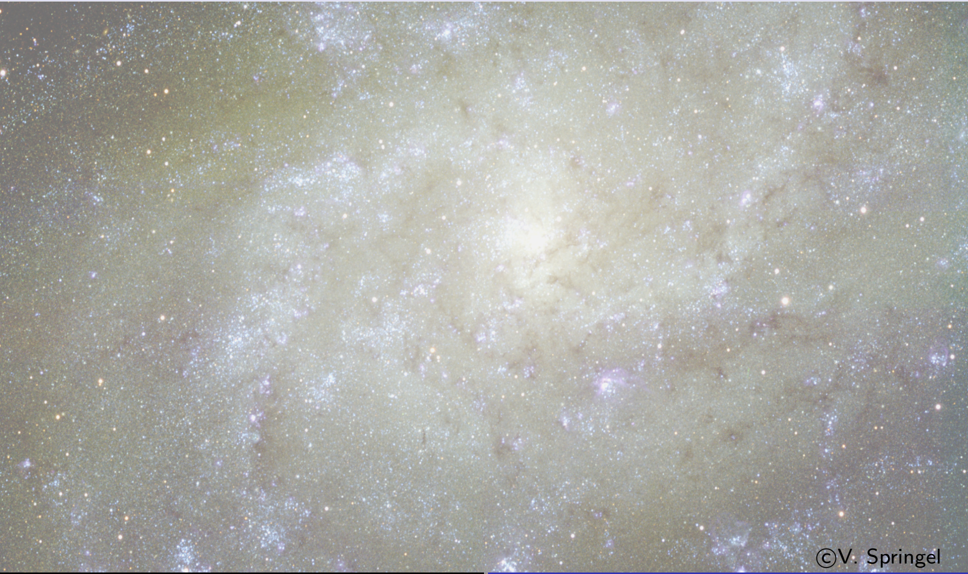
Seeding
↓
Accretion
↓
Feedback
↓
Merging
↓
Positioning

* Free floating
* Pinning to potential minimum

# Black Hole Growth



# AGN Jets



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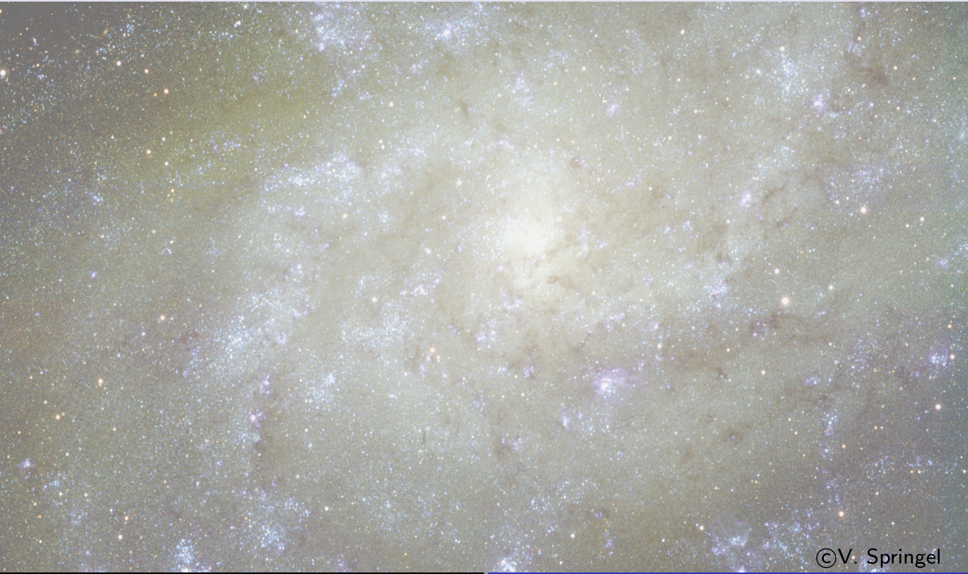
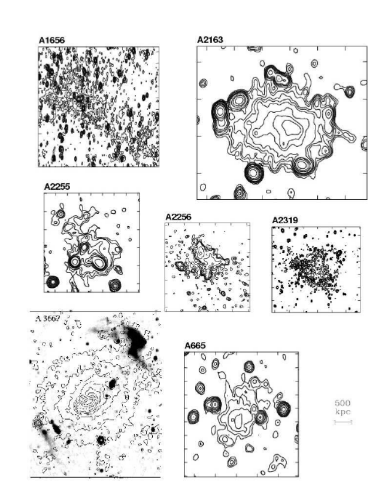


# Outline

# Observational evidence: Radio clusters




©K. Dolag

Diffuse Synchrotron emission

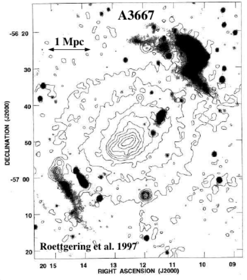
Radio halo

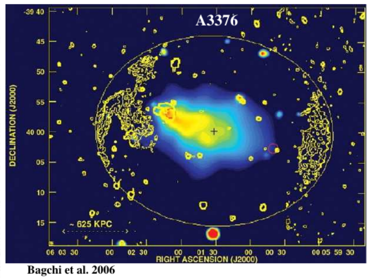
⇒ Relativistic electrons

⇒ Cluster magnetic fields

Cosmic rays (transport see later)

