

The cosmological implications of high-redshift, massive galaxy clusters.

**Ben Hoyle, Raul Jimenez, Licia Verde, ICC-IEEC University of Barcelona,
Shaun Hotchkiss University of Helsinki.**

Hoyle et al (2011, PRD) & (2011, JCAP) & in prep.



4/04/2012

Overview

- **Cluster catalogues to constrain cosmology.**
- **Individual ‘rare/extreme’ clusters of galaxies.**
- **The (biased) $>M, >z$ analysis.**
- **The high- z cluster samples.**
- **The XMM Cluster Survey.**
- **The (biased) $>M, >z$ analysis & implications.**
- **Unbiasing the $>M, >z$ analysis & exclusion curves.**
- **The (unbiased) $>M >z$ analysis and other tests.**
- **Conclusions + future work.**

The theoretical cluster mass function

The mass function describes the number of clusters per unit mass, per unit redshift as a function of cosmological parameters.

$$n_G(M, z) = \sqrt{\frac{2}{\pi}} \frac{\bar{\rho}}{M^2} \left| \frac{d}{d \ln M} \ln \sigma_M \right| \nu \exp -\nu^2/2. \quad \nu = \delta_{sc}/\sigma(M, z)$$
$$\sigma = \int P(k) \hat{W}(kR) k^2 dk,$$

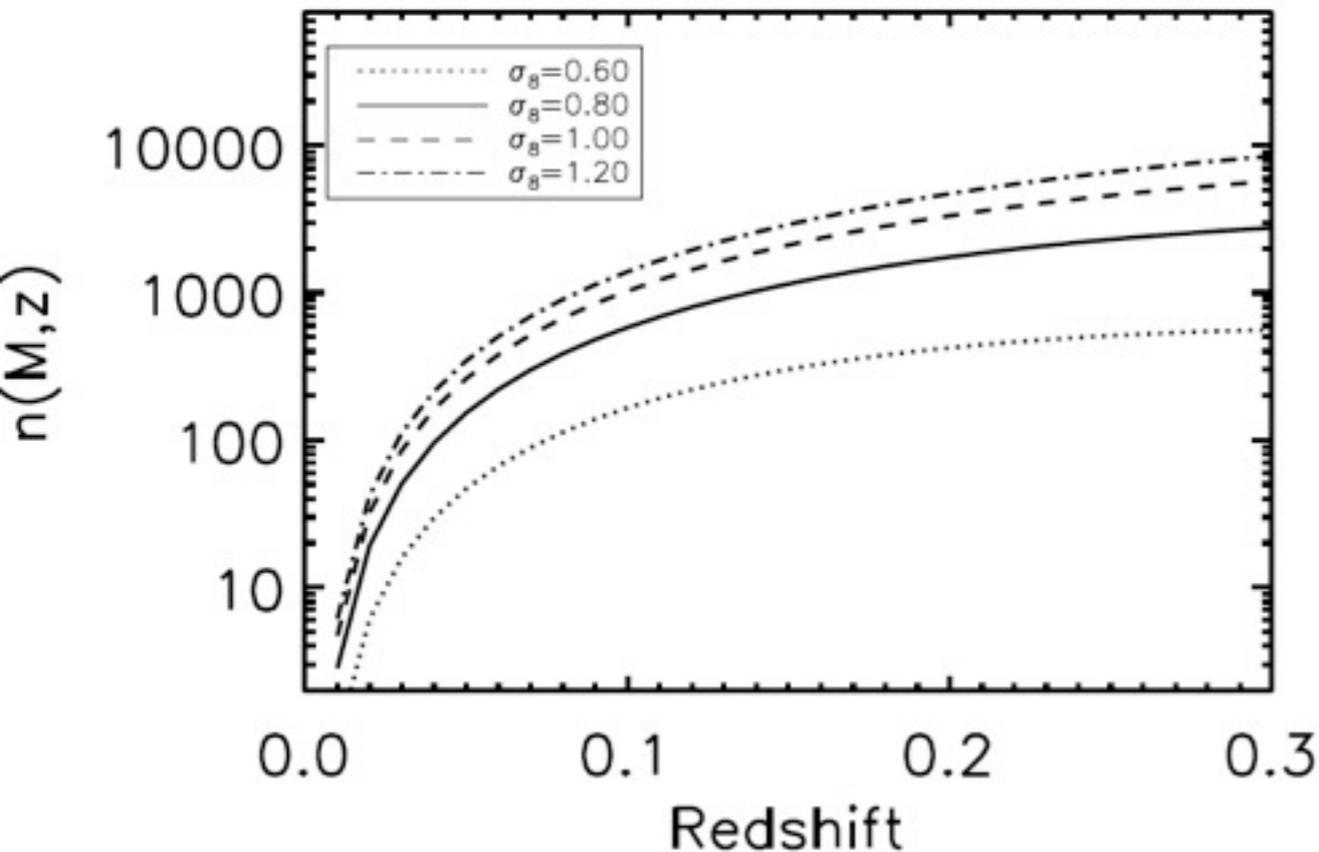
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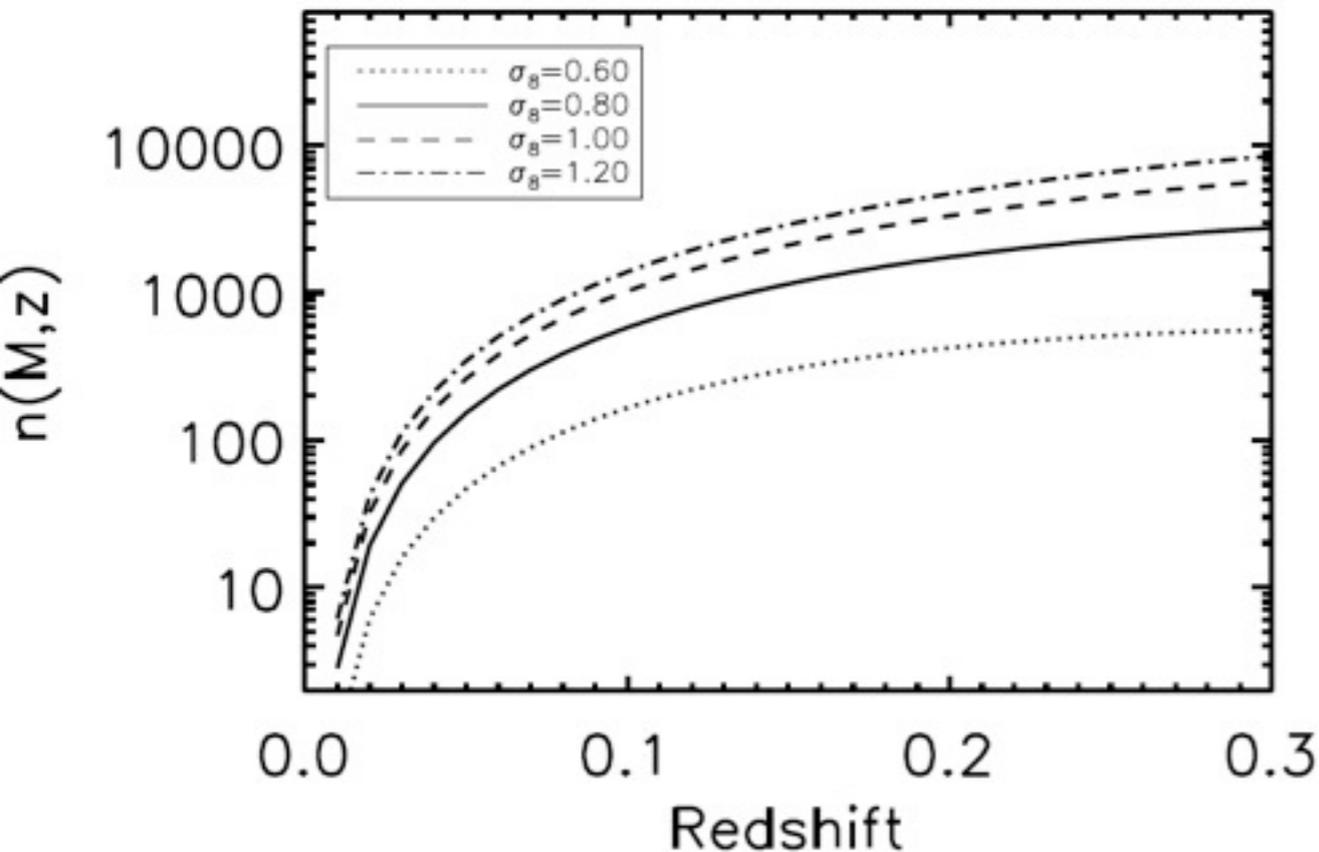
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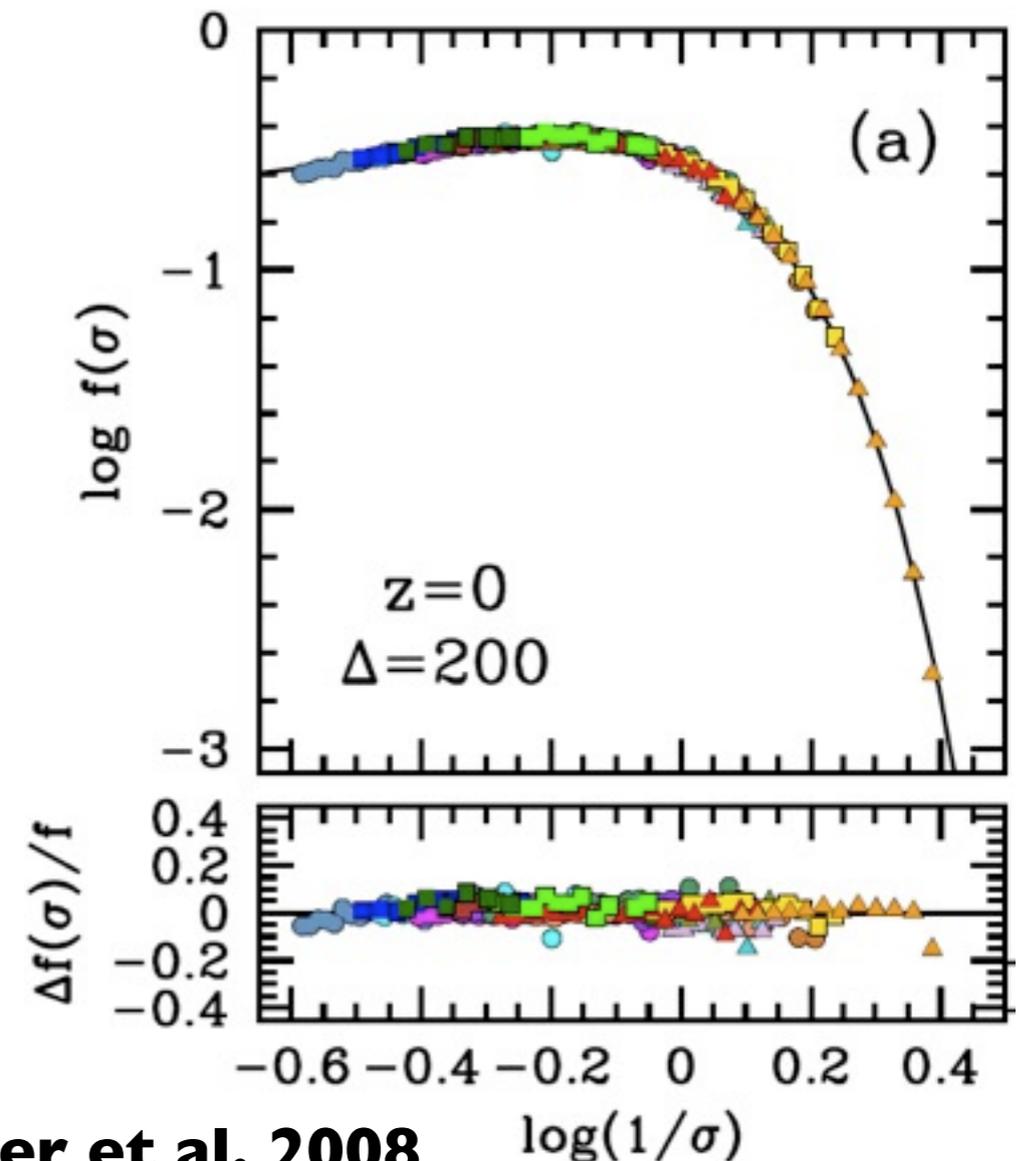


Press & Schechter 1974 and then extended (e.g., Sheth & Tormen 2001)

Now, fitting functions are calibrated to large N-body dark matter only simulations (e.g., Jenkins et al 2002)

$$\frac{dn}{dM} = f(\sigma) \frac{\bar{\rho}_m}{M} \frac{d \ln \sigma^{-1}}{dM}.$$

$$f(\sigma) = A \left[\left(\frac{\sigma}{b} \right)^{-a} + 1 \right] e^{-c/\sigma^2}$$



Tinker et al. 2008

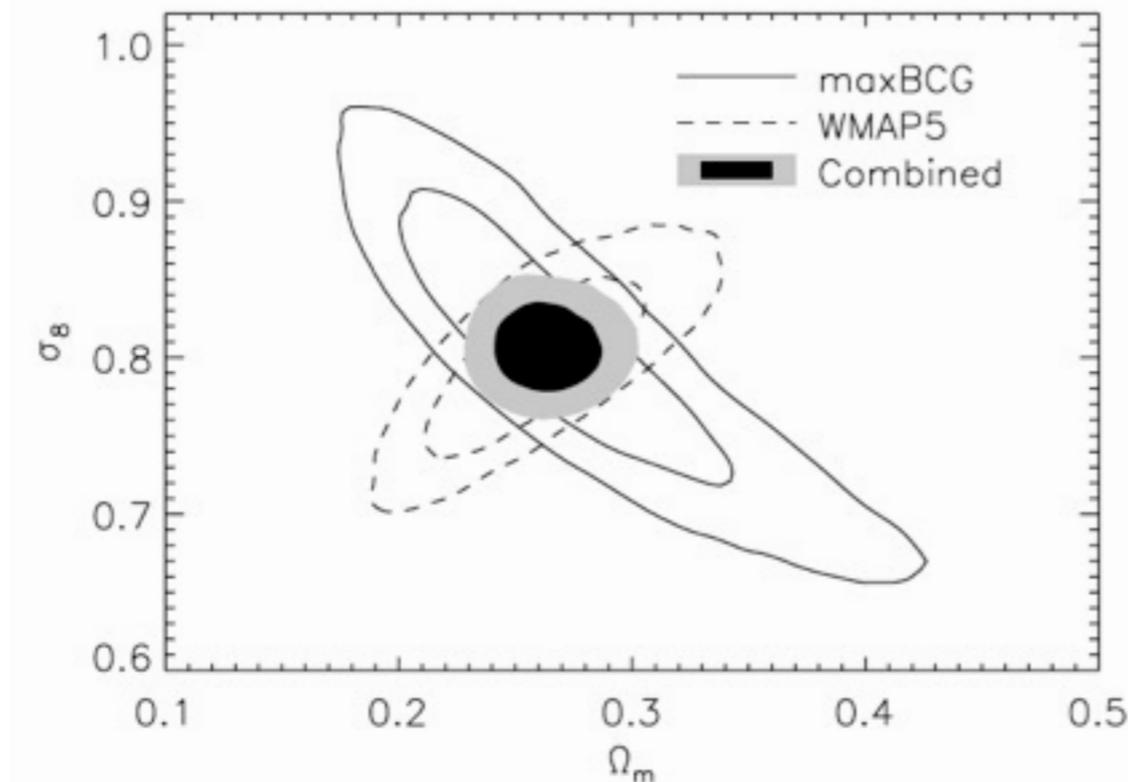
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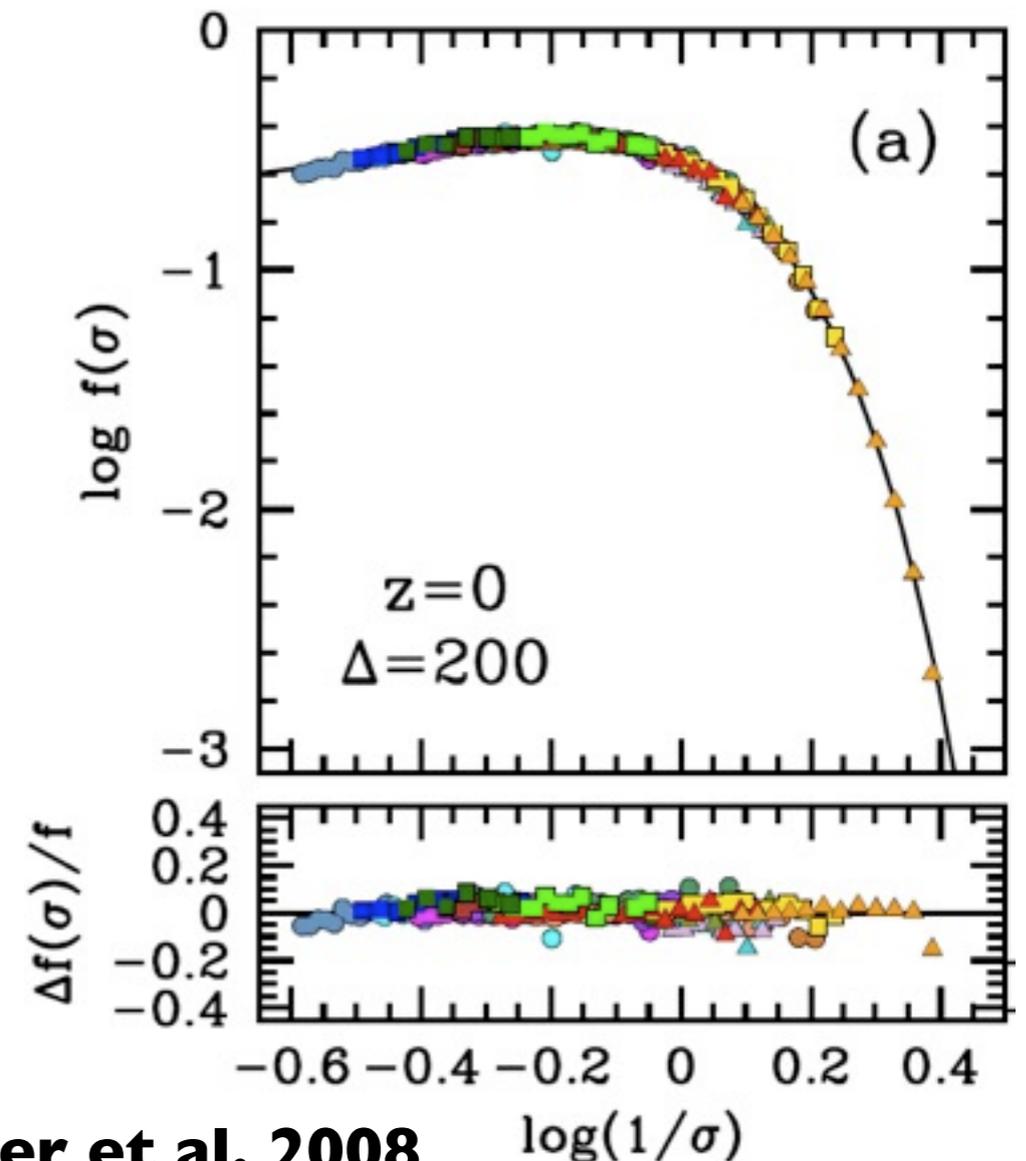


