Statistical methods – an introduction (SS 2022)

Problem set 1

Note: The 'points' assigned to each problem denote its (relative) difficulty. For each problem set, a maximum value of 12 points is foreseen.

Problem 1 [3 points] *Too old a teacher ...*

The ages (in years) of a class of 20 students are as follows:

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23.5,23.2,21.8,23.7,23.4,23.5,24.7,22.4,23.1,23.9
23.8,23.7,23.2,25.0,23.1,24.2,24.4,25.5,24.0,24.5
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a) Calculate the mean and the standard deviation of this distribution.

b) Repeat the above analysis, after including the lecturer (age: 56 years) into the data-set. Compare with the results from a), and interprete the difference.

Problem 2 [3 points] *Card games without and with jokers: Non-exclusive, dependent and independent events*

a) Calculate the probability that a card taken from a well-shuffled pack of 52 cards (no jokers!) is a '10' or a spade.

b) Are the events T ('10') and S (spade) that a card drawn from such a pack is a '10' or a spade independent?

c) Now add 4 jokers to the pack. Are the events T and S still independent?

d) Consider situation c). Determine, from first principles, the probabilities P(T|S) and P(S|T), and convince yourself about the validity of Bayes' theorem,

$$P(T|S)P(S)=P(S|T)P(T)=P(T\cdot S)$$

To answer b) and c), use the definition of independent events.

Problem 3 [2 points] *A thief in the night*

In the yard of a paint shop, 8% of the cars have scratches, and independently, 10% of the cars have color errors. What is the probability that a thief in the night (no torchlight, no cell phone etc.) steals a car with

a) both defects?

b) with at least one defect?

c) with only one defect?

Problem 4 [4 points] *Weather report and beer-garden*

On average, and independent of season, the weather report for Munich predicts, for the following day, good weather in 40% of all cases and bad weather in 60% of all cases. The hit rate for predicted good weather (i.e., both forecast and actual weather are good) is 80%, and the hit rate for bad weather (i.e., both forecast and actual weather are bad) is 90%.

a) What is the probability that 'tomorrow is good weather'? Use the rule of total probability, and draw a corresponding probability tree (check internet for 'probability tree').

   Note that the resulting probability is not only the probability that 'tomorrow is good weather', but also the probability for good weather in general (it does not depend on the forecast from the previous day).

   Solution: P(good weather) = 0.38

b) Today is good weather, but your friend did not show up to an appointment in the beer-garden, claiming that yesterday's weather report was bad. What is the probability that this was only a lame excuse, i.e., that actually yesterday's report predicted good weather for today?

Problem 5 [2 points] *Cards in a hat*

If you had fun with the above problems, you might look also in the following one (somewhat similar to the "goat problem", see lecture manuscript).

A hat contains three different cards, one with a red color on both sides, one with a blue color on both sides, and one with a red color on one side and a blue color on the other.

Some guy is closing his eyes, picks one of the cards from the hat, and puts it on the table (still with closed eyes). After opening the eyes, the guy sees that the upper side of the card is red. What is the probability that also the backside of this card is red? Derive this probability both from simple arguments (counting of all possibilities), and from applying the definition of the conditional probability.

Have fun, and much success!