# Observational evidence: Radio clusters




Peripheral Synchrotron emission

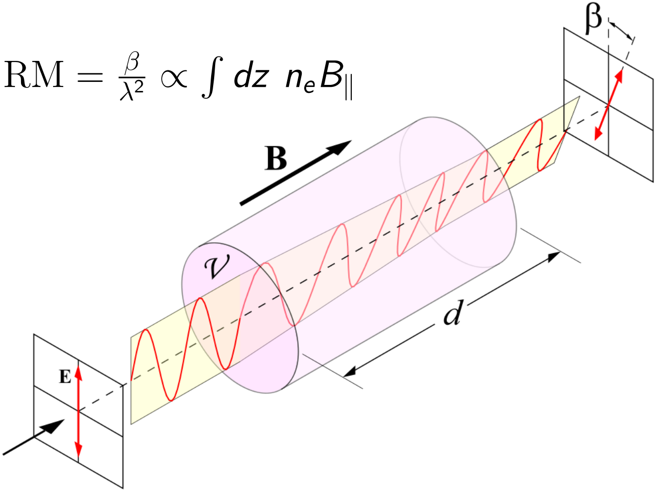
Radio relic

⇒ Related to merger or accretion shock

⇒ Shock acceleration

# Observational evidence: Rotation Measure

$$\mathrm{RM} = \frac{\beta}{\lambda^2} \propto \int dz \; n_e B_{\|}$$

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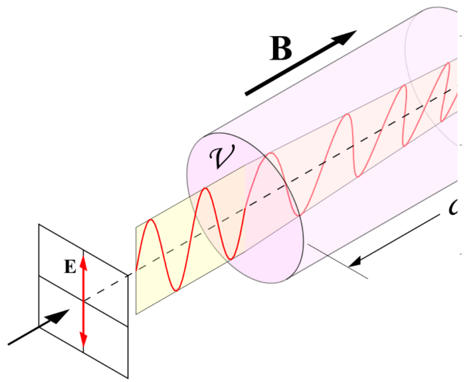


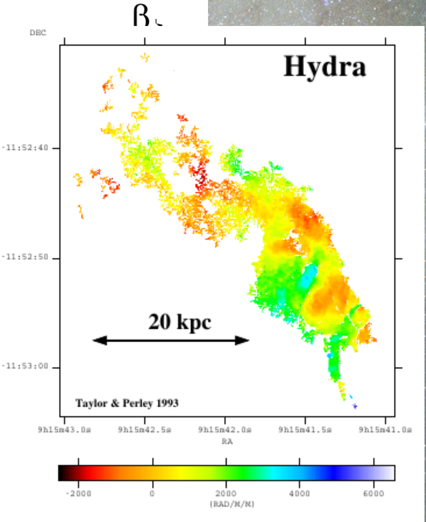

# Ideal MHD equations

$*$ **Hydro equations** (remember yesterday)

$+$ Magnetic pressure terms $p_B = B^2/8\pi$

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$+$ Infinite conductivity

$\Rightarrow$ Ideal induction equation $\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left(\vec{v} \times \vec{B}\right)$

& No magnetic monopoles $\vec{\nabla} \cdot \vec{B} = 0$ (more in a bit)

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$\Rightarrow$ Field lines flux frozen in fluid

# Origin of magnetic fields

∗ **Primordial seed field**

∗ Biermann battery

∗ Dynamo (turbulence)

∗ Stars

∗ **Supernovae**

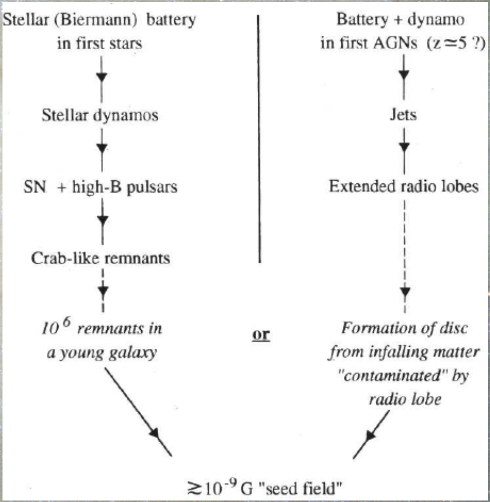
∗ **Galactic Winds**

∗ AGN, Jets

∗ Shocks

+ Structure formation

- Dissipation




©Rees 1994

# Origin of magnetic fields

* **Primordial seed field**
* Biermann battery
* Dynamo (turbulence)

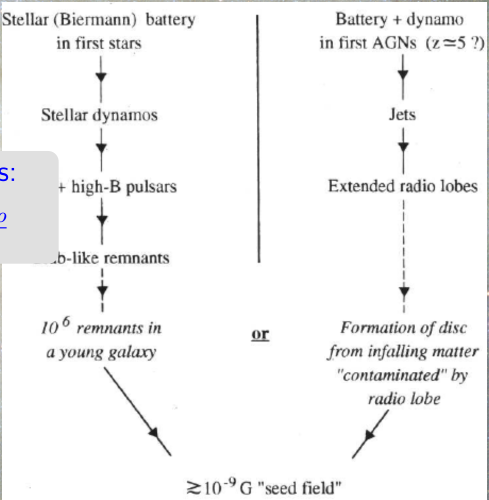
Relative motion of electrons and ions:

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left( \vec{v} \times \vec{B} \right) + \mathrm{const} \cdot \frac{\vec{\nabla} p \times \vec{\nabla} \rho}{\rho^2}$$

* **Galactic Winds**
* AGN, Jets
* Shocks

\+ Structure formation

\- Dissipation

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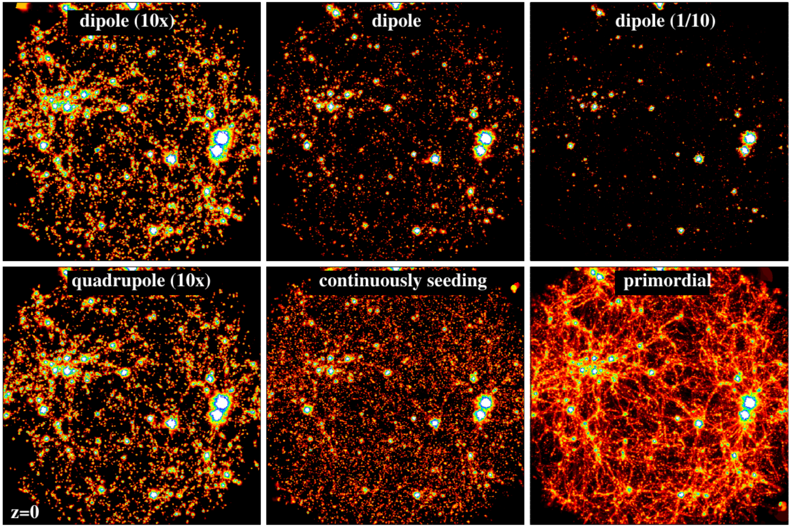
# Galactic Wind Seeding