**maxBCG optically selected clusters:
Rozo et al. 2009**

$$N \sim 13 \times 10^3$$

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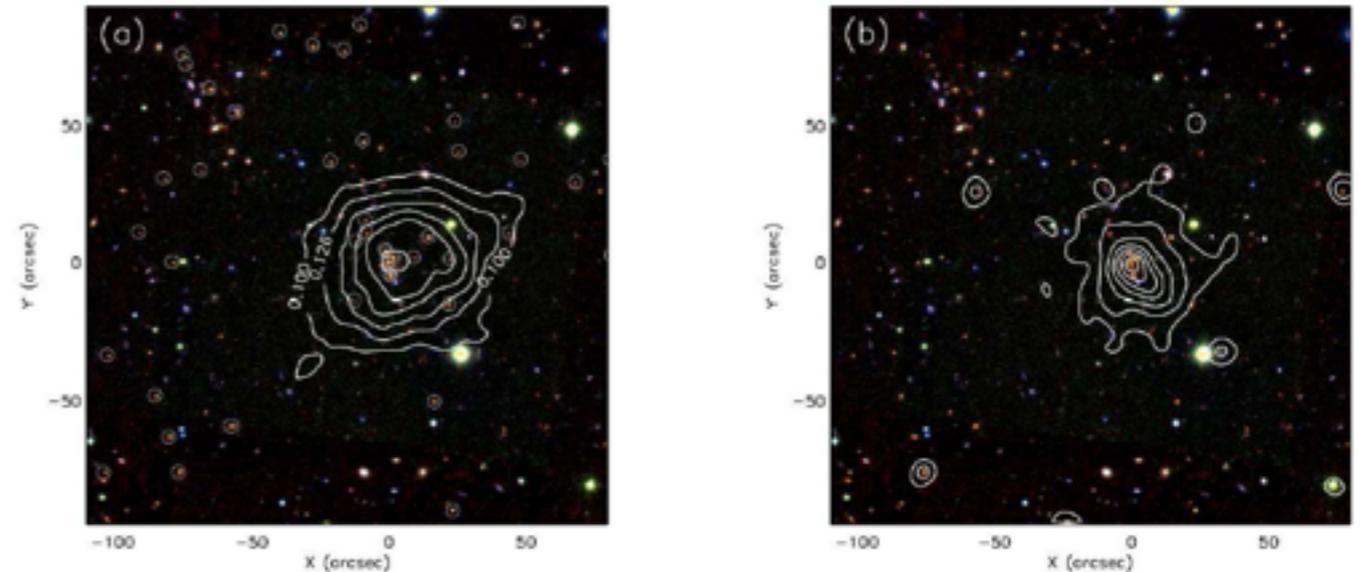
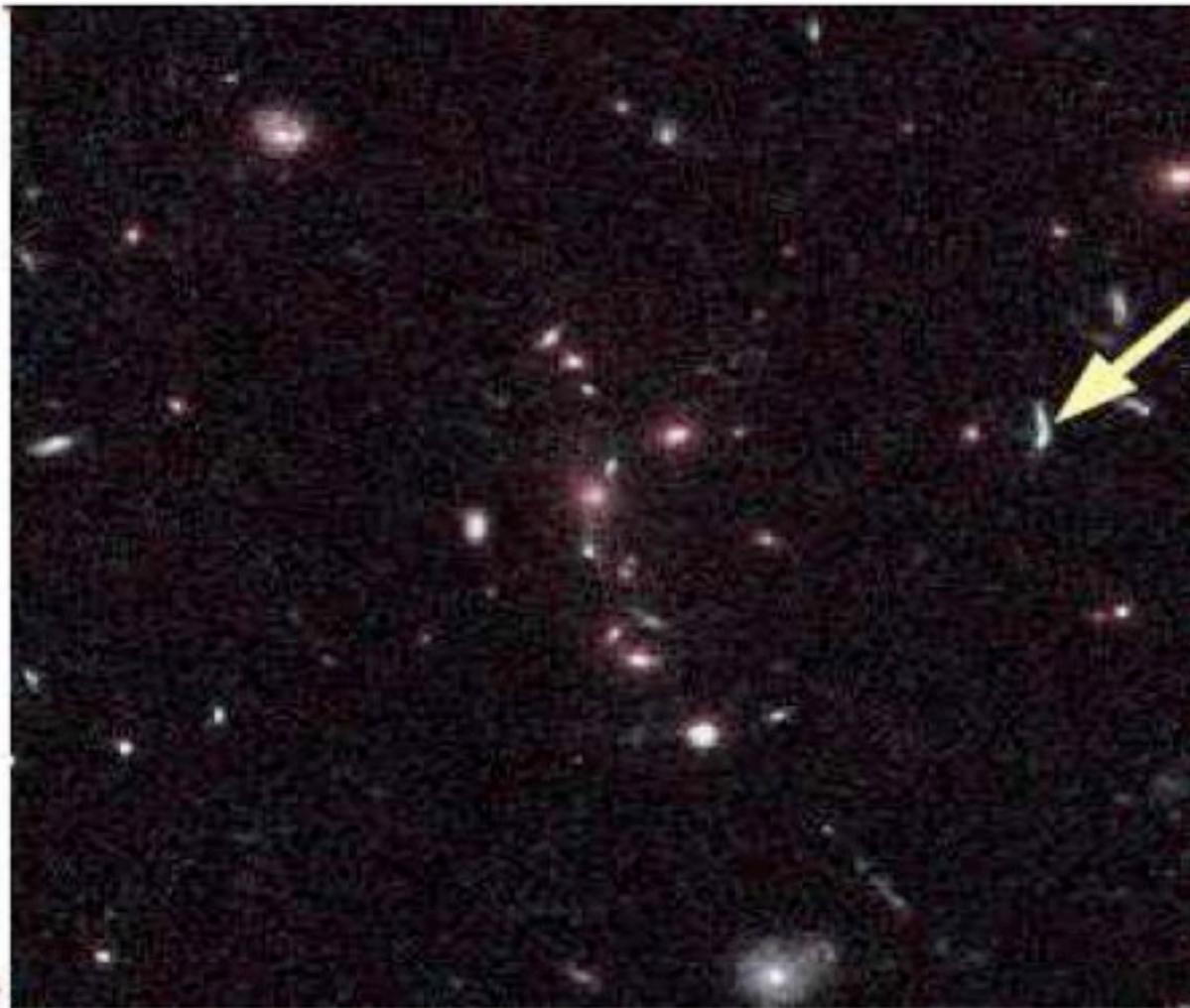
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Tinker et al. 2008

Motivation: observations of an extreme object

The observations of XMMJ2235 appeared to cause tension with the LCDM model + WMAP priors on the cosmological parameters. A very massive clusters of galaxies at high redshift, was statistically unlikely to have been observed.

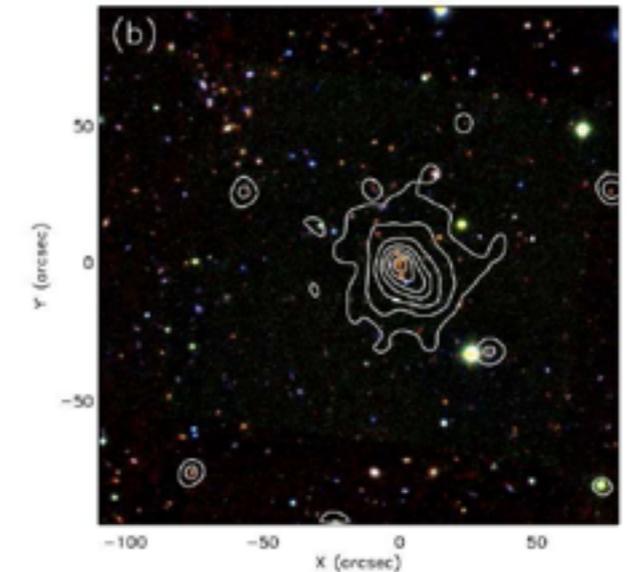
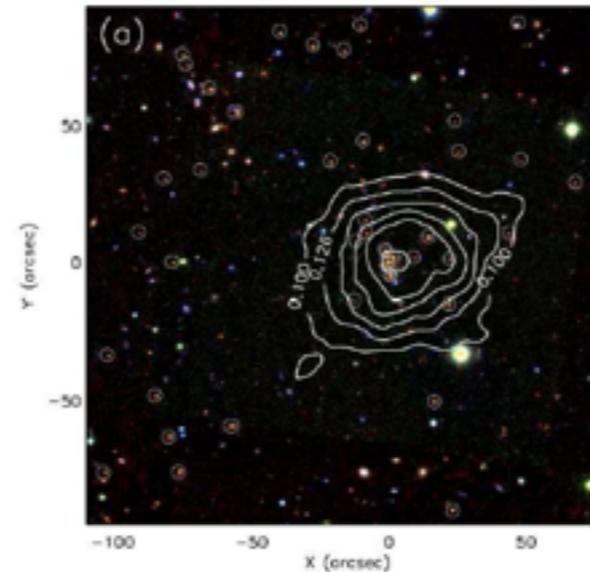
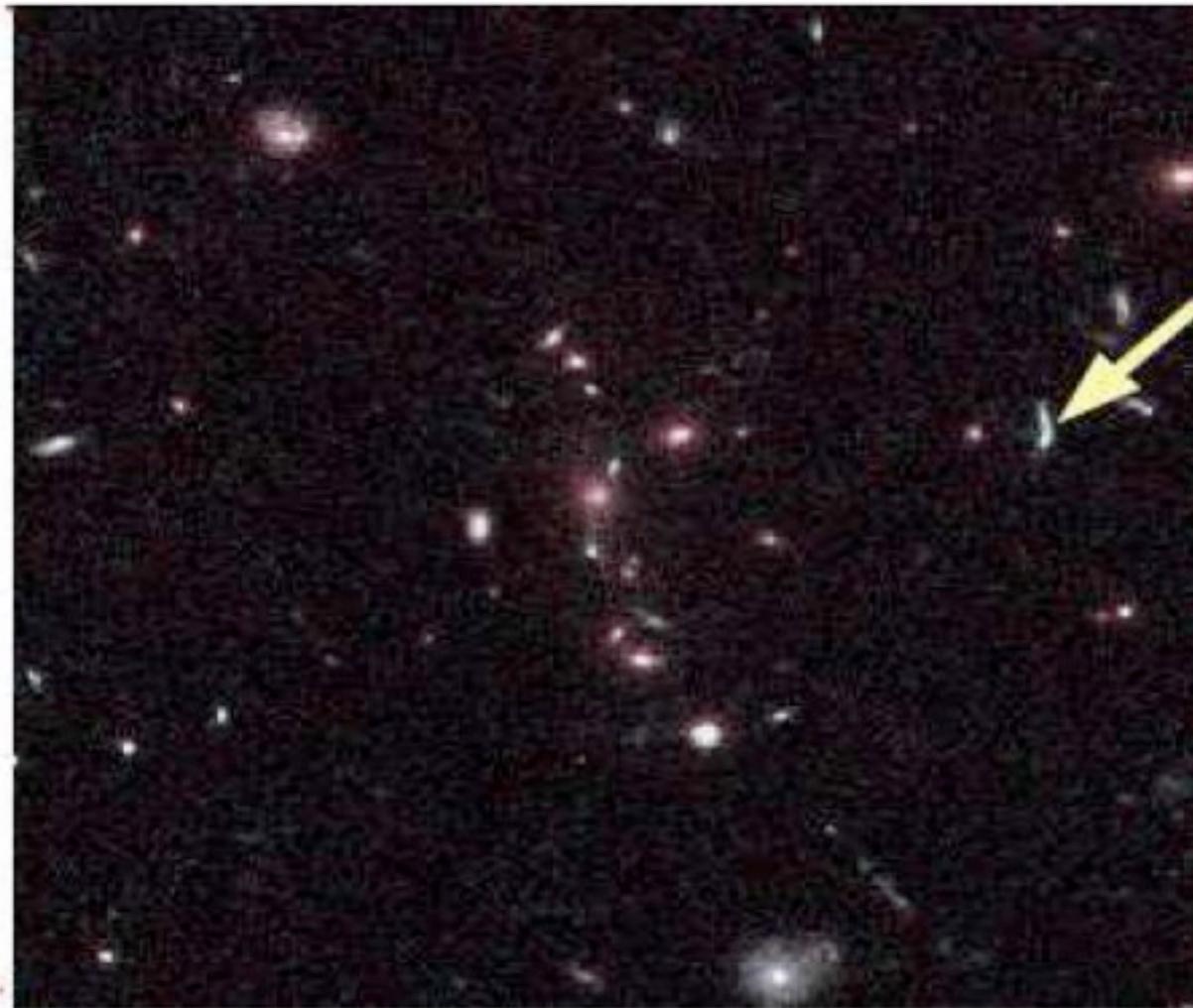


$$M_{200} = 7.7 \pm 1.3 \times 10^{14} M_{\odot}$$
$$z = 1.4 \quad M_{200} = 7.7^{+4.4}_{-3.3} \times 10^{14} M_{\odot}$$

Jee et al 2009

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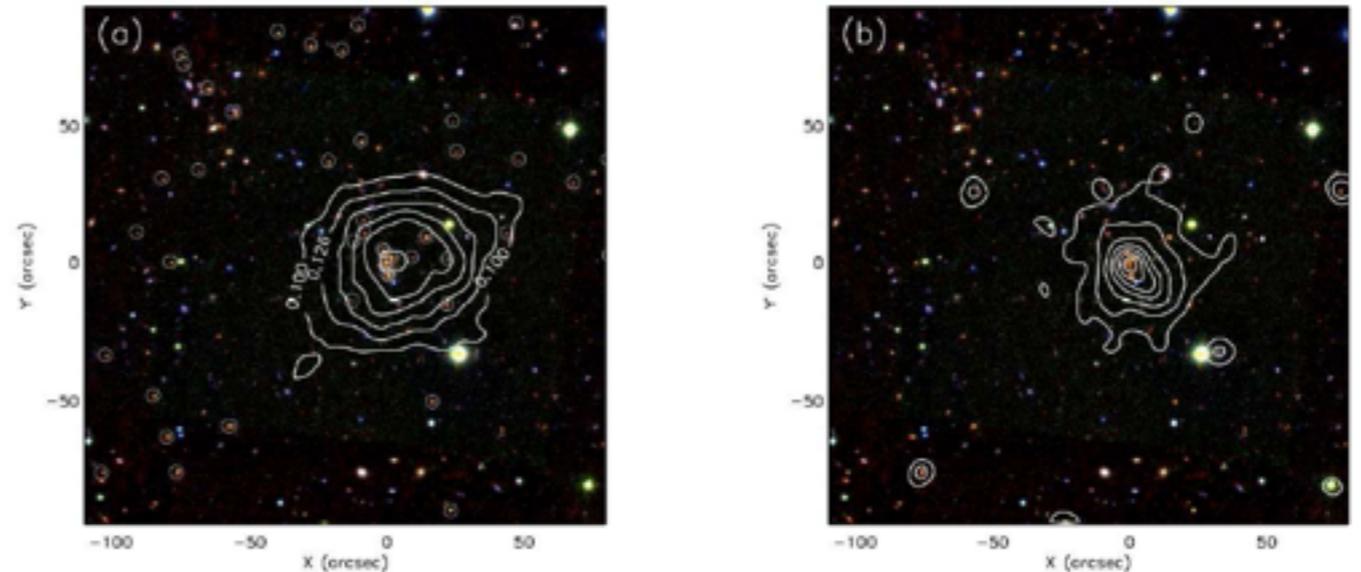
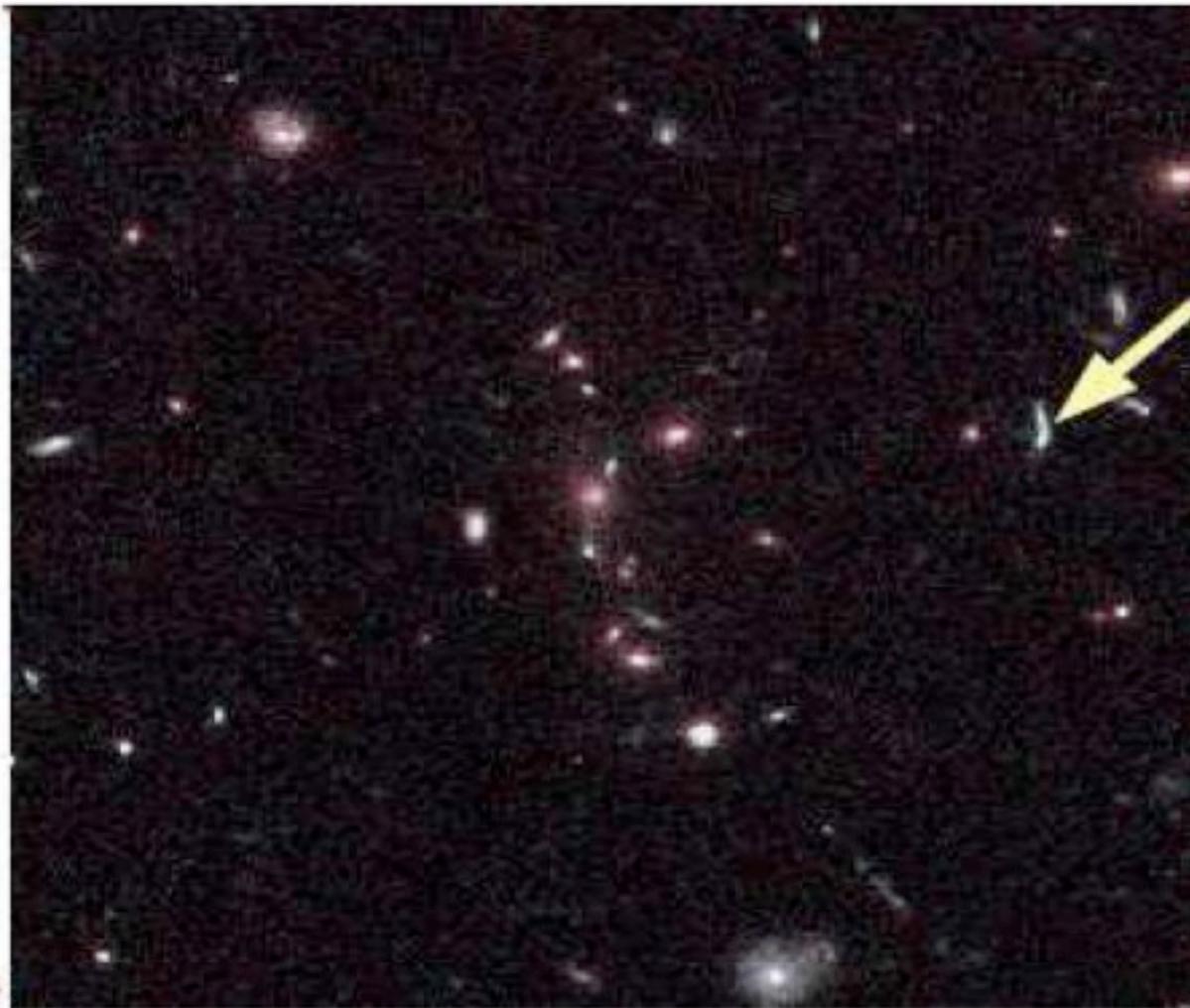
How likely was this cluster to exist $>M >z$?

