Radiative processes, stellar atmospheres and winds (WS 2021/2022)

Problem set 8

Problem 1 [2 points] *Adiabatic temperature gradient*

Derive, from $\nabla_{\rm ad} = 1 - 1/\gamma$, the corresponding spatial temperature gradient $(dT/dr)_{\rm ad}$, for an atmosphere in hydrostatic equilibrium and neglecting radiative acceleration. Your result should depend, among other factors, on g, μ, and γ.

Problem 2 [2 points] *Ratio of* $\nabla_{\rm rad}$ *and* $\nabla_{\rm ad}$

Derive (within the standard convection scenario) the following result for the ratio

$$\frac{\nabla_{\rm rad}}{\nabla_{\rm ad}} = \frac{3}{16}\Big(\frac{T_{\rm eff}}{T}\Big)^4 \frac{\bar{\chi}_{\rm Ross} H}{1-\frac{1}{\gamma}},$$

by explicitly using the corresponding spatial temperature gradients (cf. problem 1).

Problem 3 [2 points] *Pressure scale height and opacity*

Show that the product $\bar{\chi}_{\rm Ross} H$ (script page 110) is on the order of $\tau_{\rm Ross}$, i.e., of order unity or below in a stellar photosphere. Assume that the Rosseland opacity depends linearly on density, i.e., $\bar{\chi}_{\rm Ross}(r) = {\rm const} \cdot \rho(r)$. Hint: consider the dependence of density vs. column density in a hydrostatic atmosphere.

Problem 4 [6 points] *Convective flux at the base of the solar convection zone*

On page 113 of the script we have seen that the convective flux depends on the difference $(\nabla_a - \nabla_i)$, where 'a' denotes the ambient medium and 'i' the bubble(s). We ask now how large the difference in ∇ needs to be that the *total* energy flux is transported by convection under typical conditions. Normalized in terms of $\nabla_{\rm ad}$, this would measure the required degree of 'superadiabaticity' of the ambient medium, if we approximate $\nabla_i \approx \nabla_{\rm ad}$.

a) Derive $(\nabla_a - \nabla_i)$ from the condition that the convective flux is the total energy flux, and that the total 'luminosity' at r is given by L_r.

b) Calculate $(\nabla_a - \nabla_i)$ at the base of the solar convection zone, from the condition outlined in problem 4a), and from the following parameters:

   $L_r = L_\odot$, $r = 0.82 R_\odot$, $M_r = 0.9965 M_\odot$. The density is 0.036 g/cm^3, and the temperature $T = 1.13 \cdot 10^6$ K (all this from stellar structure calculations).
   Mean molecular weight, μ (measured in units of $m_{\rm H}$), from a solar H/He mixture (cf. problem set 7).

To calculate the thermodynamical quantities, assume a mono-atomic ideal gas, and no ionization effects.

Note that C_p needs to be defined per unit mass, and thus C_p(per unit mass) = C_p(per mol)$/\mu$. Show that under these conditions C_p(per unit mass) $= 3.41 \cdot 10^8$ in cgs-units. What are these units?

Finally, assume a mixing length parameter of unity.

Hint: Try to calculate all quantities (for this and the following problems) by using an adequate program code/script, and print out important intermediate results. This diminishes potential error sources quite a lot.

c) Determine the required degree of superadiabaticity, $(\nabla_a - \nabla_i)/\nabla_{\rm ad}$. What do you conclude?

d) Calculate the average convection velocity, $\bar{v}$, and convince yourself that the resulting value makes sense. Otherwise, you made some error, presumably in the conversion of units.

e) Finally, convince yourself that at the considered location (base of convection zone) $\nabla_{\rm ad} \approx \nabla_{\rm rad}$, when $\log_{10}(\bar{\chi}_{\rm Ross}/\rho) = 1.28$ (in cgs) and $\bar{\chi}_{\rm Ross}$ increases strongly with height, thus enabling convection.

Have fun, and much success!