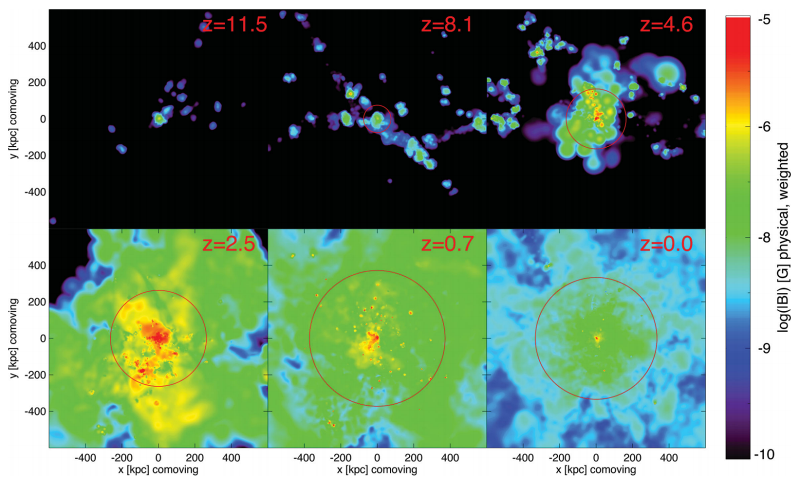



©Donnert et al. 2009

# Supernova Seeding

* Attach to supernova feedback of star formation model
* Bubbles around supernova events
* Inject dipoles to satisfy vanishing divergence

# Supernova Seeding




©Beck et al. 2013

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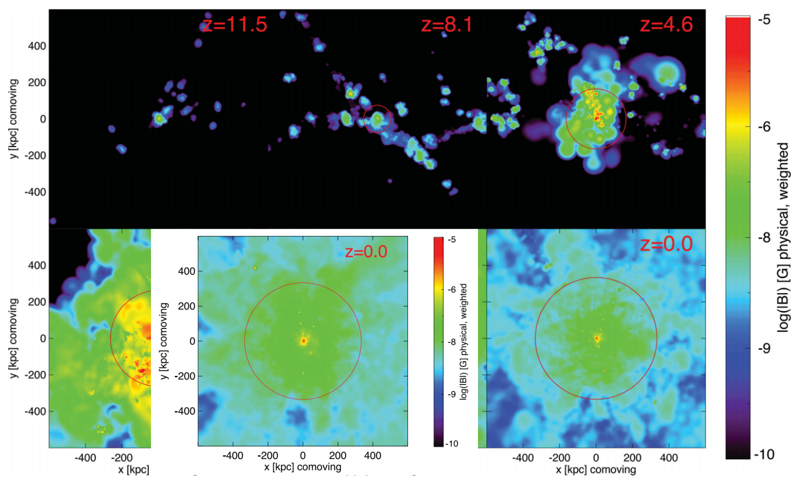


Primordial seed

©Beck et al. 2013

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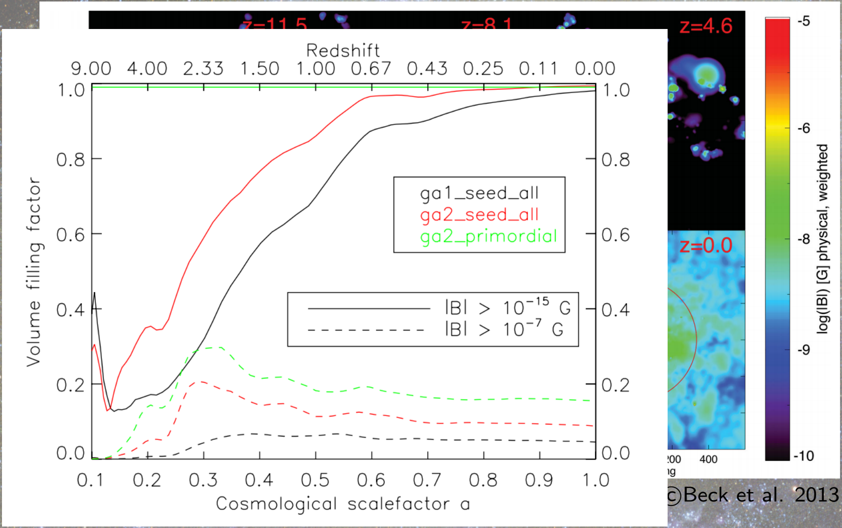


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# The $\nabla \cdot \vec{B}$ problem

* **Physics:** $\vec{\nabla} \cdot \vec{B} = 0$ always given
* **Numerics:** Discretisation & finite computation accuracy

$\Rightarrow$ $\nabla \cdot \vec{B} > 0$; Small error but accumulates!