Jee et al 2009

- How many clusters would we expect to find at $>M, >z$
- The expected number in the full sky ~ 7 .
- Footprint was 11 square degrees XMM X-ray survey, 0.02% of sky.
- Poisson sample from $(0.0002 * 7) > 1$ only 1.4%

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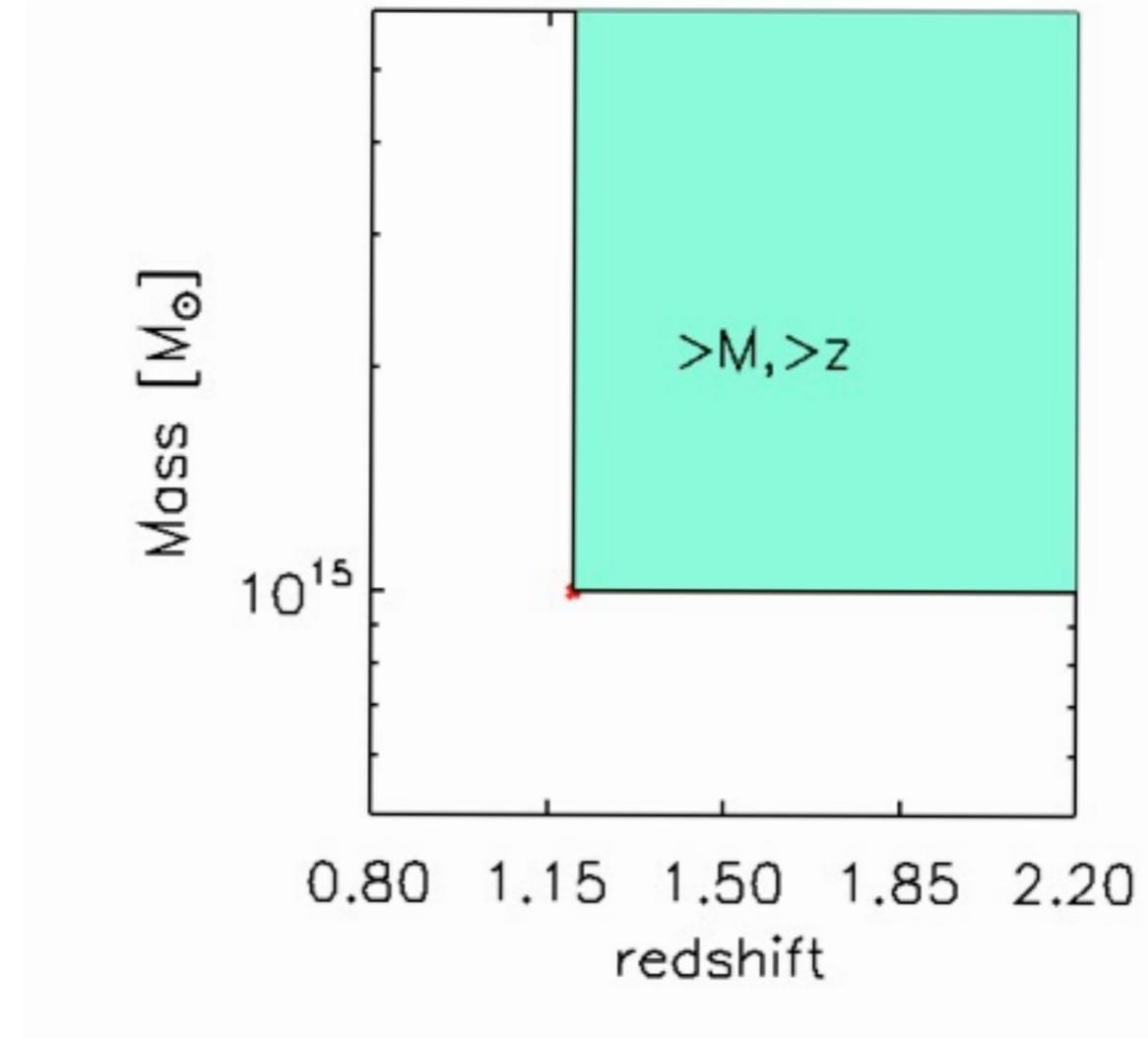
Jimenez & Verde 2009 showed $f_{nl} \sim 150$ relieves tension.

Cayon et al 2010 $f_{nl} = 360, f_{nl} > 0$ at 95%

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The (biased) $>M, >z$ analysis

The $>M, >z$ analysis begins by assuming that we would have observed any cluster with greater mass, or greater redshift than an observed cluster.

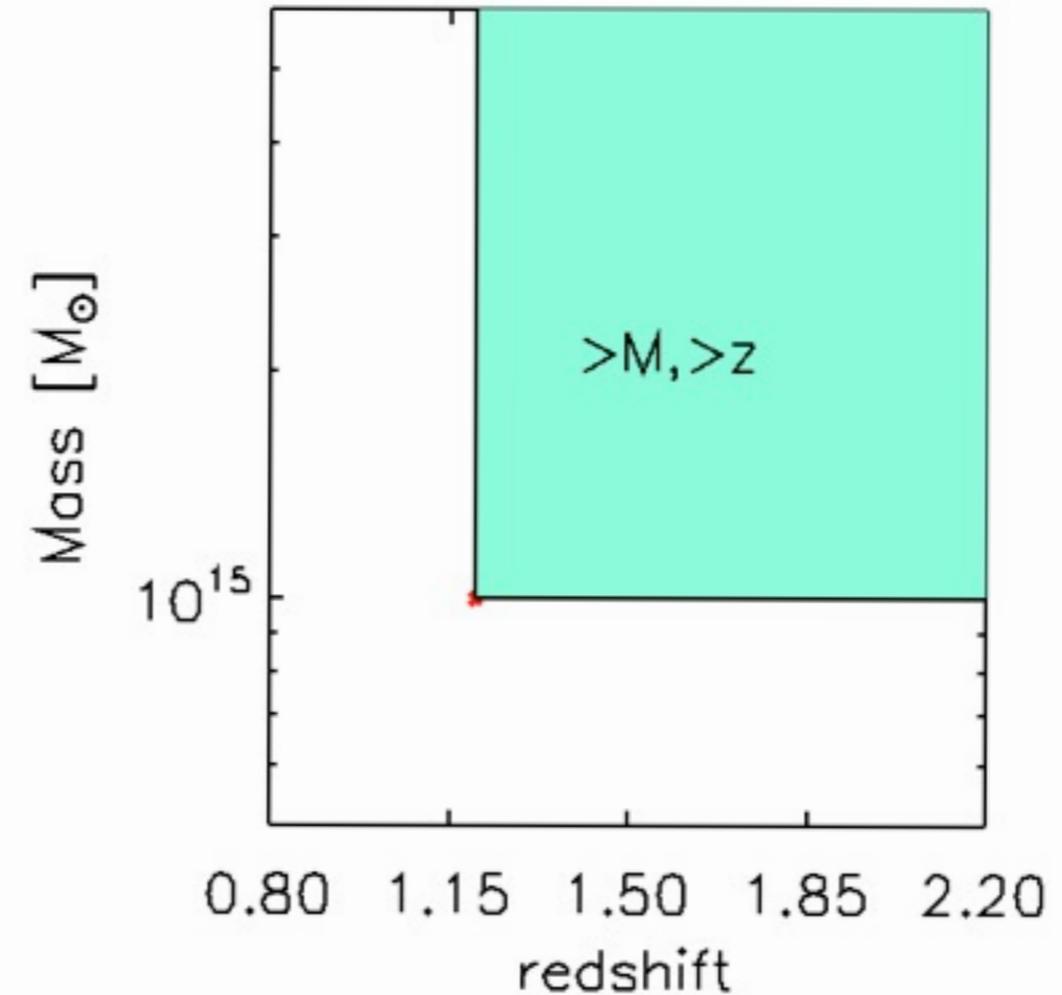


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$$A_s = \int_{M_S}^{\infty} \int_{z=z_{cluster}}^{z=2.2} n(m, z, f_{NL}, C) dm dz$$



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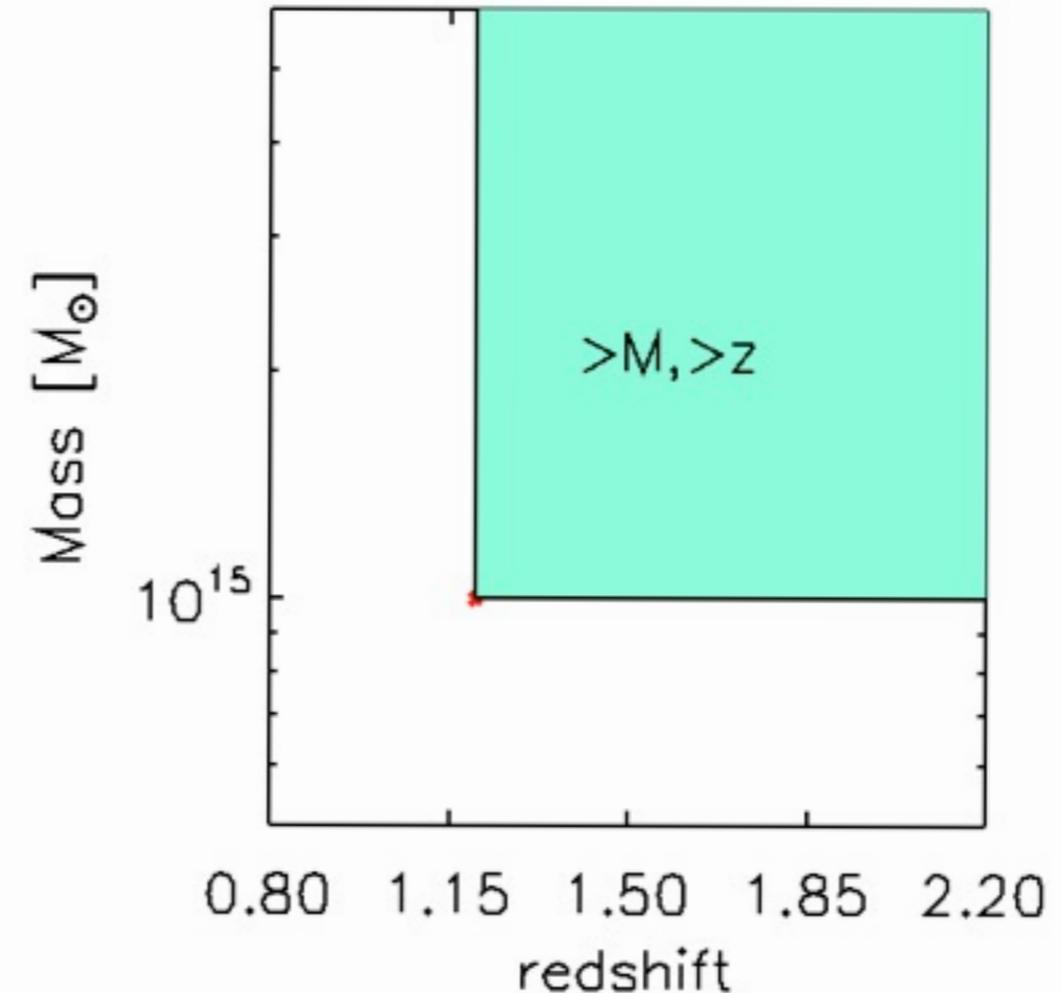
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If the Poisson sample is >1 , the cluster exists in this realisation.

If the Poisson sample is <1 the cluster does not exist in this realisation.



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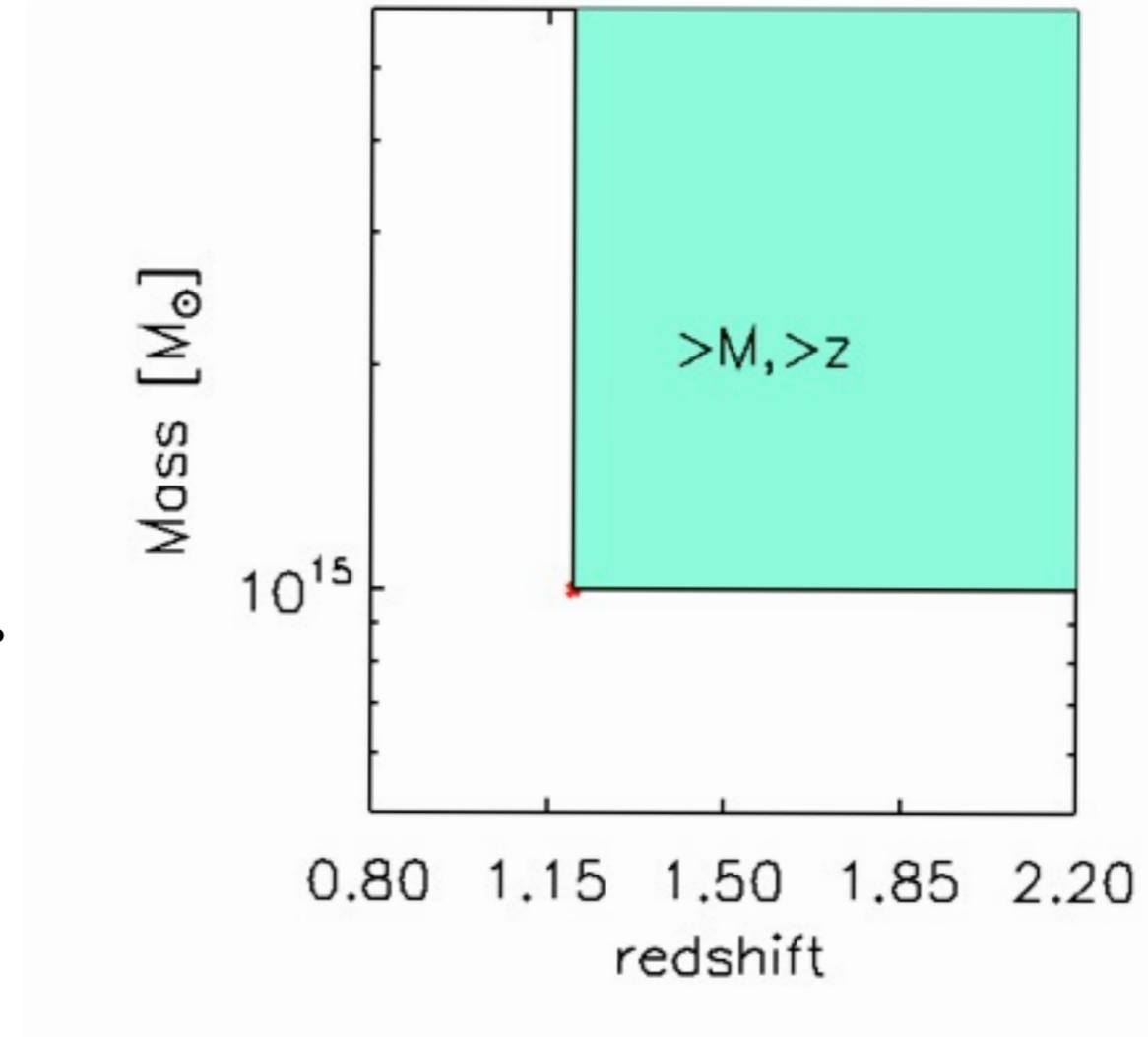
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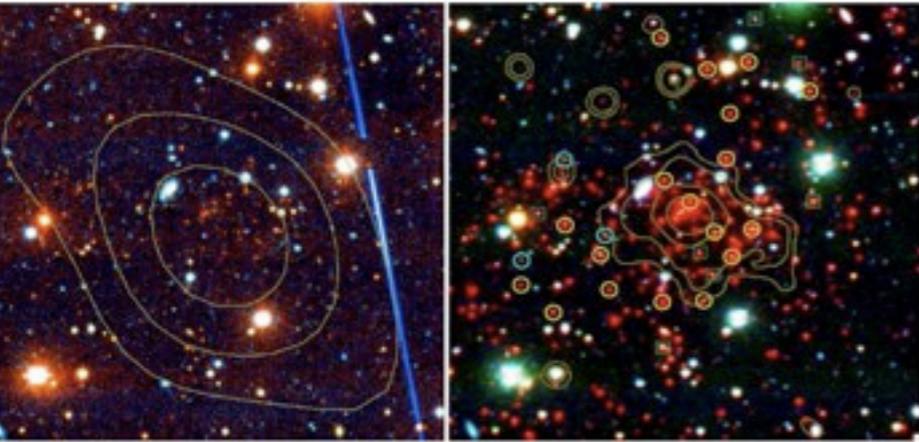
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The “existence probability” R , is given by

$$R = \text{Number}(P^O(A_s) \geq 1) / 10^4$$



Motivation: observations II - More “rare” clusters



SPT CL J0546-5345

$$M_{200} \sim 10^{15} M_{\odot}$$

$$z = 1.05$$

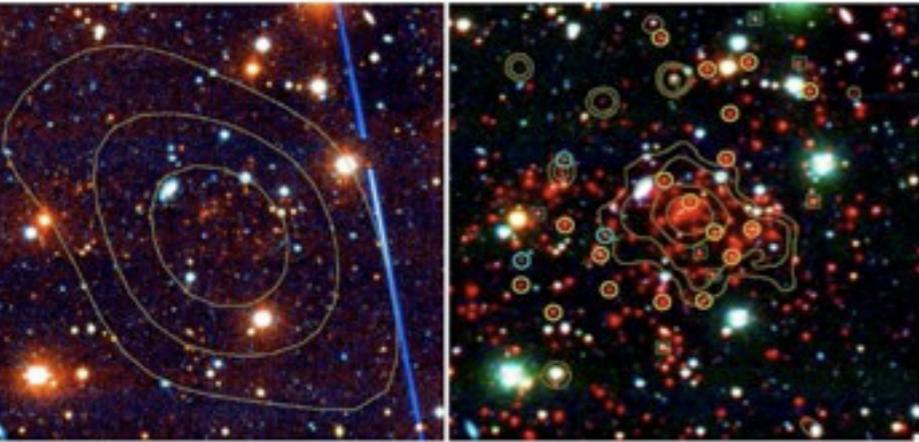
Brodwin et al 2010

- **Expect to see one
18% of time in the
>M,>z sense**

Are we just getting lucky?

Left: Optical $4' \times 4'$ color image (grz) of SPT-CL J0546-5345, with SZE significance contours overlaid ($S/N = 2, 4,$ and 6).
Right: Color optical (ri) + IRAC ($3.6 \mu m$) image of SPT-CL J0546-5345, with Chandrab X-ray contours overlaid ($0.25, 0.4, 0.85$ and
 $2'' \times 2''$ pixel per 55.6 ks in the $0.5-2$ keV band). North is up, east is to the left. Due to its high angular resolution, Chandrab
reveals substructure to the SW, which may be evidence of a possible merger. These images highlight the importance of IRAC
in identifying the galaxies in high redshift, optically faint clusters. Spectroscopic early-type (late-type) members are indicated with
circles. Green squares show the spectroscopic non-members.

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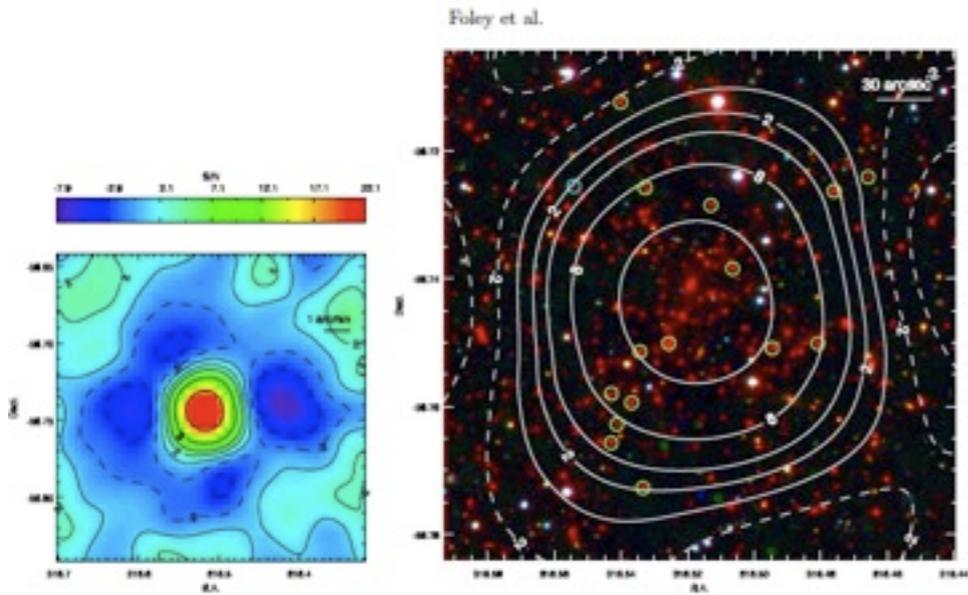
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Foley et al.

