# Radiative processes, stellar atmospheres and winds (WS 2021/2022)

# Problem set 2

**Problem 1** [5 points] *Color indices*

[To understand the physical background of this problem, you might read, already now, p. 63/64 of the script.]

Apparent magnitudes from stars might be approximated by the relation

$$m_\lambda \approx -2.5\log_{10}(\pi B_\lambda(T)\Delta_\lambda(R_*/d)^2) + C_\lambda,$$

if one (i) neglects interstellar absorption ("reddening"), (ii) assumes that the star is a black-body radiator with temperature $T$, and (iii) approximates the sensitivity function of the photometer by a rectangular shape with amplitude '1' around 'effective wavelength' $\lambda$ within the 'effective bandwidth' $\Delta_\lambda$, and '0' elsewhere. $m_\lambda$ is the magnitude at band $\lambda$, $B_\lambda$ is the Planck-function at effective wavelength $\lambda$, $R_*$ is the stellar radius, and $d$ is the distance to the star. $C_\lambda$ is a constant for the considered band, set from certain conditions (roughly: all magnitudes of the A0V star Vega shall be zero).

In the above approximation for $m_\lambda$, the following issues were considered:

- Since the star has been approximated by a black-body radiator, its (outward directed) flux is given by $F_\lambda = \pi B_\lambda(T)$ (p. 64, right).
- The integral over the response function $S(\lambda)$, $\int F(\lambda)S(\lambda)\mathrm{d}\lambda$, collapses to $F_\lambda\Delta_\lambda$ in the above approximation.
- the factor $(R_*/d)^2$ arises because of the quadratic dilution of the radiation field. Since, in the absence of interstellar absorption, the total energy/time remains conserved, i.e., $L = F_* \cdot 4\pi R_*^2 = F_{\mathrm{earth}} \cdot 4\pi d^2$, we find $F_{\mathrm{earth}} = F_*(R_*/d)^2$.

Now to the actual problem. For the blue supergiant $\zeta$ Pup (Naos, O4If, surface temperature $\approx$ 39,000 K), the brightest O-star on the southern sky, $U$ = 0.88, $B$ = 1.97, and $V$ = 2.25.

Table 1: Effective wavelengths and bandwidths (in Å) for the UBV system

| Magnitude | effective wavelength | bandwidth |
|---|---|---|
| U | 3640 | 680 |
| B | 4420 | 980 |
| V | 5400 | 890 |

a) Approximate, from these numbers, the constants $C_{U-B} = C_U - C_B$ and $C_{B-V} = C_B - C_V$ for the color indices $U - B$ and $B - V$ (for effective wavelengths and bandwidths, see Table 1).

b) Using the constants derived in a), approximate the color indices $U - B$ and $B - V$ for stars with surface temperatures between 10,000 K and 60,000 K, with stepsize 10,000 K, and one additional point at 5,000 K.

c) Plot a color-color diagram ($U - B$ vs. $B - V$, with x-axis $B - V$) for these 'stars'. Draw your axes in such a way that the bluest stars appear in the upper left corner. What do you conclude? (Compare Problem 1.3).

d) What could be the reason that the approximate $U - B$ color index at 10,000 K is significantly different from zero as it should be because of the above normalization convention (Vega)? (Hint: flux distribution of A-stars). Which temperature would be required in the $U$ range (if $B$ is evaluated for $T = 10,000$ K) that $U - B$ becomes (close to) zero?

**Problem 2** [4 points] *Number density of blackbody photons*

a) Show that the number density of blackbody photons, $n_\lambda d\lambda$, is given by

$$n_\lambda d\lambda = \frac{8\pi}{\lambda^4} \frac{1}{\exp(hc/(\lambda kT)) - 1} d\lambda$$

b) Calculate the total number of (blackbody) photons inside a kitchen oven at 250°C, assuming a volume of 0.5 m$^3$.
Hint: $\int_0^\infty x^2/(\exp(x) - 1)\mathrm{d}x = 2\mathrm{Zeta}(3) = 2.40411$, with Zeta($z$) the Zeta-function.

**Problem 3** [3 points] *Average blackbody photon energy*

a) Use the results from Problem 2b) and the relations from the script to show that the average energy per blackbody photon, $u/n$, is given by

$$\frac{u}{n} = 2.70kT,$$

where $u$ is the total energy density and $n$ the total number density of black body photons of all wavelengths.

b) Calculate the average energy per blackbody photon at the center of the Sun ($T \approx 1.57 \cdot 10^7$ K) and in the solar photosphere ($T = 5777$ K). Express your results in units of electron Volt (eV).

Have fun, and much success!