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$\Rightarrow$ Cleaning scheme of some sorts

- Powell 1999: Source in momentum, induction and energy eqs.
- Dedner et al. 2002: Evolve scalar potential which contains $\vec{\nabla} \cdot \vec{B}$ and subtract it's gradient; pump difference in internal energy
- Mocz et al. 2014: Constrained transport technique

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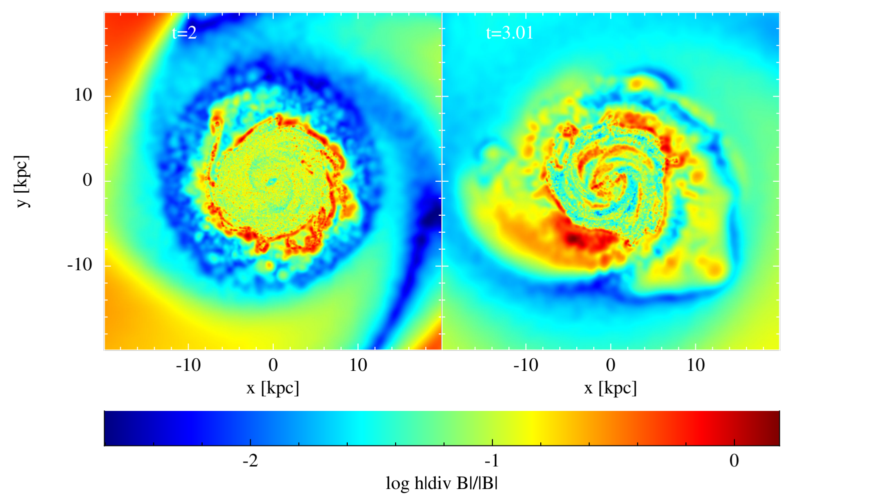


©Steinwandel, Arth et al. 2018

# Non-ideal MHD

Three additional source terms $\frac{\partial \vec{B}}{\partial t}$:

* **Ohmic resistivity**: Drift electrons - ions; Collisionally coupled to neutral gas

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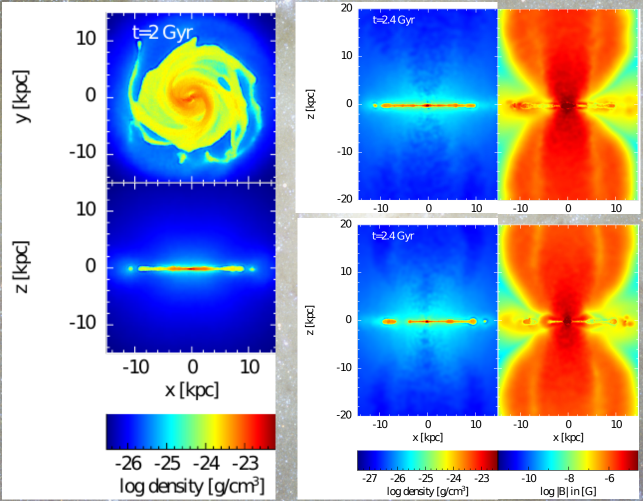
Possibly important if $P_m = \frac{\nu_{visc}}{\nu_M} = \frac{Re_m}{Re_h} \approx 10^{-5} \frac{T[K]^4}{n_H[cm^{-3}]} < 1$

$\Rightarrow$ cold, dense systems

Consistent with low ionisation fractions from Saha-Boltzmann

# Galaxy simulations with SPMHD (Gadget3)



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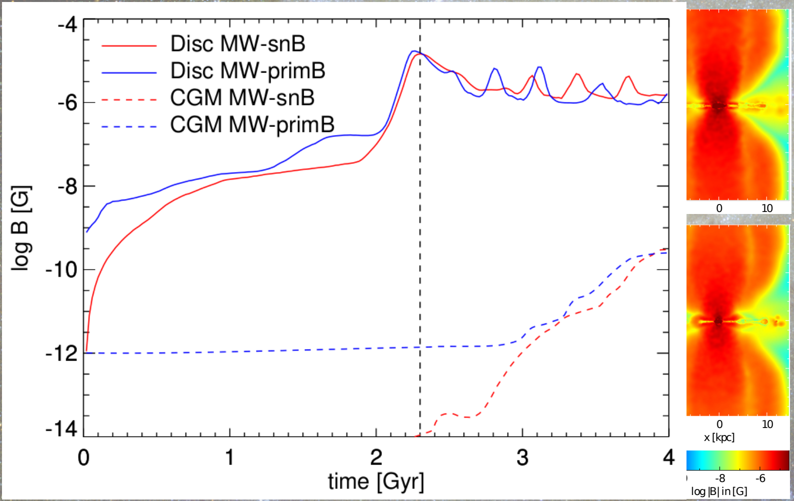

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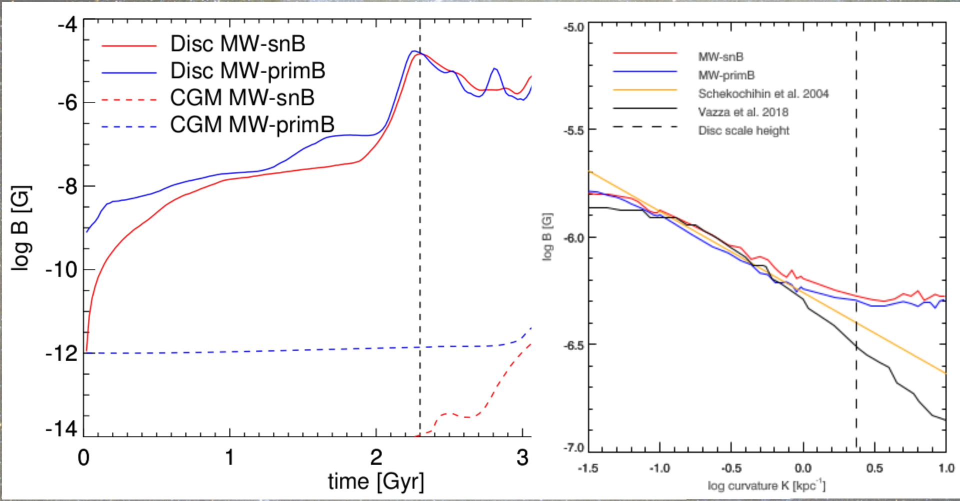