SPT-CL J2106-5844

$$M_{200} = 1.27 \times 10^{15} h^{-1} M_{\odot}$$

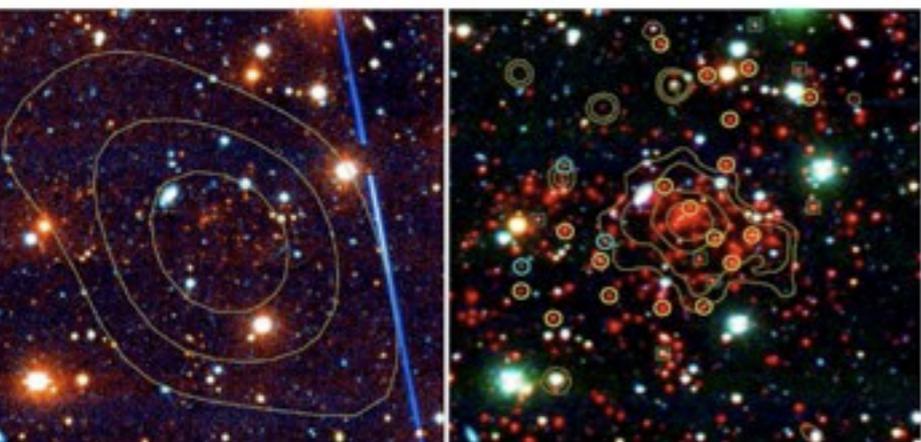
$$z = 1.13$$

Foley et al 2011

- Expect to see one
5.9% of time in the
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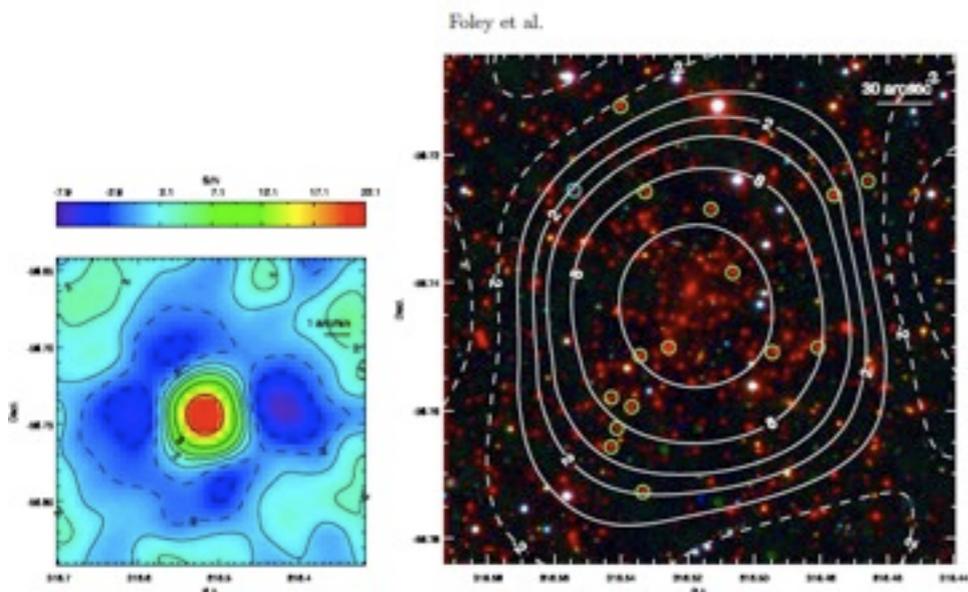
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Foley et al.

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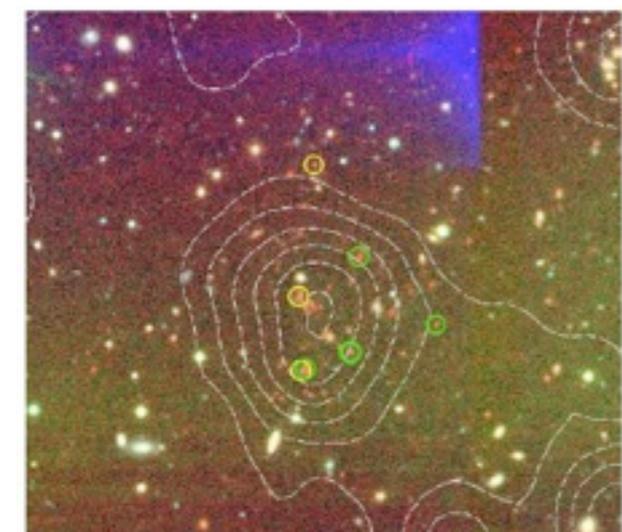
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Foley et al 2011

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XMMUJ0044.0-2033

$$3.5 < M < 5 \times 10^{14} M_{\odot}$$

$$z = 1.57$$

Santos et al 2011

- Expect to see one
<10% of time in the
>M,>z sense

Are we just getting lucky?

More clusters.

How lucky are we being? Are high-redshift, massive clusters consistent with LCDM using the $>M, >z$ test?

B.H., Jimenez, Verde 2010 PRD.83.103502

• $z > 1, M > 10^{14}$

• 3 SZ detected ‘*’

• 11 X-ray detected ‘+’

Cluster Name	Redshift	M_{200} $10^{14} M_{\odot}$	Method
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‘SPT-CLJ2341-5119’ *	1.03	$7.60^{+3.94}_{-3.94}$	Richness
‘XLSSJ022403.9-041328’ +	1.05	$1.66^{+1.15}_{-0.38}$	X-ray
→ ‘SPT-CLJ0546-5345’ *	1.06	$10.0^{+6.00}_{-4.00}$	Velocity dispersion
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‘RXJ1053.7+5735(West)’ +	1.14	$2.00^{+1.00}_{-0.70}$	X-ray
‘XLSSJ022303.0043622’ +	1.22	$1.10^{+0.60}_{-0.40}$	X-ray
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‘RXJ0849+4452’ +	1.26	$3.70^{+1.90}_{-1.90}$	X-ray
‘RXJ0848+4453’ +	1.27	$1.80^{+1.20}_{-1.20}$	X-ray
→ ‘XMMUJ2235.3+2557’ +	1.39	$7.70^{+4.40}_{-3.10}$	X-ray
‘XMMXCSJ2215.9-1738’ +	1.46	$4.10^{+3.40}_{-1.70}$	X-ray
‘SXDF-XCLJ0218-0510’ +	1.62	$0.57^{+0.14}_{-0.14}$	X-ray

The next generation of cluster samples will be found by X-ray (eRosita ~ 100,000 clusters) not SZ (ActPol ~ 1000 clusters).

All X-ray clusters detected or re-detected with XMM Cluster Survey (XCS).

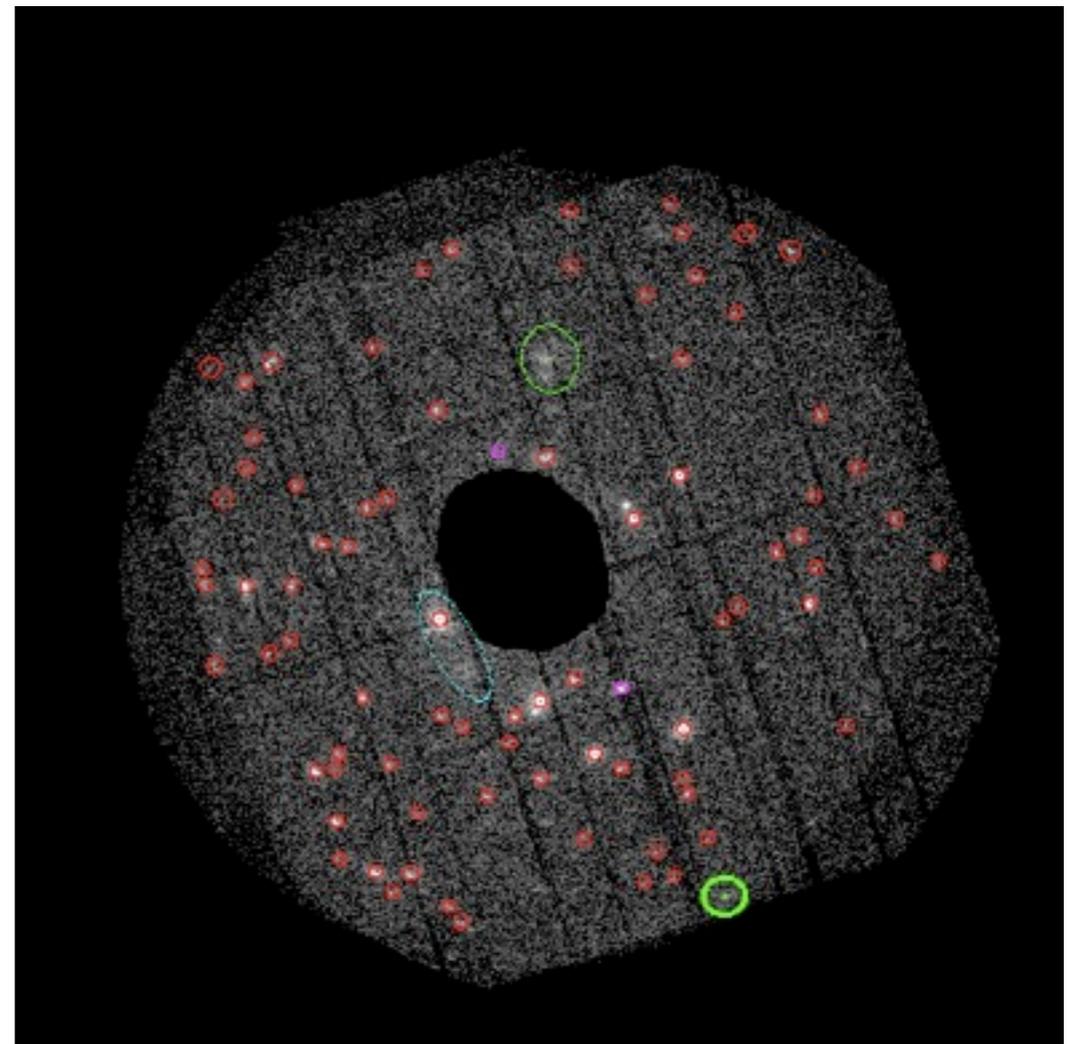
XCS: Identifying and classifying extended sources

Members: Kathy Romer [P.I], E. J. Lloyd-Davies, Mark Hosmer, Nicola Mehrrens, Michael Davidson, Kivanc Sabirli, Robert G. Mann, Matt Hilton, Andrew R. Liddle, Pedro T. P. Viana, Heather C. Campbell, Chris A. Collins, E. Naomi Dubois, Peter Freeman, Ben Hoyle, Scott T. Kay, Emma Kuwertz, Christopher J. Miller, Robert C. Nichol, Martin Sahlen, S. Adam Stanford, John P. Stott



X-ray photon map + automated pipeline to detect point sources (red) and extended sources (green).

X-ray emission is the smoking gun, but it's not enough. Need optical identification and redshifts (X-ray redshift difficult) before the fluxes can be converted to temperatures/masses.



Algorithms paper, Lloyd-Davies et al. 2010

XCS: Recent achievements

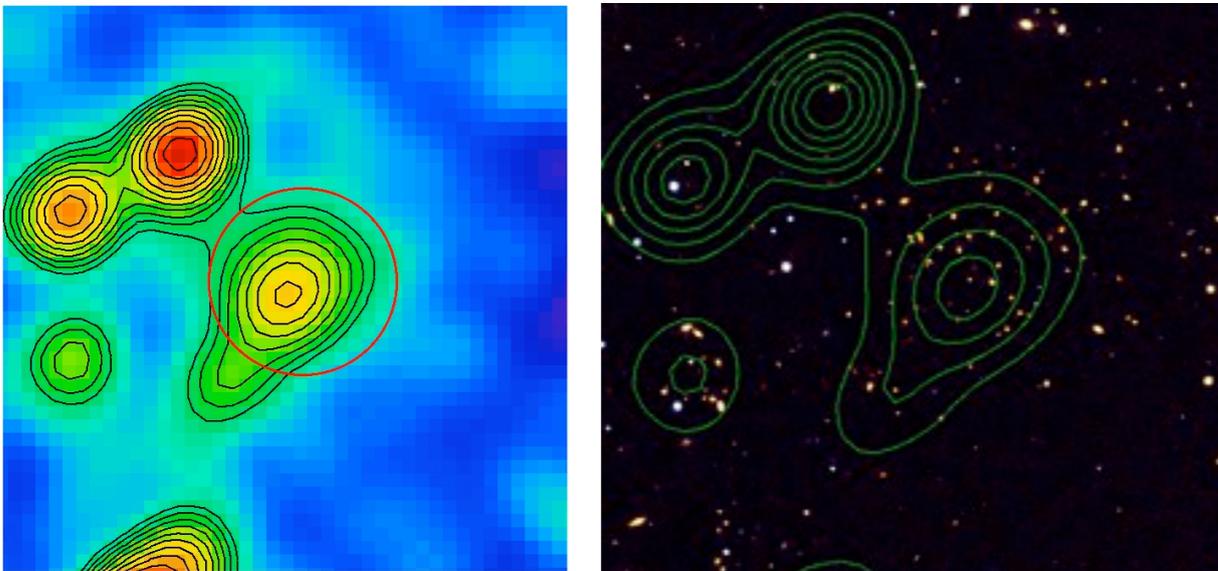
Recent Data release, Mehrrens et al. 2011

503 clusters, spanning $0.06 < z < 1.46$

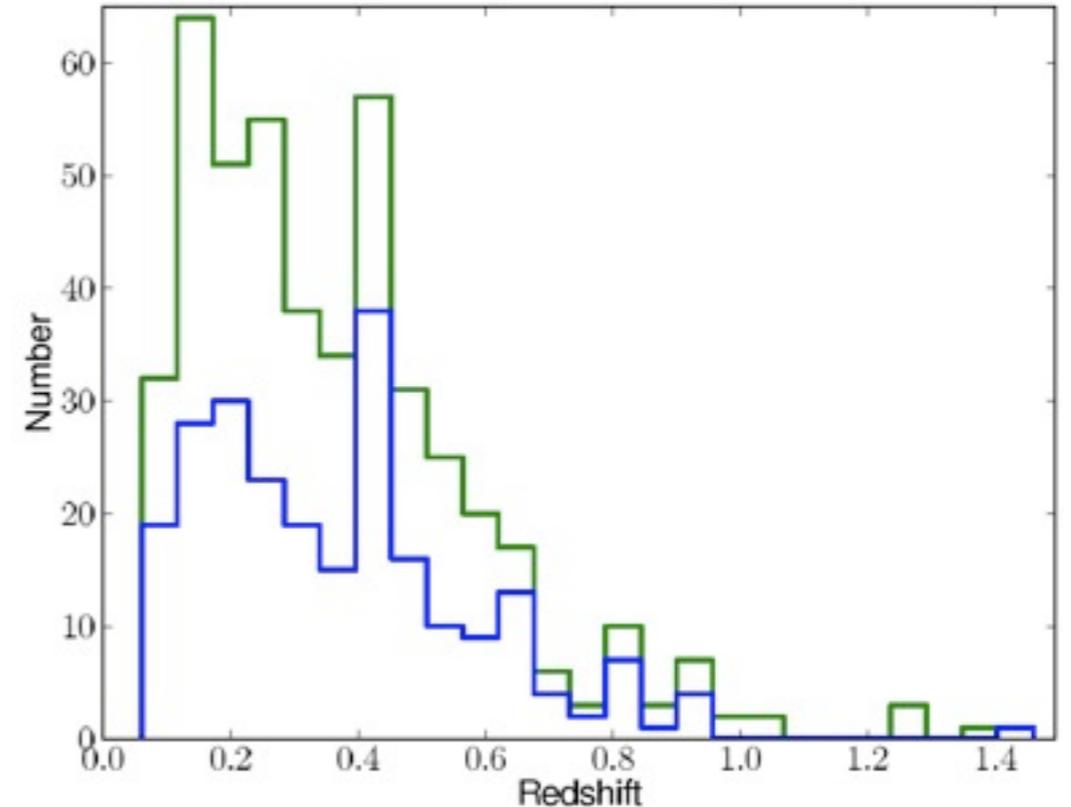
402 have X-ray temperatures

XMMXCS J2215

Was the highest redshift X-ray selected cluster, $z=1.46$ (Stanford et al. 2006, Hilton et al. 2007, 2008)



Now $z=2.07$, $M \sim 5-8 \cdot 10^{13} \text{ SolMass}$,
Gobat et al. 2011



Some XCS papers

The Stellar Mass Assembly of Fossil Galaxies:

Harrison et al. arXiv:1202.4450

The interplay between the BCG and the ICM via AGN feedback:

Stott et al. 2012

Predicted overlap with the Planck Clusters:

Viana et al. 2011

AGN and Starburst Galaxies in XMMXCS J2215.9-1738 at $z=1.46$:

Hilton et al 2010

The build up of stellar mass in BCG at high redshift:

Stott et al. 2010

Galaxy Morphologies and the Color-Magnitude Relation in J2215 at $z=1.46$:

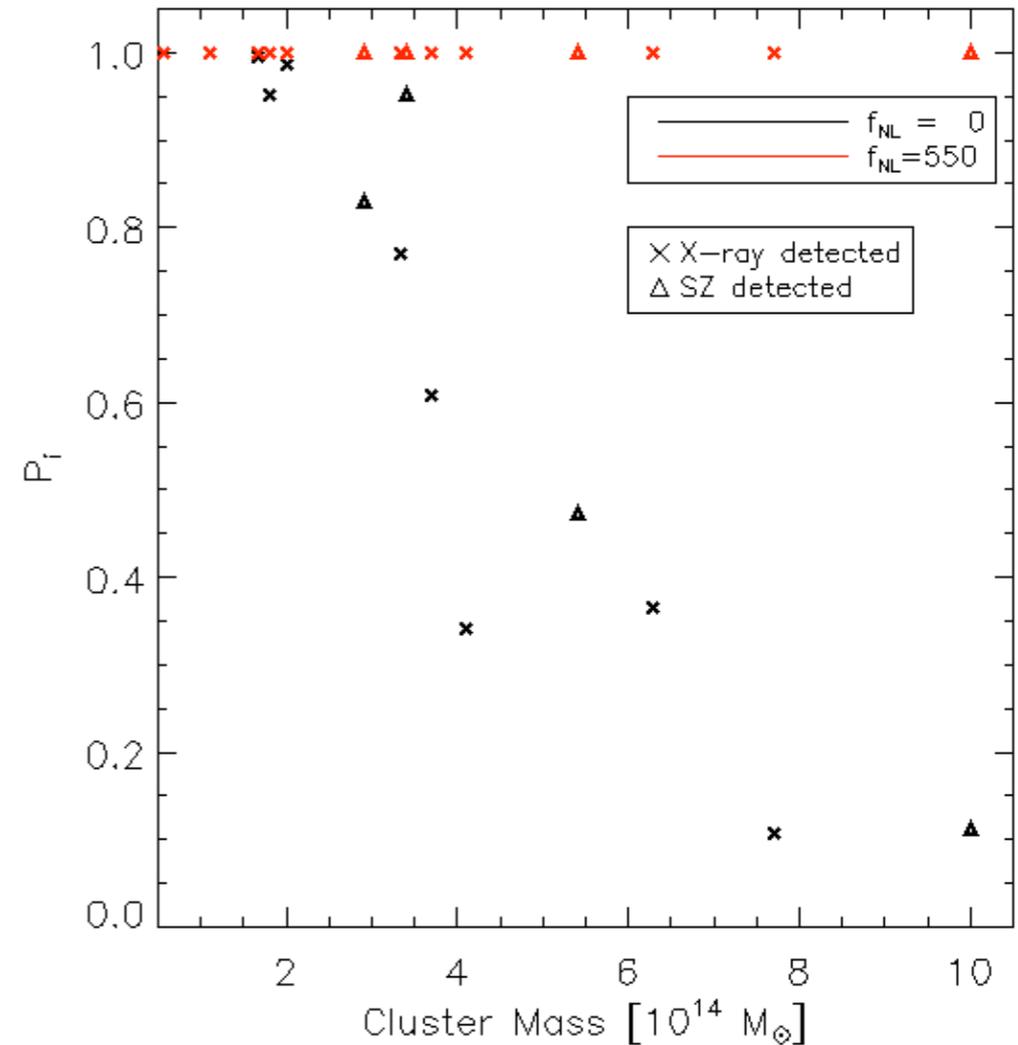
Hilton et al. 2009

Forecasting cosmological and cluster scaling-relation parameter constraints:

