# Statistical methods – an introduction (SS 2022)

# LAST problem set (10)

**Problem 1** [4 points] *Log-likelihood: Information and other expectation values*

a) From some general arguments, it can be shown that $E(d\ln L/d\lambda) = 0$ (see script, Eq. 7.10). Convince yourself about the validity of this relation, by *explicitely* calculating (i) $E(d\ln L/d\mu)$ and (ii) $E(d\ln L/d\sigma)$, for the case of a normal distribution with expectation value $\mu$ and standard deviation $\sigma$.

b) Calculate the information $I(\sigma)$, using (see script p. 187, 188)

$$I(\sigma) = -E\big[\frac{d^2 \ln L}{d\sigma^2}\big],$$

for a normal distribution with standard deviation $\sigma$. Remember that the inverse of the information, evaluated at the ML-estimator, is the asymptotic variance of this estimator, i.e., tells about its errors (at least if one assumes uniform priors).

c) Convince yourself that the alternative expressions (see Eq. 7.13a)

$$I(\lambda) = -E\big[\frac{d^2 \ln L}{d\lambda^2}\big] = -NE\Big[\big(\frac{f'(x,\lambda)}{f(x,\lambda)}\big)'\Big]$$

are indeed identical.

**Problem 2** [4 points] *Exponential distribution: ML-estimator and likelihood*

a) Calculate the ML-estimator for the life time $\tau$ of an exponential distribution, $f(t) = (1/\tau)\exp(-t/\tau)$, given a set of $N$ observed life times $t_i$.

b) Express the log-likelihood for problem a) in terms of $\tau$, its ML-estimator, $\tilde{\tau}$, and $N$.

c) Derive the corresponding asymptotic log-likelihood, $\ln L_\infty$ (script pages 195ff.), in terms of the same quantities. Until here, the offset ($\ln L_\infty(\tilde{\tau})$) does **not** need to be specified.

d) Convince yourself that the results from b) and c) are identical in the large $N$ limit, by a useful expansion about $\tilde{\tau}$. What is the value of "const" (page 197, 2nd paragraph), i.e., the offset?

**Problem 3** [4 points] *Minimum variance bound*

Calculate the MVB for the estimator $\mathrm{S}(\sigma^2) = s'^2 = \frac{1}{N}\sum(x_i - \bar{x})^2$ given a Gaussian distribution. Show that this MVB is in agreement with the fact that $Var(s'^2) = 0$ for $N = 1$.

Note: In problem 1b), you should have calculated $I(\sigma)$. Here, however, we are dealing with an estimator for $\sigma^2$!

Have fun, and much success!