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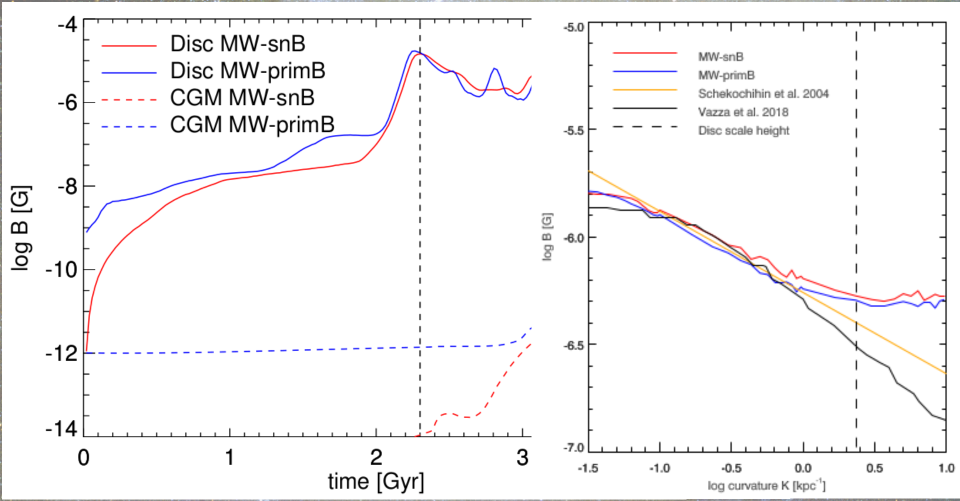


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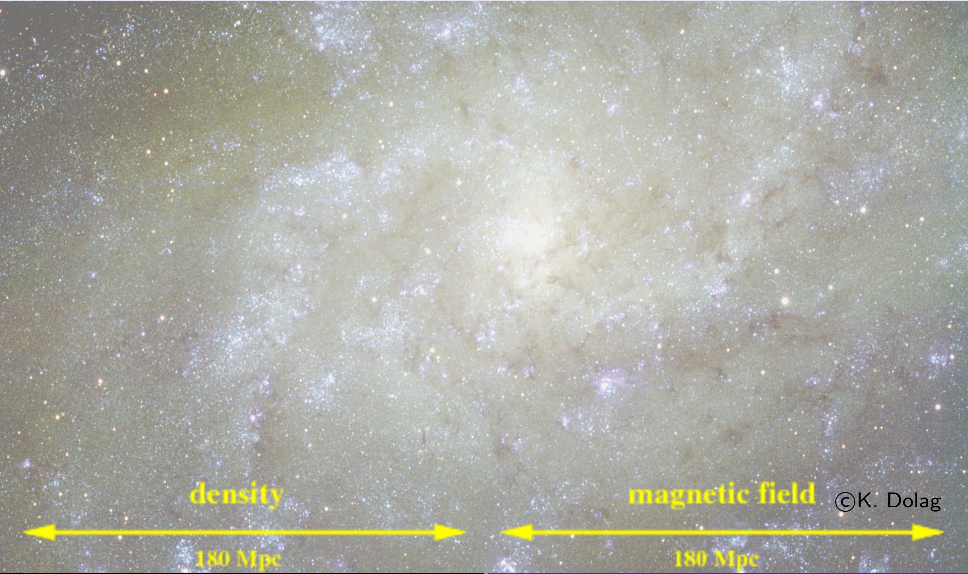
# Cosmological simulations with SPMHD (Gadget3)



# Outline

# Basics of thermal conduction

* Macroscopic: Transport of heat energy along a temperature gradient without movement of gas
* Microscopic: Exchange of energy due to collisions of particles
* Isotropic transport if particle movement is unrestricted

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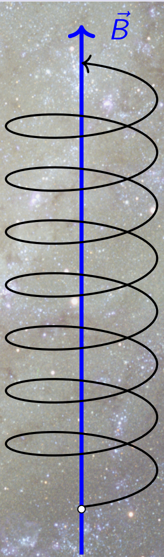
- Macroscopic: Transport of heat energy along a temperature gradient without movement of gas
- Microscopic: Exchange of energy due to collisions of particles
- Isotropic transport if particle movement is unrestricted
- Consider charged particles in magnetic field: Movement along field lines introduces an anisotropy
- ⇒Suppressed conduction perpendicular to $\vec{B}$



# The conduction equation

**General diffusion equation**

$$\frac{\partial \Phi}{\partial t} = \vec{\nabla} \cdot \left( -\kappa \vec{\nabla} A \right)$$

* $\kappa = const$

$$\frac{\partial \Phi}{\partial t} = -\kappa \Delta A$$

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**Jubelgas et al. 2004**

$$\frac{\partial u}{\partial t} \propto -\vec{\nabla} \cdot \left[ T^{5/2} \vec{\nabla} T \right]$$

**Arth et al. 2014**

$$\frac{\partial u}{\partial t} \propto -\vec{\nabla} \cdot \left[ T^{5/2} \vec{B} \left( \vec{B} \cdot \vec{\nabla} T \right) \right]$$

# Adding a perpendicular component

## Final conduction equation

$$\frac{\partial u}{\partial t} \propto -\vec{\nabla} \cdot \underbrace{\left[ \overbrace{\kappa_{\parallel} \vec{\nabla}_{\parallel} T}^{\text{normal}} + \overbrace{\kappa_{\perp} \vec{\nabla}_{\perp} T}^{\text{suppressed}} \right.}_{\text{collisional}} + \underbrace{\left. \overbrace{\kappa_{\Lambda} \vec{B}_{\text{norm}} \times \vec{\nabla} T}^{\text{vanish for large } \vec{B}} \right]}_{\text{non collisional}}$$

with Spitzer like coefficients $\kappa \propto T^{5/2}$

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**Final conduction equation**