Sahlen et al. 2008

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We assumed that the probability, that an ensemble of N clusters exists is

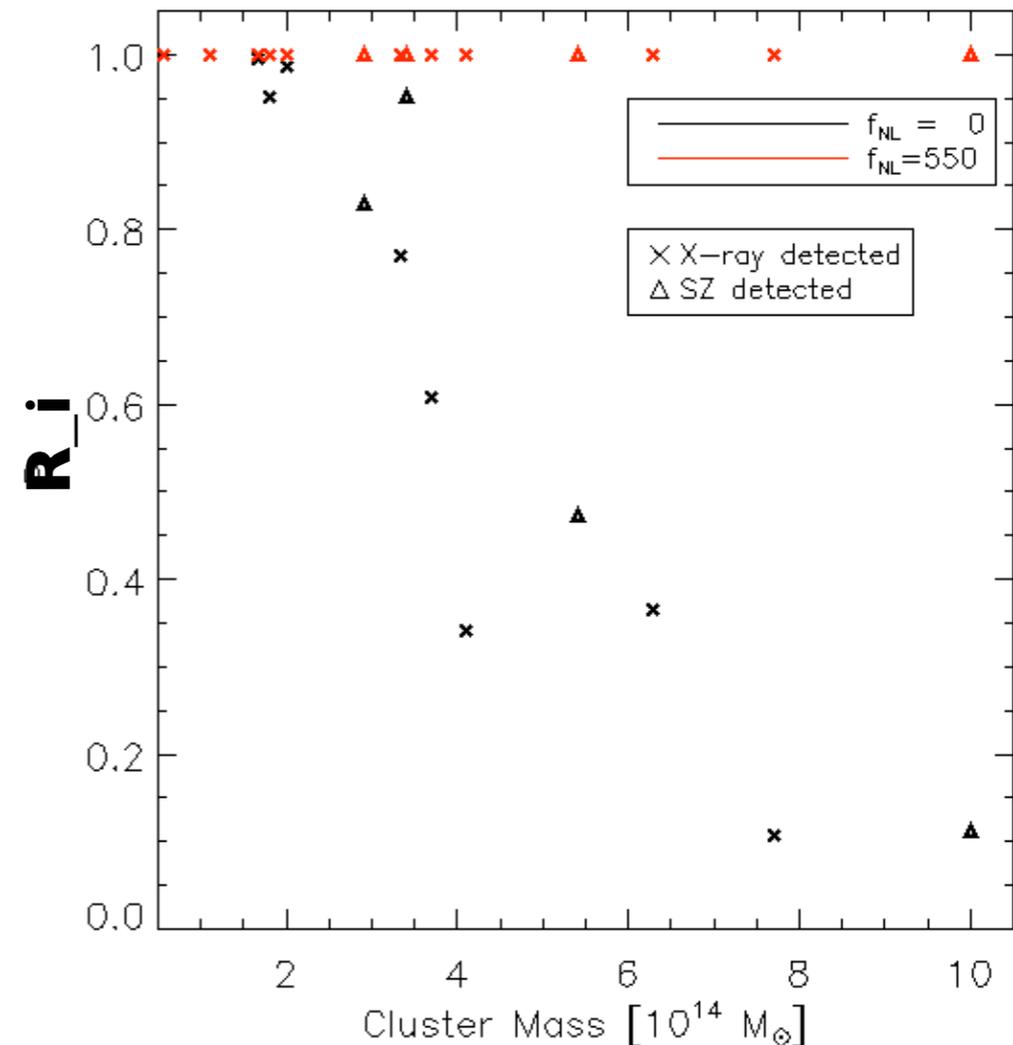
$$R_N = \prod_N R_i$$

$$f_{NL} > 123; \quad \sigma_8 \geq 0.9;$$

corroborated by Enqvist et al. 2010

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$$R_N = \prod_N R_i$$

Using the $>M, >z$ analysis, it appeared as though these clusters were unlikely.

Possible explanations:

$$f_{\text{NL}} > 123; \quad \sigma_8 \geq 0.9;$$

corroborated by Enqvist et al. 2010

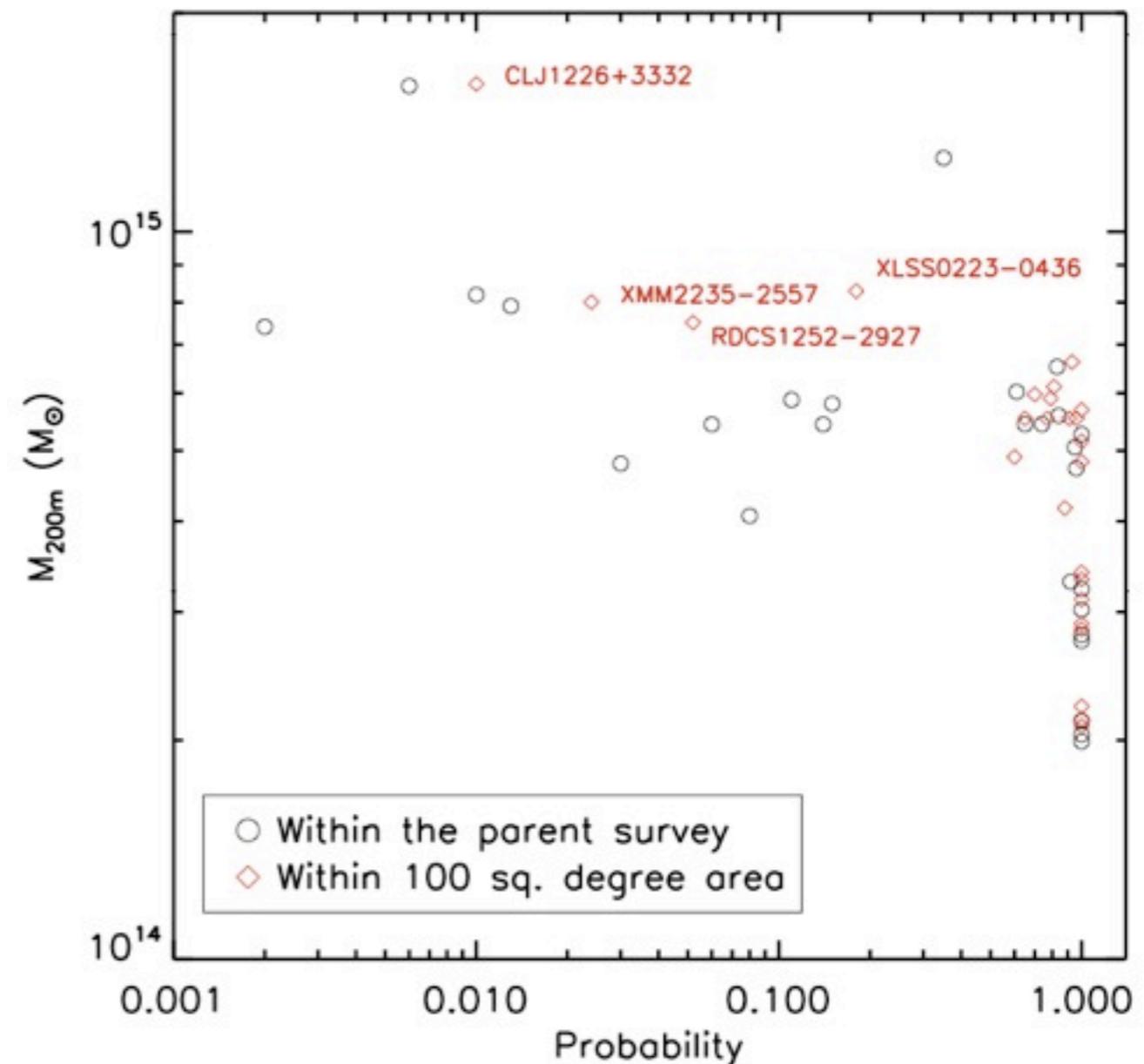
The (biased) $>M, >z$ analysis II

TABLE 3
DISCOVERY PROBABILITY OF GALAXY CLUSTERS

Cluster name	Within Parent Survey
XMMXCS J2215-1738	0.96
XMMU J2205-0159	1
XMMU J1229+0151	0.61
WARPS J1415+3612	0.65
ISCS J1432+3332	0.14
ISCS J1429+3437	0.15
ISCS J1434+3427	1
ISCS J1432+3436	0.11
ISCS J1434+3519	1
ISCS J1438+3414	0.92
RCS 0220-0333	0.74
RCS 0221-0321	1
RCS 0337-2844	0.84
RCS 0439-2904	0.95
RCS 2156-0448	1
RCS 1511+0903	1
RCS 2345-3632	1
RCS 2319+0038	0.83
XLSS J0223-0436	0.01
RDCS J0849+4452	0.03
RDCS J0910+5422	0.06
RDCS J1252-2927	0.002
XMMU J2235-2557	0.013
CL J1226+3332	0.006
MS 1054-0321	0.35
CL J0152-1357	1
RDCS J0848+4453	0.08

Jee et al 2011

Improved (HST WL) cluster mass estimates & less conservative (more realistic) survey footprints.



The ensemble of clusters was 'unlikely' to have been observed

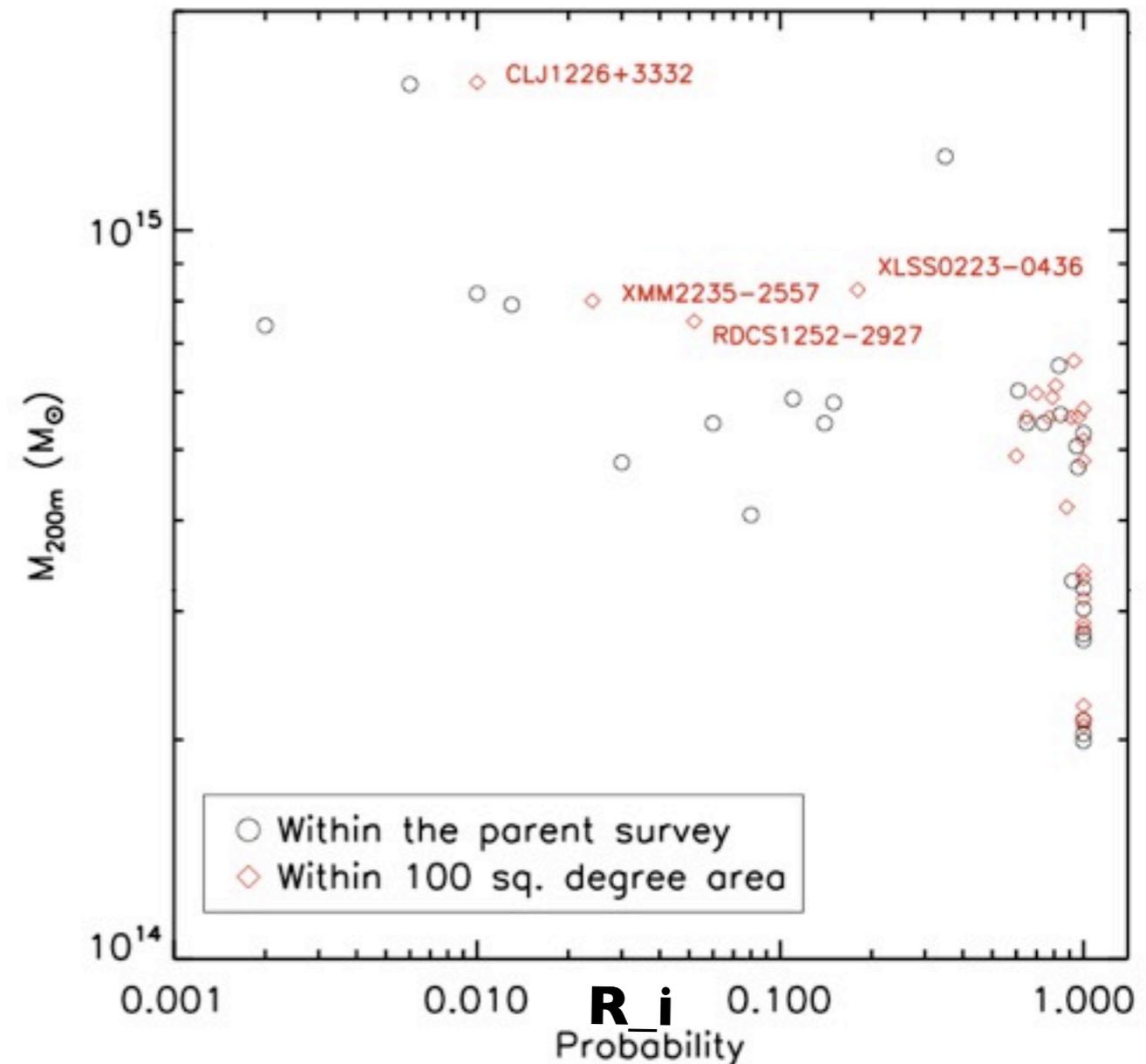
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(Biased) exclusion curves

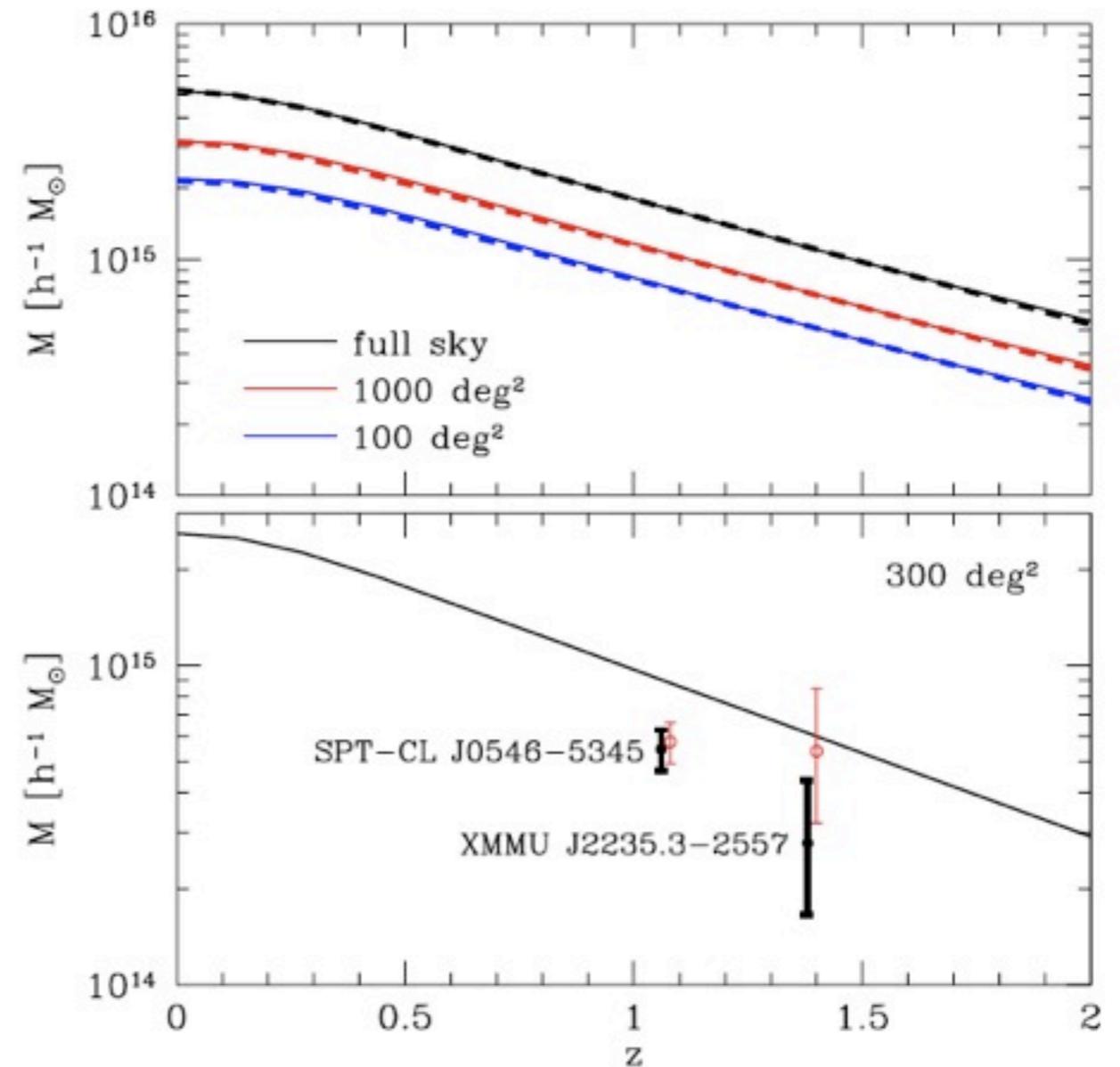
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Given the (w)LCDM model with WMAP7 cosmological priors, we do not expect any cluster to sit above the curve at 95% or some other specified confidence, (after fixing for selection functions and bias)

These lines were created by tracing constant values of $>M, >z$ existence probability R .



Mortonson et al 2010

Hotchkiss 2011, Hoyle et al 2011 identified a $>M, >z$ bias.

Unbiasing the $\langle M, \rangle z$ statistic I

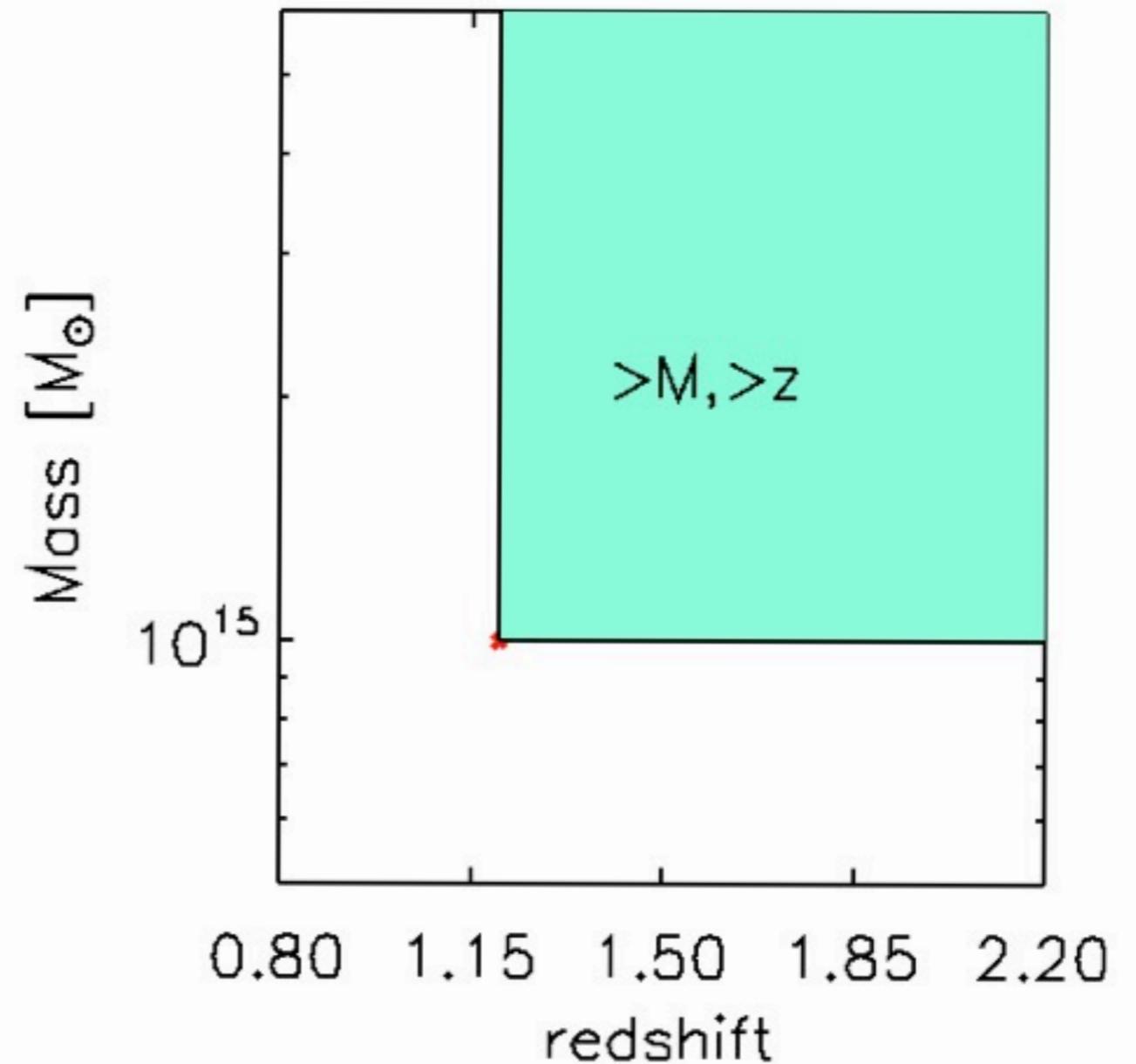
The bias in a nutshell: In previous literature, the question, a) What is the probability of finding a cluster(s) in this $\langle M, \rangle z$ box? referred to as “existence probability” R has been used as a proxy for what we actually want to know, b) “What level of tension with a model is caused by the existence of this cluster(s)?”

