Radiative processes, stellar atmospheres and winds (WS 2021/2022)

Problem set 3

Problem 1 [5 points] *Mean intensity and Eddington/radiative flux*

a) Calculate the mean intensity and the Eddington flux for an isotropic radiation field with specific intensity $I_\nu(\mu, \phi) = I_0$

b) Calculate the mean intensity and the radiative flux (density) for an extremely anisotropic radiation field, with specific intensity $I_\nu(\mu, \phi) = I_0 \delta(\mu - \mu_0)\delta(\phi - \phi_0)$ and Dirac δ-function $(-1 < \mu_0 < 1)$.

   Note: This situation corresponds to a monochromatic planar wave propagating into direction $\vec{n_0}$ (specified by angles θ_0, ϕ_0), where in vacuum $I_0 = cE_0^2/(8\pi)$, with E_0 the amplitude (absolute value) of the corresponding electric field,

$$\vec{E}(\vec{r}, t) = \vec{E_0} \cos[2\pi(k\vec{n_0} \cdot \vec{r} - \nu t)].$$

c) Make a sketch for the flux through an area element dS when (i) $\mu_0 \to 1$ and (ii) $\mu_0 = 0$, and ϕ_0 has an arbitrary value $0 \leq \phi_0 < 2\pi$. The surface-normal $\vec{dS}$ shall be parallel to the z-axis of the coordinate-system.

d) Repeat problem 1b), now with $I_\nu(\mu, \phi) = I_0 \delta(\mu - \mu_0)$, where I_ν shall be independent of azimuth (i.e., plane-parallel or spherical symmetry). Calculate also the Eddington flux. Compare mean intensity and Eddington flux for the two cases $\mu_0 \to 1$ and $\mu_0 = 0$, and sketch both situations again, for the flux through an area element dS as in problem 1c.

Problem 2 [3.5 points] *Solar constant*

The total solar radiative flux (at solar minimum), as measured above the absorbing and reflecting terrestrial atmosphere, is given by

$$\int_0^\infty f_\nu \mathrm{d}\nu = 1.361 \cdot 10^6 \,\mathrm{erg\,cm^{-2}\,s^{-1}},$$

and called the solar constant or solar irridiance. [Note: at solar maximum, it iss roughly $1.362 \cdot 10^6 \,\mathrm{erg\,cm^{-2}\,s^{-1}}$].

a) Express the solar constant in [W m^{-2}].

b) Calculate the effective temperature of the sun from this (and other) quantity/quantities, and evaluate the corresponding wavelengths where B_ν and B_λ obtain their maxima.

c) At which distance from a 100-W light bulb (assuming the light bulb to be 100% efficient) is the radiative flux equal to the solar irridiance?

d) Consider a model of the star ζ Puppis (the brightest O-star on the southern sky), with an effective temperature of roughly 40,000 K, a stellar radius of 19 $R_{\odot}$, and a distance of 430 pc. Determine the total radiative flux above the terrestrial atmosphere, in units of the solar constant (neglecting interstellar extinction).

Problem 3 [3.5 points] *Light in your eye*

a) Calculate the energy of the blackbody photons in your eye. Approximate your eye by a hollow sphere of radius 1.5 cm, at a temperature of 37° C.

b) Compare this with the energy inside your eye while looking at a 100-W light bulb that is 1 m away from you (again assuming the light bulb to be 100% efficient). Adopt the area of your eye's pupil as 0.1 cm^2.
Hint: To convert from energy per time to energy, make use of the speed of light.

c) Why is it dark when you shut your eyes?

Have fun, and much success!