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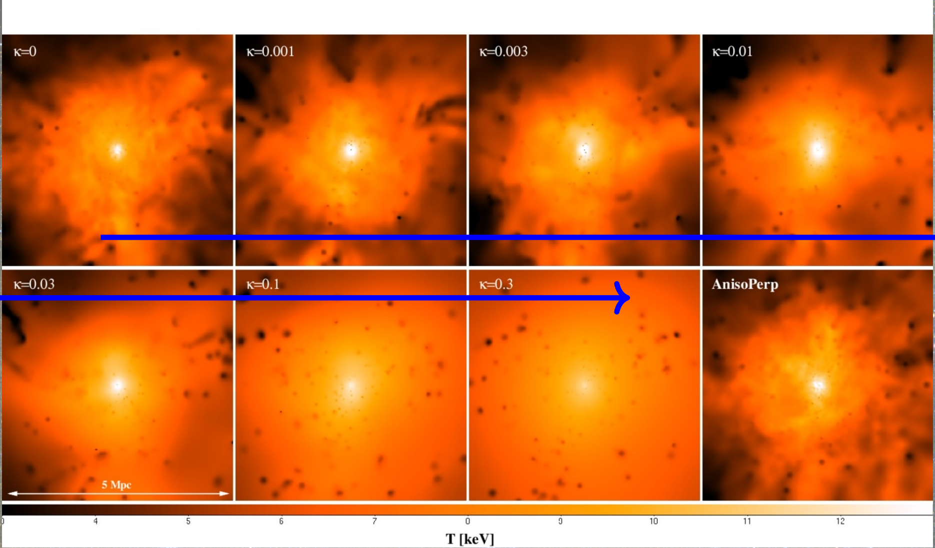
with Spitzer like coefficients $\kappa \propto T^{5/2}$

**How are these coefficients related?**

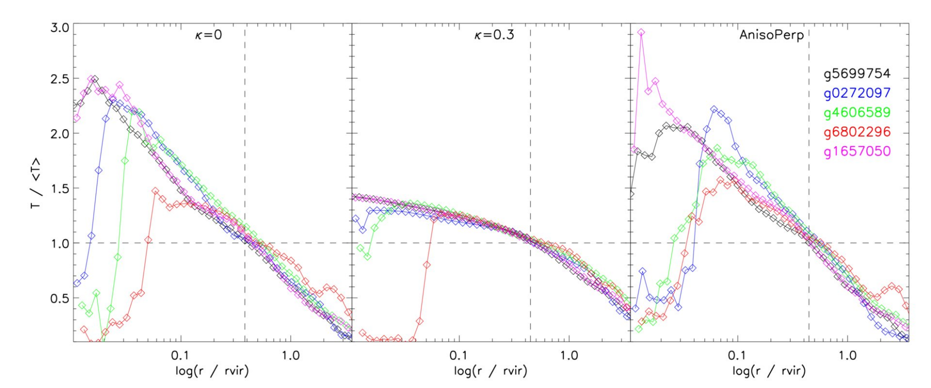
$$\kappa_{\|} / \kappa_{\perp} \approx \left[ (\omega_g \tau)^{\alpha} + 1 \right]^{-1} \propto B^{-\alpha}$$

with $\alpha = 1$ or $2$ and $\omega_g = \frac{eB}{mc}$

# Temperature maps for different settings



# Radial temperature profiles



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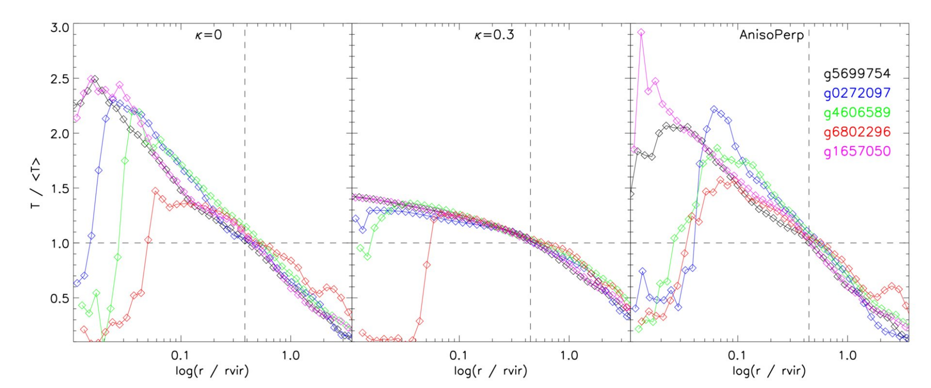
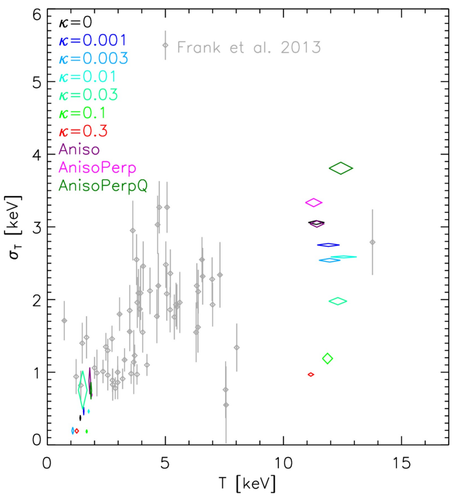