When stated like this, one can see that a) does not imply b).

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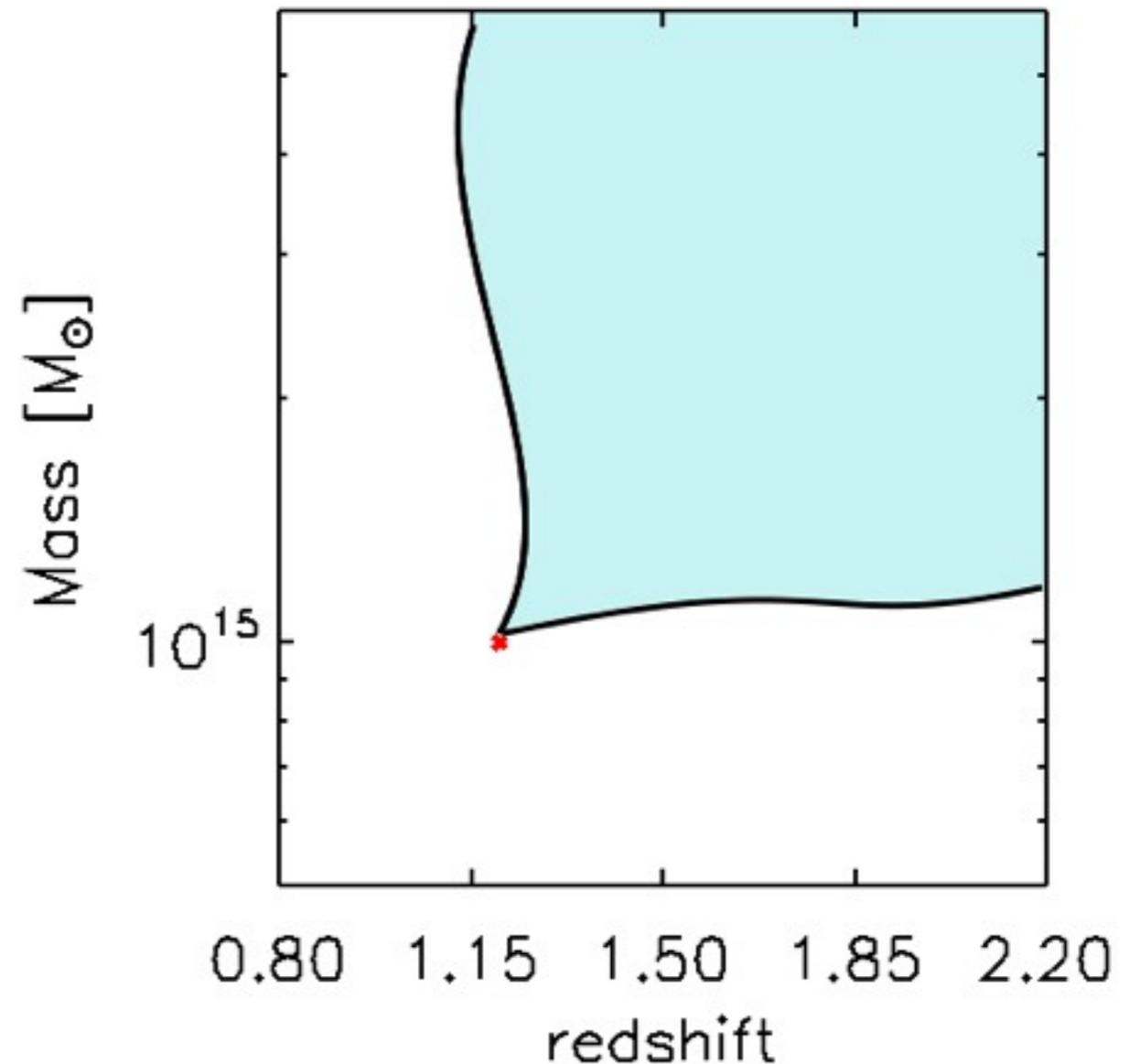
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Why this is wrong

Why should we restrict ourselves to the easily calculated, but arbitrary, $>M,>z$ contours, e.g, what dictates that the box should be placed at right angles to the (M,z) axis, or have straight instead of curved boundaries? One could simply modify the $>M,>z$ box and obtain a new “existence probability” R^* which would be equally as ‘justified’ as the original existence probability R .

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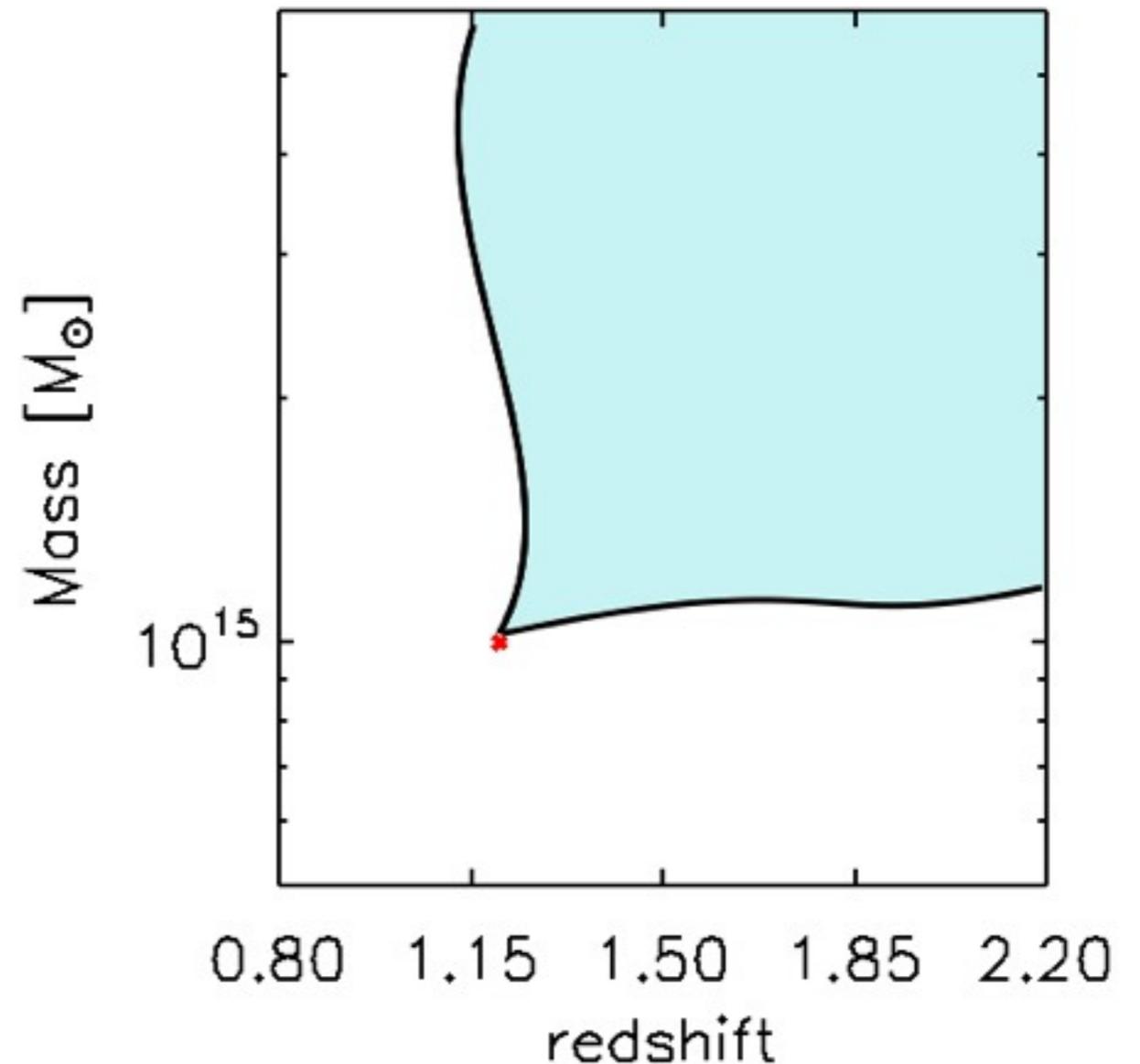
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Once the above is understood, we can calibrate R on simulations, and then use it to test for tension with Λ CDM.



Unbiasing the $>M, >z$ statistic II

Only necessary if we don't know the selection function (sf) of a survey. X-ray/ Weak lensing (actually SNe) Jee et al 2011 sample of clusters have a very complicated sf. Only the existence, not the absence, of clusters can constrain cosmology (as opposed to e.g., SPT, maxBCG, R400d).

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Extended sources not followed up => no redshifts or
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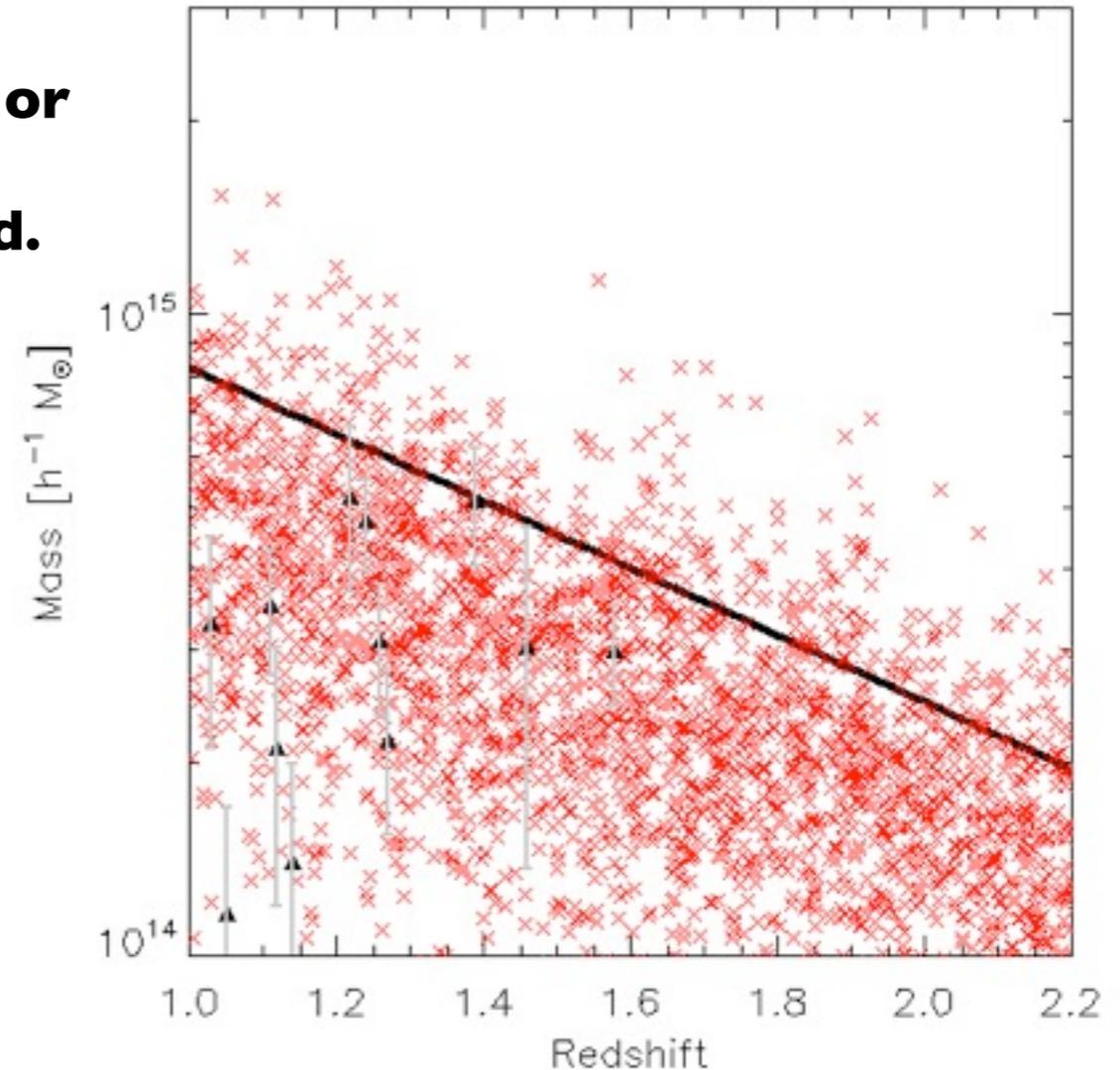
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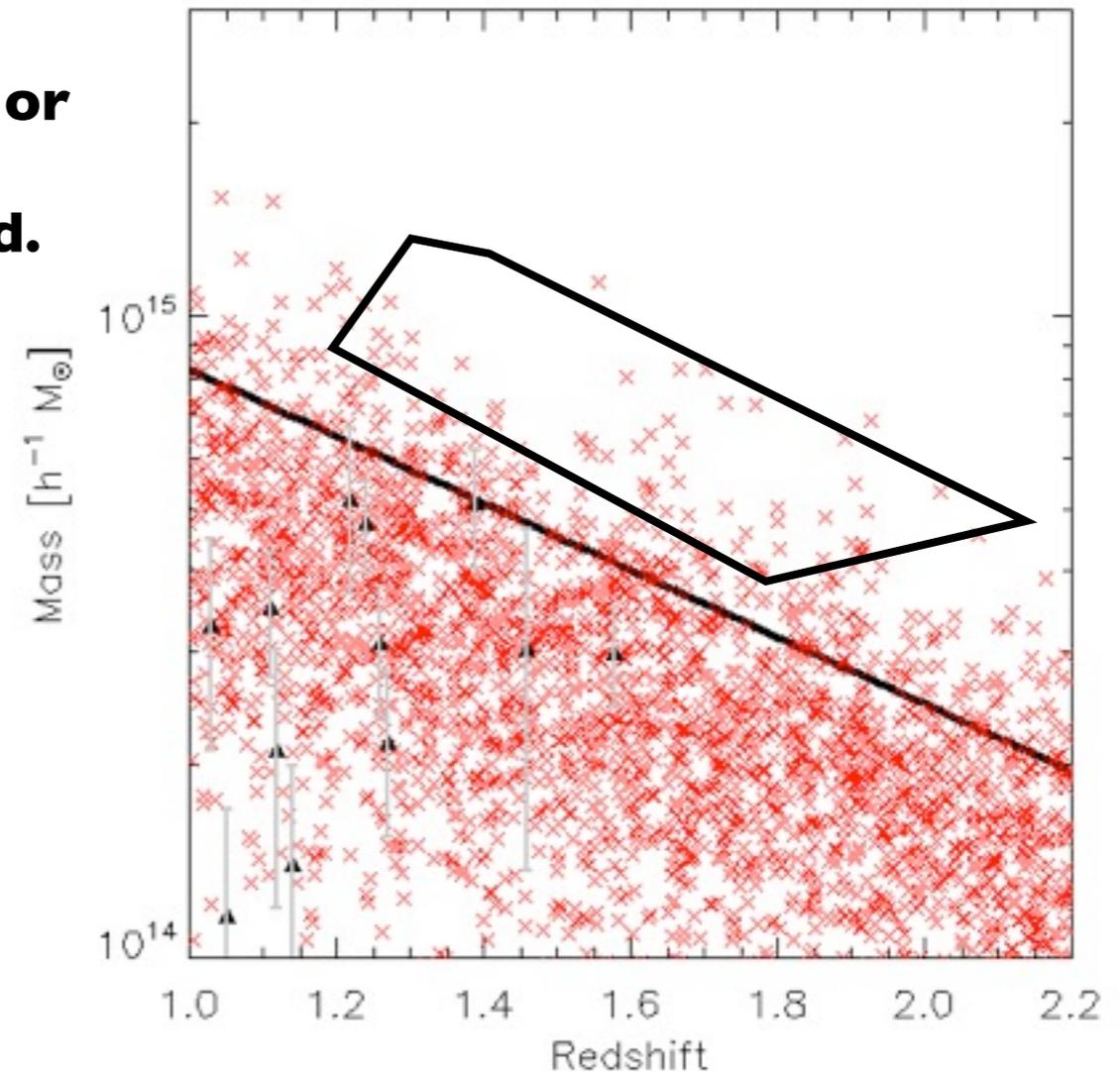
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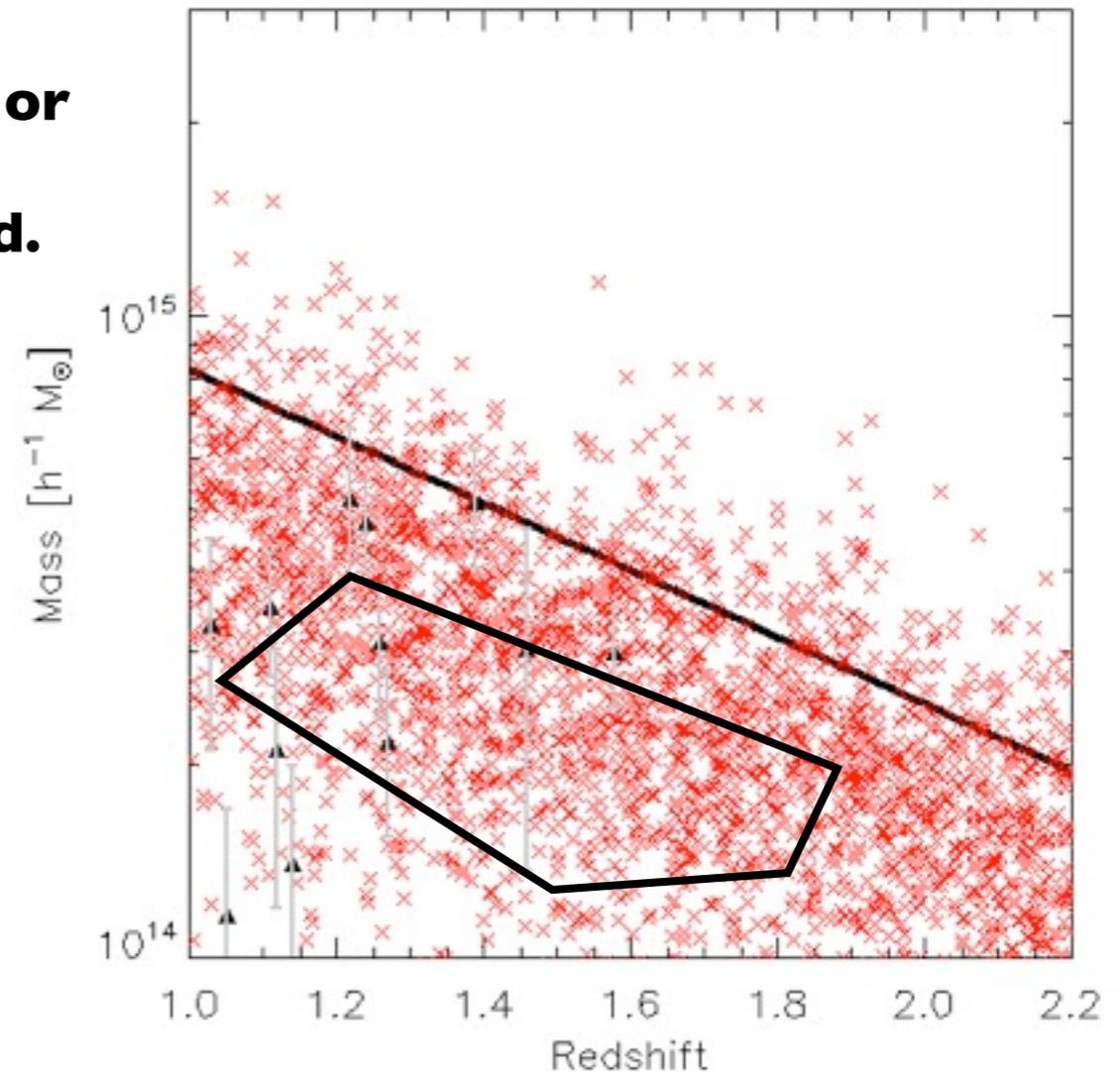
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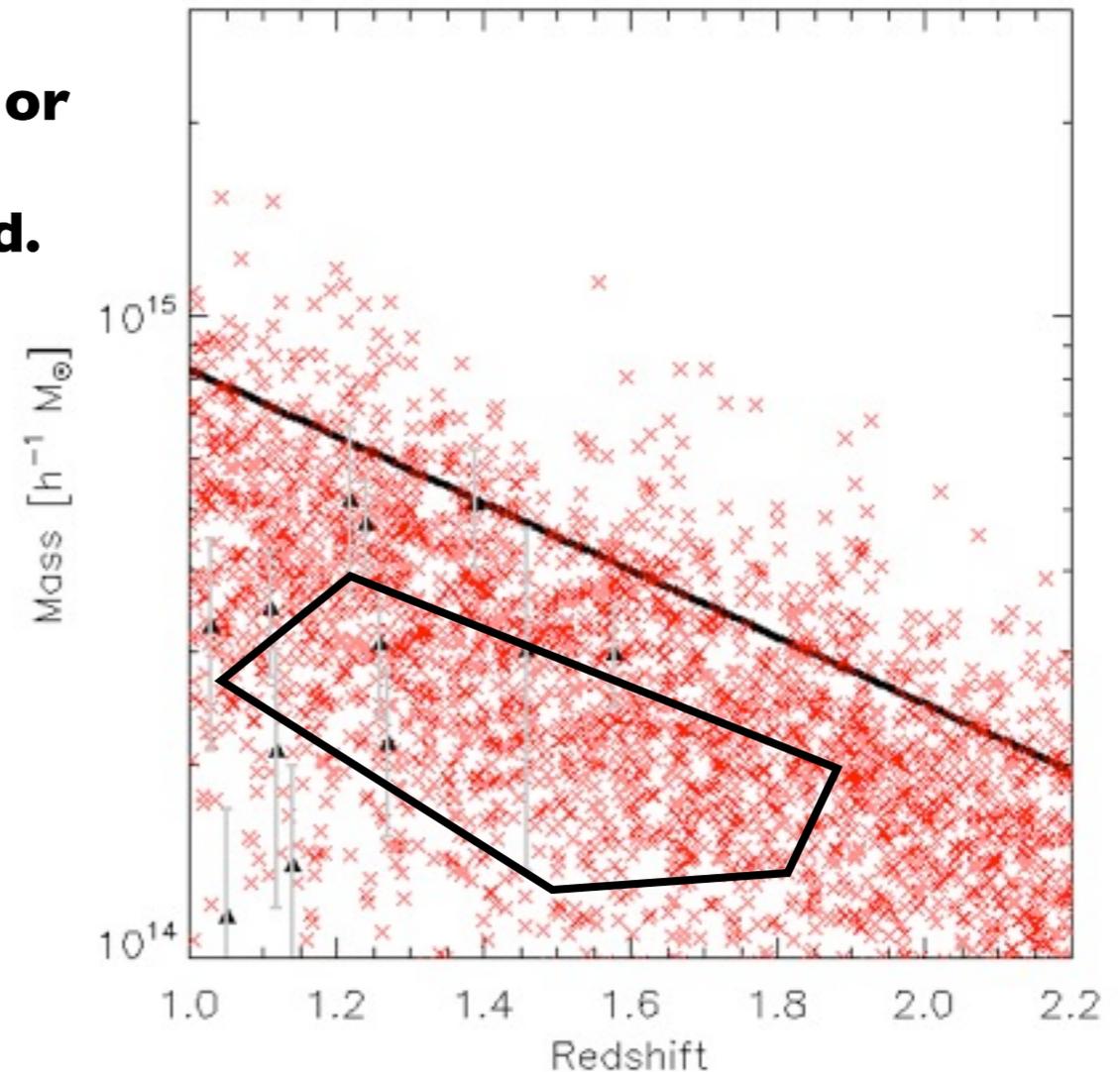
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To unbias $>M, >z$ analysis using simulated clusters, we must assume which part of the (M, z) plane has been "observed" (i.e., a sf).

Ongoing work to recover cosmological constraints using weaker assumptions about the selection function (Hoyle et al, in prep)



(Unbiased) γ_M, γ_z exclusion curves

Once unbiased, exclusion curves can be used to test for tension using only one cluster. But assumptions about survey geometry & σ also have to have been made.

(Unbiased) λ , μ exclusion curves

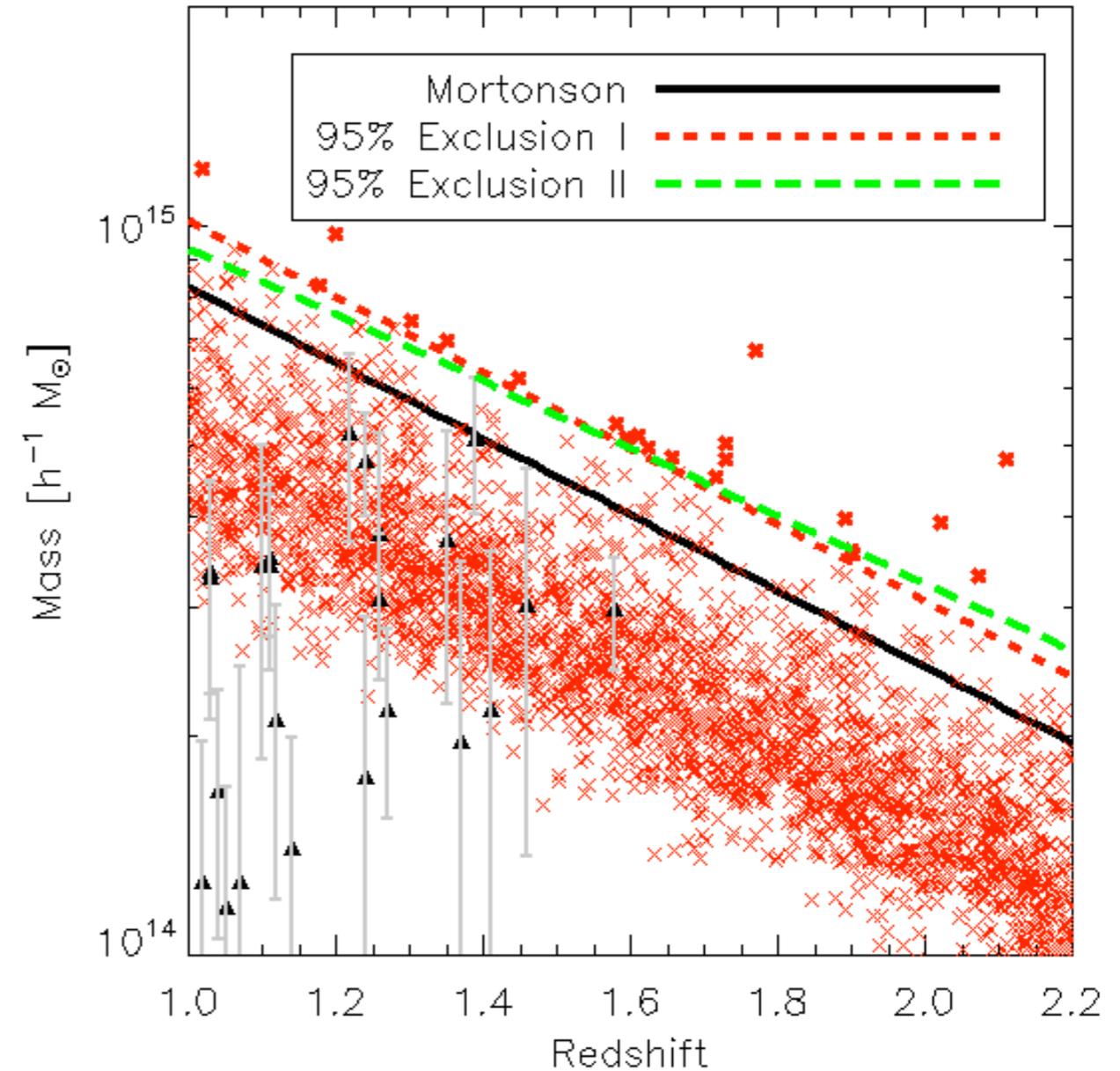
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- **Assume a λ / geometry**

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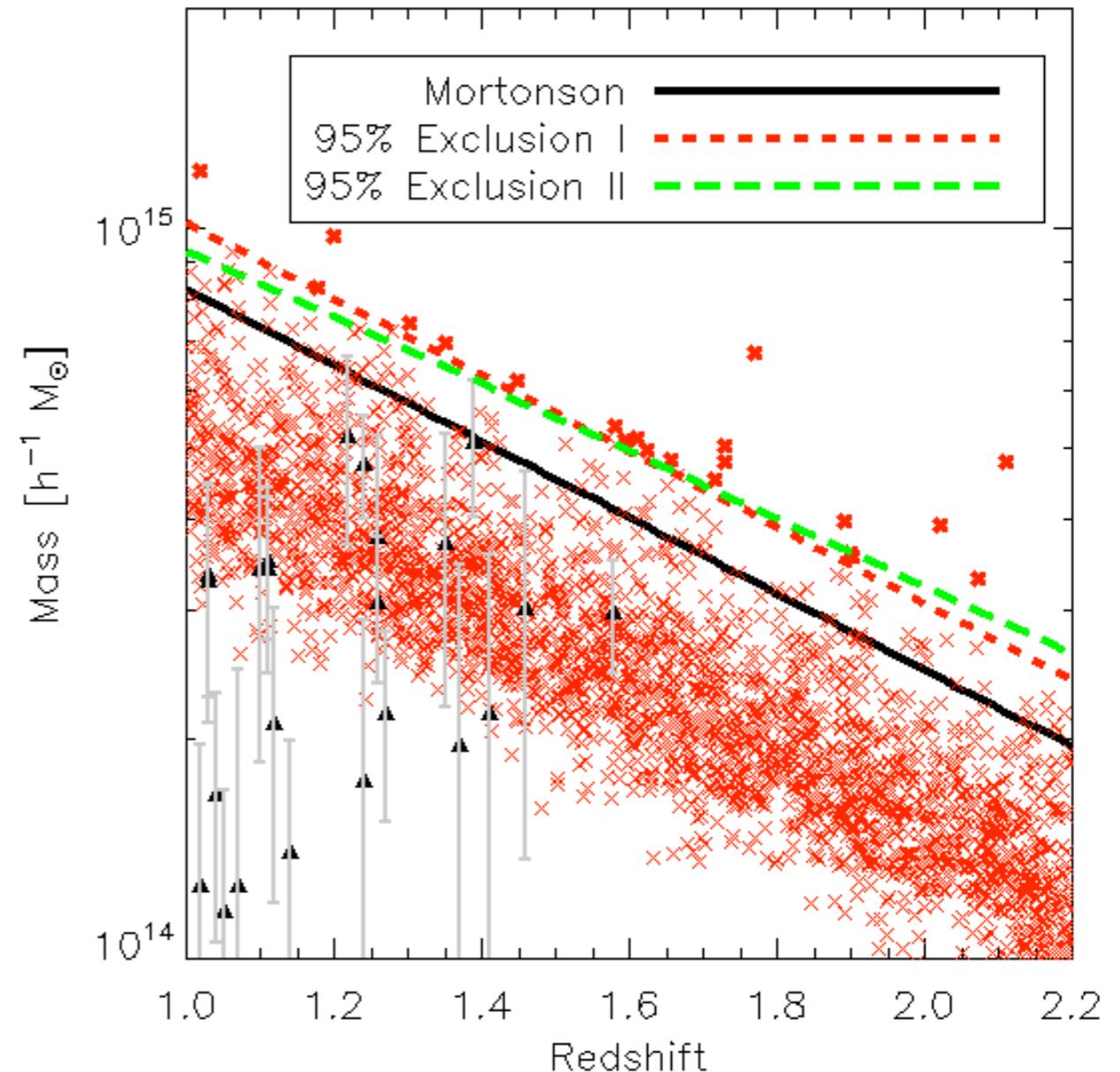
- Assume a sf / geometry
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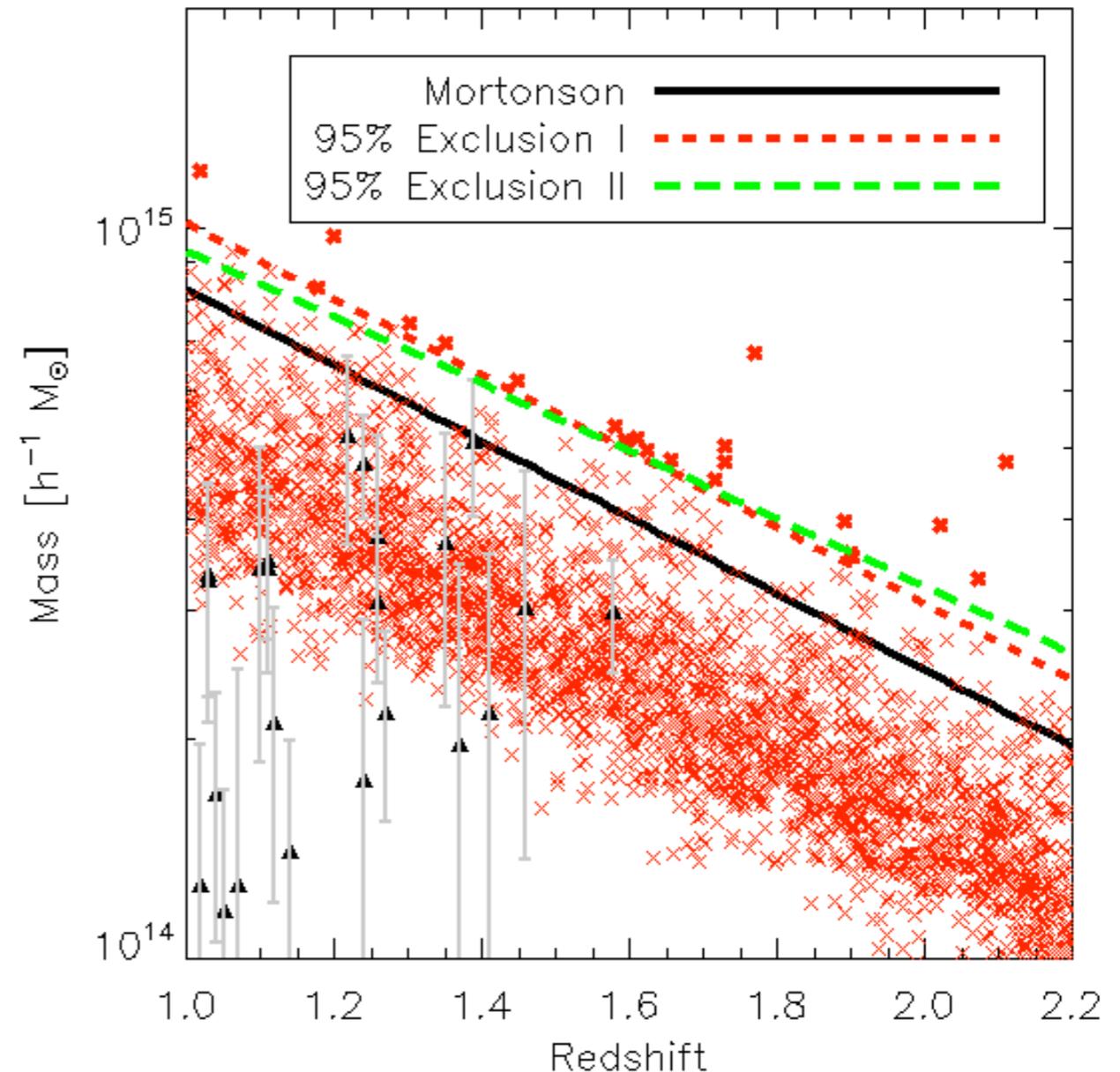


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- Draw a line which correctly excludes (e.g.) 95% of the simulated clusters
- But, this line is arbitrary!

Any inferred exclusion significance must be quoted together with the metric.



(see also Hotchkiss 2011)

Unbiased analysis/comparison with sim.