## Cool Core VS Non-Cool Core

Treatment of perpendicular conduction promotes bimodality

# Temperature fluctuations



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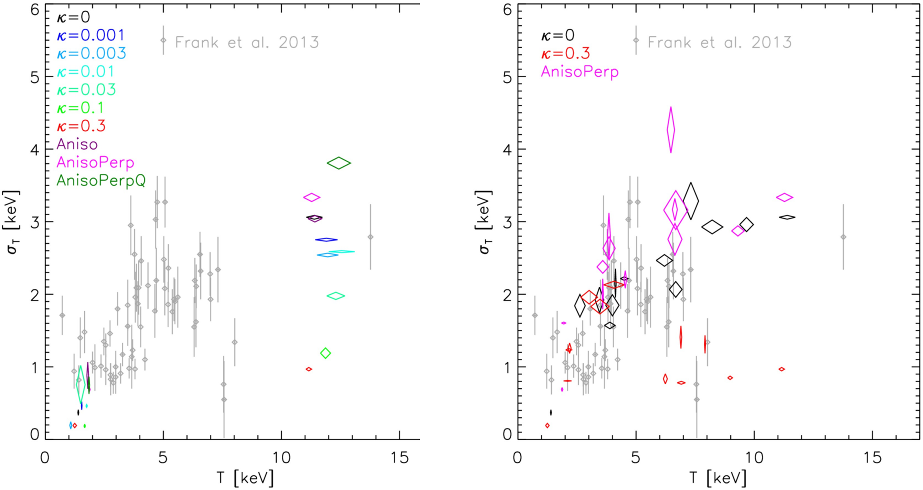
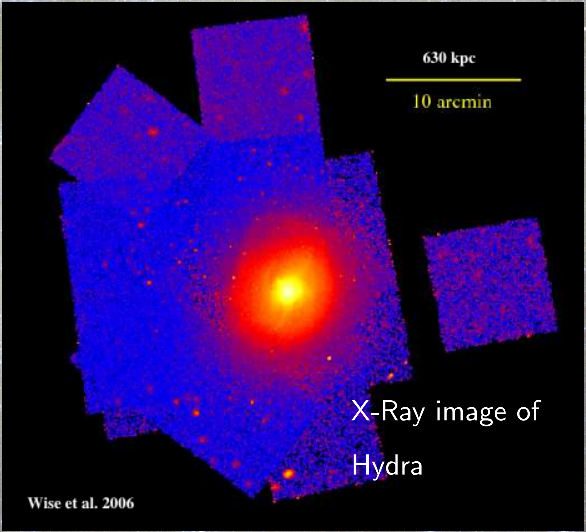
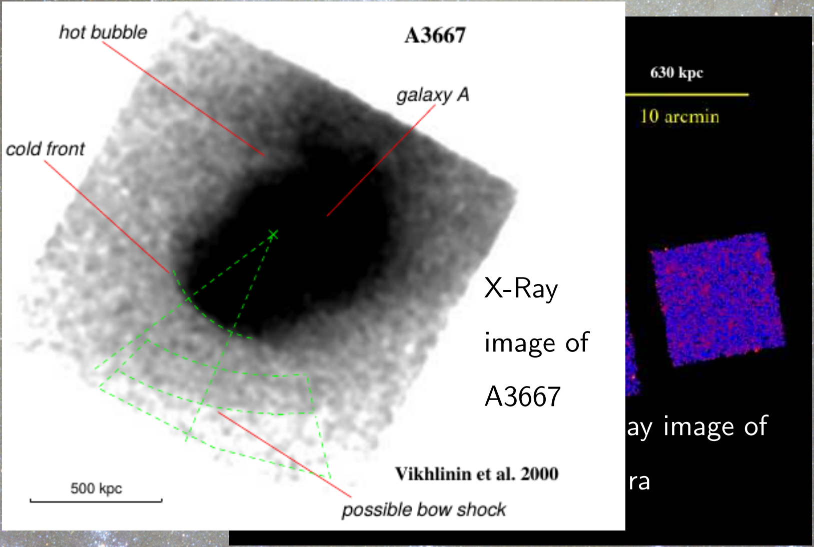


# Observational evidence

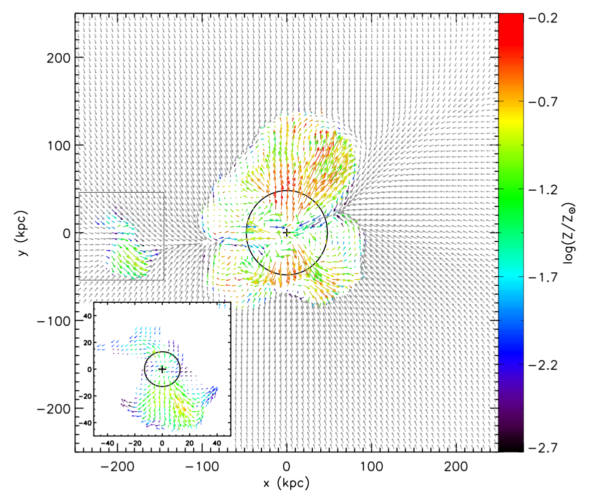



X-Ray image of Hydra

# Observational evidence



# Other examples for diffusion equations

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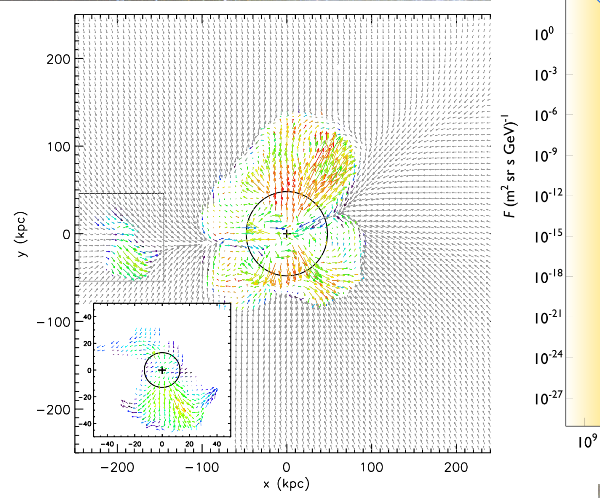
# Other examples for diffusion equations

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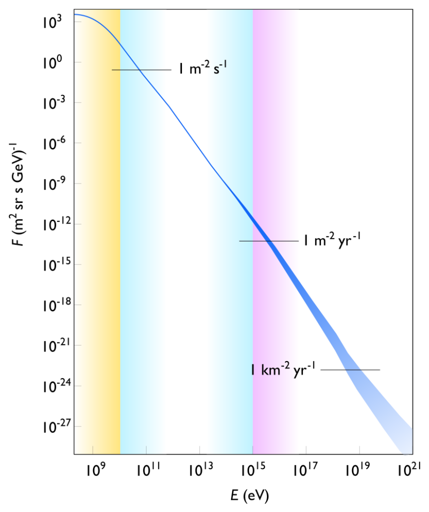


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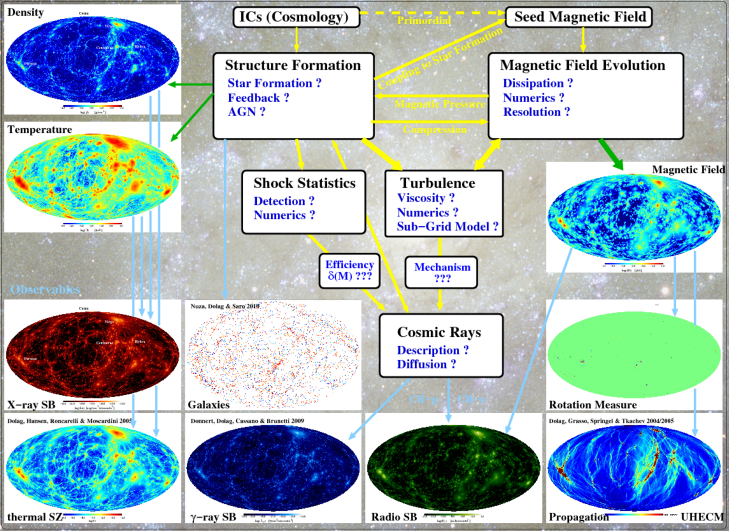
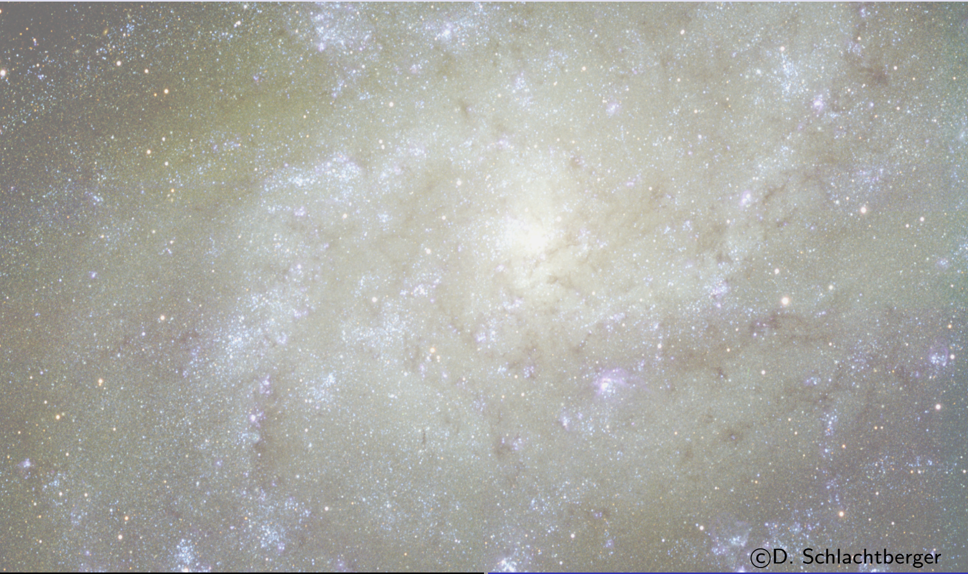
# Outline

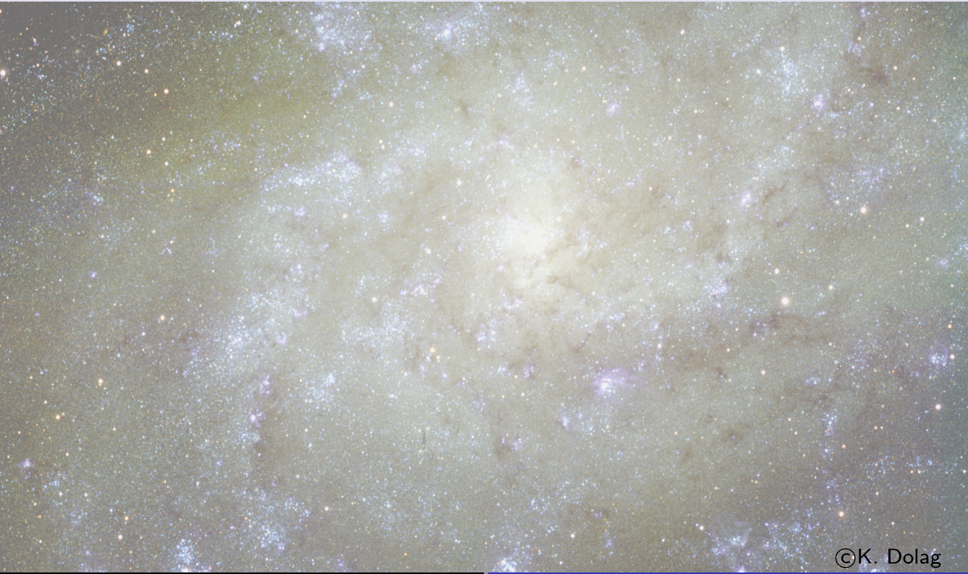
# Overview


Density
Temperature
ICs (Cosmology)
Primordial
Seed Magnetic Field
Coupling to Star Formation
Structure Formation
Star Formation ?
Feedback ?
AGN ?
Magnetic Field Evolution
Dissipation ?
Numerics ?
Resolution ?
Magnetic Pressure
Compression
Shock Statistics
Detection ?
Numerics ?
Turbulence
Viscosity ?
Numerics ?
Sub−Grid Model ?
Magnetic Field
Efficiency
δ(M) ???
Mechanism
???
Observables
Cosmic Rays
Description ?
Diffusion ?
X−ray SB
Nuza, Dolag & Saro 2010
Galaxies
Rotation Measure
Dolag, Hansen, Roncarelli & Moscardini 2005
thermal SZ
Donnert, Dolag, Cassano & Brunetti 2009
γ−ray SB
Radio SB
Dolag, Grasso, Springel & Tkachev 2004/2005
Propagation
UHECM

# Galaxy formation over time



# Fly through simulation



# Sources

* Lecture of Volker Springel
* Lectures of Klaus Dolag
* The Encyclopedia of Cosmology
* My PhD thesis ☺
* Several papers as mentioned . . .

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**Now, break and tutorials!**