Cluster Name	Redshift	M_{200} $10^{14}M_{\odot}$	Method	\bar{R}	Mass reference
RCS0221-0321	1.02	$1.80^{+1.30}_{-0.70}$	WL	0.992	[15]
WARPSJ1415+3612	1.03	$4.70^{+2.00}_{-1.40}$	WL	0.706	[15]
RCS0220-0333	1.03	$4.80^{+1.80}_{-1.30}$	WL	0.709	[15]
RCS2345-3632	1.04	$2.40^{+1.10}_{-0.70}$	WL	0.989	[15]
XLSSJ022403.9-041328*	1.05	$1.66^{+1.15}_{-0.38}$	X-ray	0.997	[31]
RCS2156-0448	1.07	$1.80^{+2.50}_{-1.00}$	WL	0.916	[15]
RCS0337-2844	1.10	$4.90^{+2.80}_{-1.70}$	WL	0.567	[15]
RDCSJ0910+5422	1.11	$5.00^{+1.20}_{-1.00}$	WL	0.595	[15]
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XMMUJ2205-0159	1.12	$3.00^{+1.60}_{-1.00}$	WL	0.888	[15]
RXJ1053.7+5735(West)	1.14	$2.00^{+1.00}_{-0.69}$	X-ray	0.989	[31]
XLSSJ0223-0436	1.22	$7.40^{+2.50}_{-1.80}$	WL	0.119	[15]
RDCSJ1252-2927	1.24	$6.80^{+1.20}_{-1.00}$	WL	0.094	[15]
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ISCSJ1429+3437	1.26	$5.40^{+2.40}_{-1.60}$	WL	0.327	[15]
RDCSJ0849+4452	1.26	$4.40^{+1.10}_{-0.90}$	WL	0.517	[15]
RDCSJ0848+4453	1.27	$3.10^{+1.00}_{-0.80}$	WL	0.839	[15]
ISCSJ1432+3436	1.35	$5.30^{+2.60}_{-1.70}$	WL	0.265	[15]
ISCSJ1434+3519	1.37	$2.80^{+2.90}_{-1.40}$	WL	0.636	[15]
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ISCSJ1438+3414	1.41	$3.10^{+2.60}_{-1.40}$	WL	0.584	[15]
XMMXCSJ2215-1738	1.46	$4.30^{+3.00}_{-1.70}$	WL	0.335	[15]
XMMUJ0044.0-2033**	1.57	$4.25^{+0.75}_{-0.75}$	X-ray	0.152	[30]

Cluster mass measurements from Jee et al 2009, 2011, Santos et al 2011, Stott et al 2010

Realistic assumptions

X-ray survey footprint 100 sq. deg. $1.0 < z < 2.2$ (Jee et al 2011)

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Compare to simulations

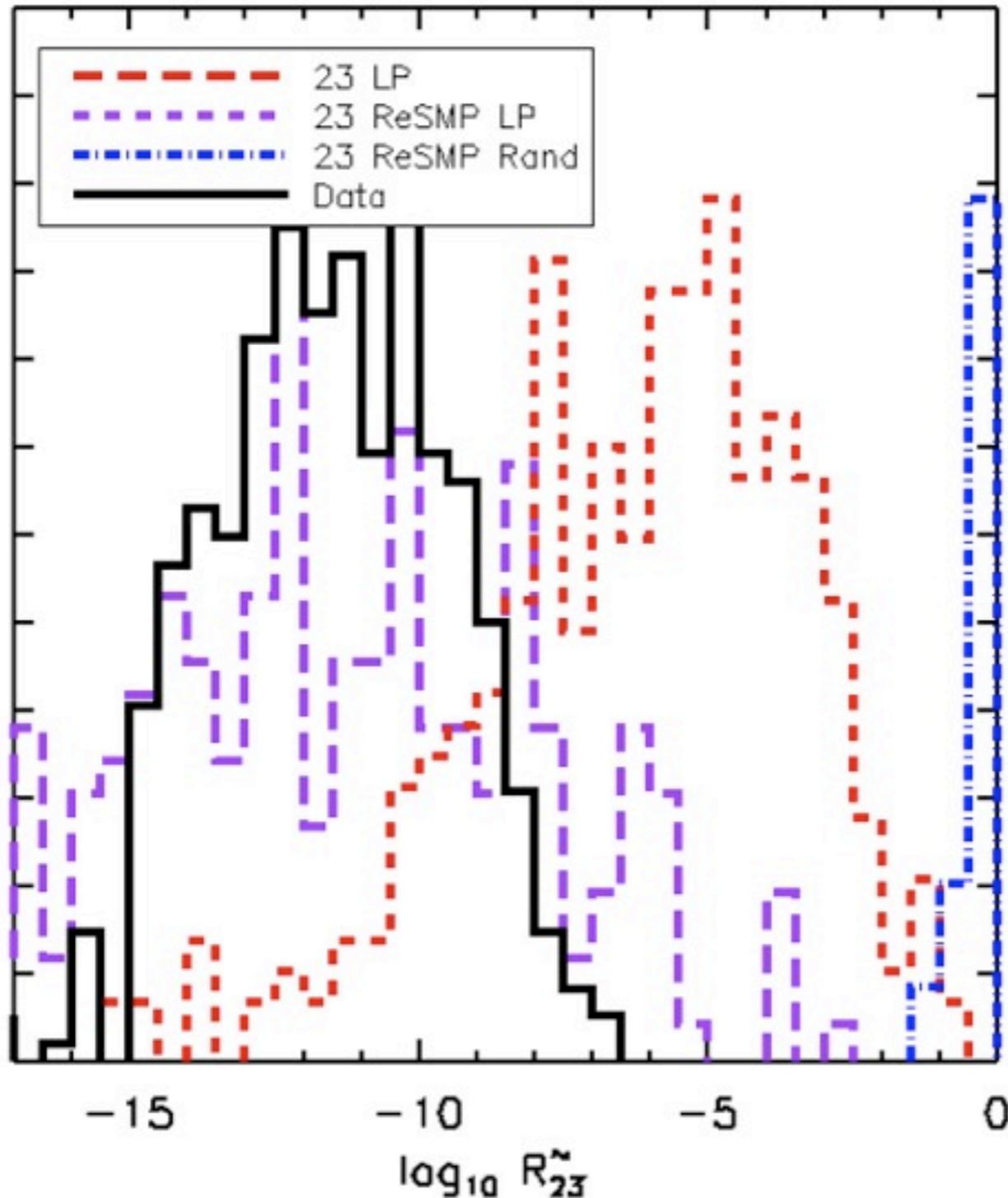
• 450 sets of simulated clusters; Poisson samplings from mass function, vary cosmological parameters within WMAP7 priors.

B.H., Jimenez, Verde, Hotchkiss (2011, JCAP)

• Calculate R for each simulated cluster.
• Identify sets of LP clusters and random clusters.

The (unbiased) $>M, >z$ analysis

Compare the distribution of ensemble probabilities R_{23} , from the data, with the ensemble probabilities from sets of simulated clusters.



No R_{23} tension if the observed clusters are consistent with being drawn from the LP re-sampled clusters.

Massive tension if the observed clusters are drawn from a random sample.

=>

We can't claim tension, but we can't also immediately rule it out without determining which sample of simulated clusters (LP or rand or other) the observed clusters are consistent with being drawn from.

The 2d K-S test

To compare two 2-d distributions we can use 2d Kolmogorov-Smirnov test to calculate the probability that two 2d data sets are drawn from the same parent population. We compare the distribution in the (M,z) plane of the 23 LP clusters from each simulation with each other (varying WMAP7 cosmology) and with the data (after sampling from the mass and error), and 23 randomly selected simulated clusters with the data.

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S1(M,z)	S2(M,z)	$\langle \log P \rangle_{f_{NL}^{-200}}$	$\langle \log P \rangle_{f_{NL}^0}$
Sim P_{LP}	Sim P_{LP}	-0.79 ± 0.67	-0.81 ± 0.72
D^x	Sim P_{LP}	-3.24 ± 0.97	-3.33 ± 0.96
D^x	Sim P_{RAND}	-5.09 ± 1.08	-4.94 ± 1.08
S1(M,z)	S2(M,z)	$\langle \log P \rangle_{f_{NL}^{200}}$	$\langle \log P \rangle_{f_{NL}^{400}}$
Sim P_{LP}	Sim P_{LP}	-0.82 ± 0.70	-0.84 ± 0.73
D^x	Sim P_{LP}	-3.36 ± 0.94	3.50 ± 0.91
D^x	Sim P_{RAND}	-4.85 ± 1.186	-4.70 ± 1.13

- **The simulated LP clusters are consistent with each other ($P=0.2$, $10^{-0.7}$)**
- **The simulated LP clusters are not consistent with the observed clusters ($P=0.001$)**
- **But, the observed clusters are less likely still to be consistent with a randomly selected simulated clusters.**

Recall: If LP no Rn tension, if random lots of Rn tension

Main results

The (unbiased) χ^2 statistic tells us that if the observed clusters are consistent with being the LP clusters (compared with simulations), all tension has been removed. But the 2dK-S test probability, show that this is very unlikely.

The observed clusters are inconsistent with a random selection of clusters (from simulations). The χ^2 R statistic is very discrepant, and the 2dK-S test probabilities are very low.

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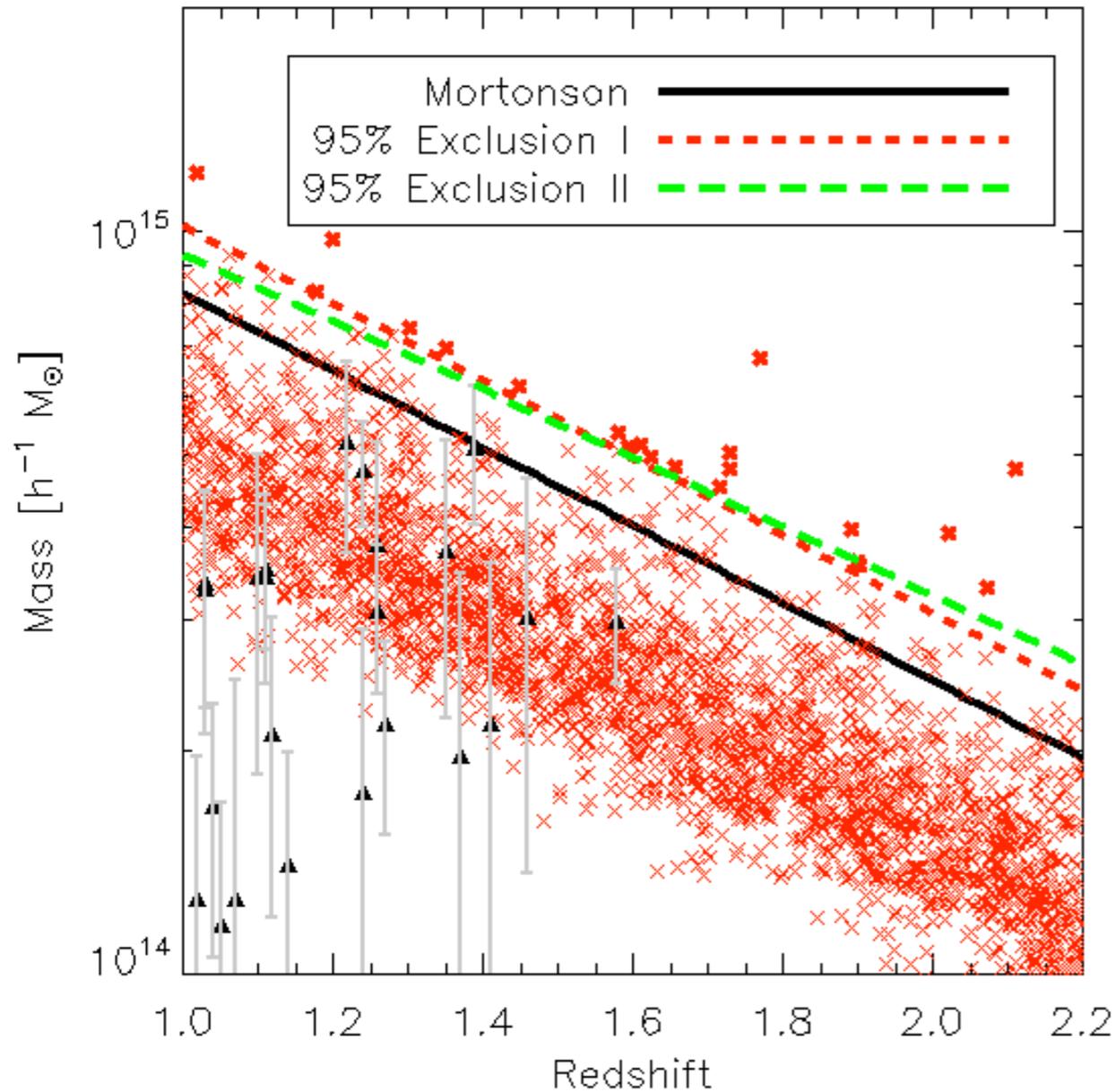
What could alleviate this discrepancy? Selection function/survey geometry?

Possible causes.

All clusters have $z > 1.6$. If we modify the assumed survey geometry, by imposing a hard cut to our simulations, the comparison between observations and simulations begins to agree.

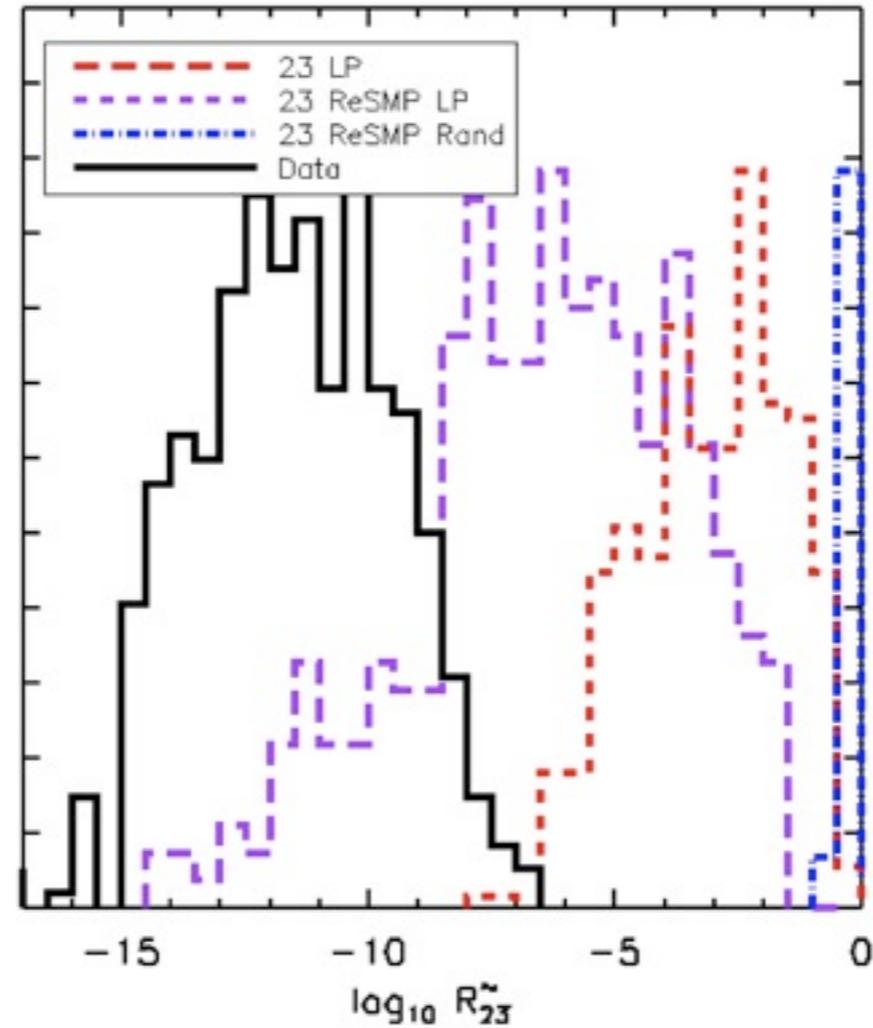
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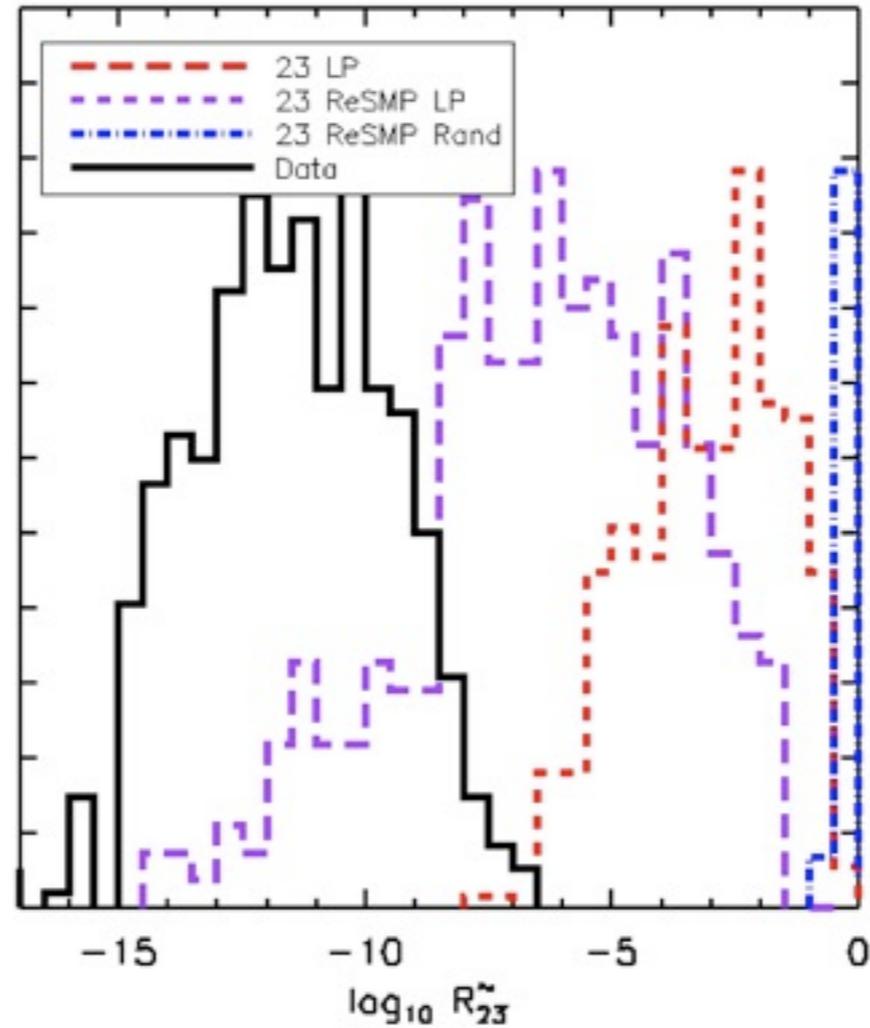
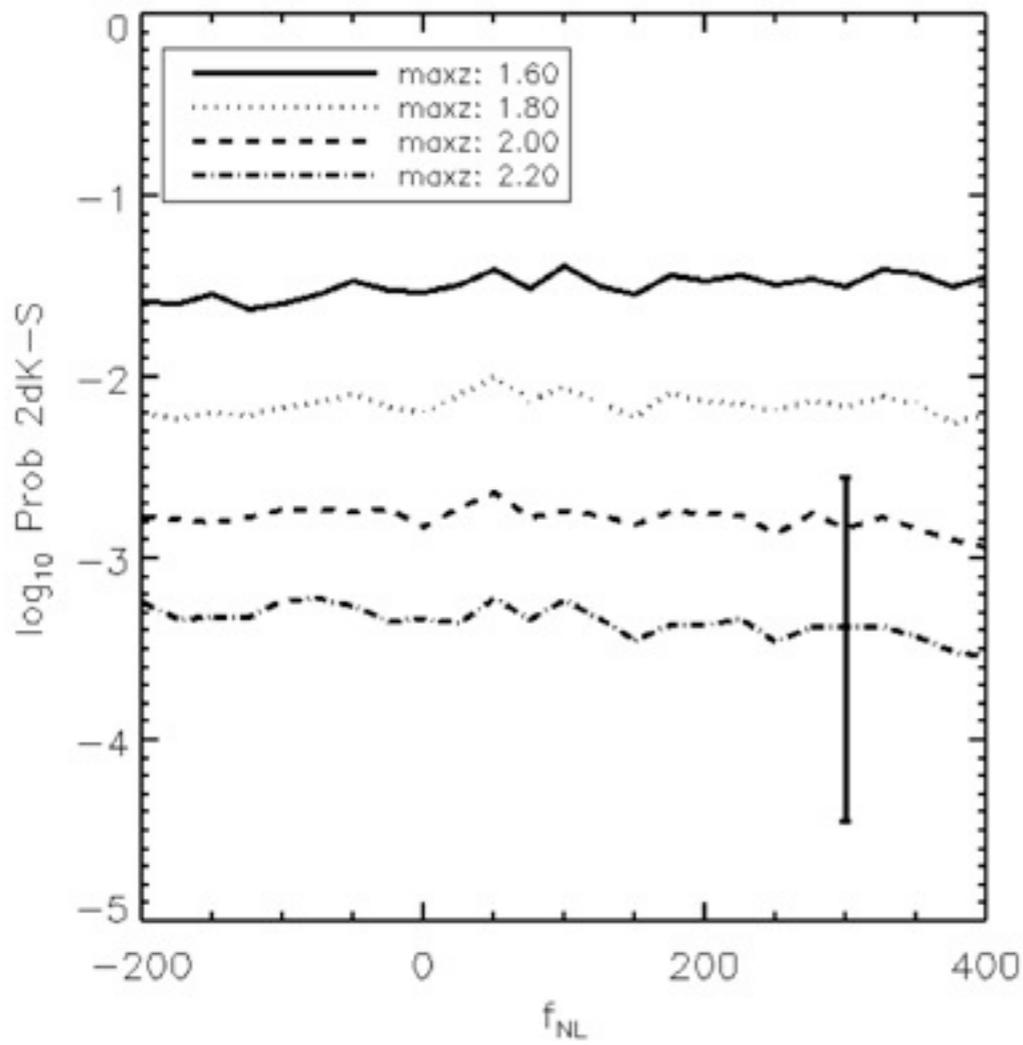
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All clusters have $z > 1.6$. If we modify the assumed survey geometry, by imposing a hard cut to our simulations, the comparison between observations and simulations begins to agree.



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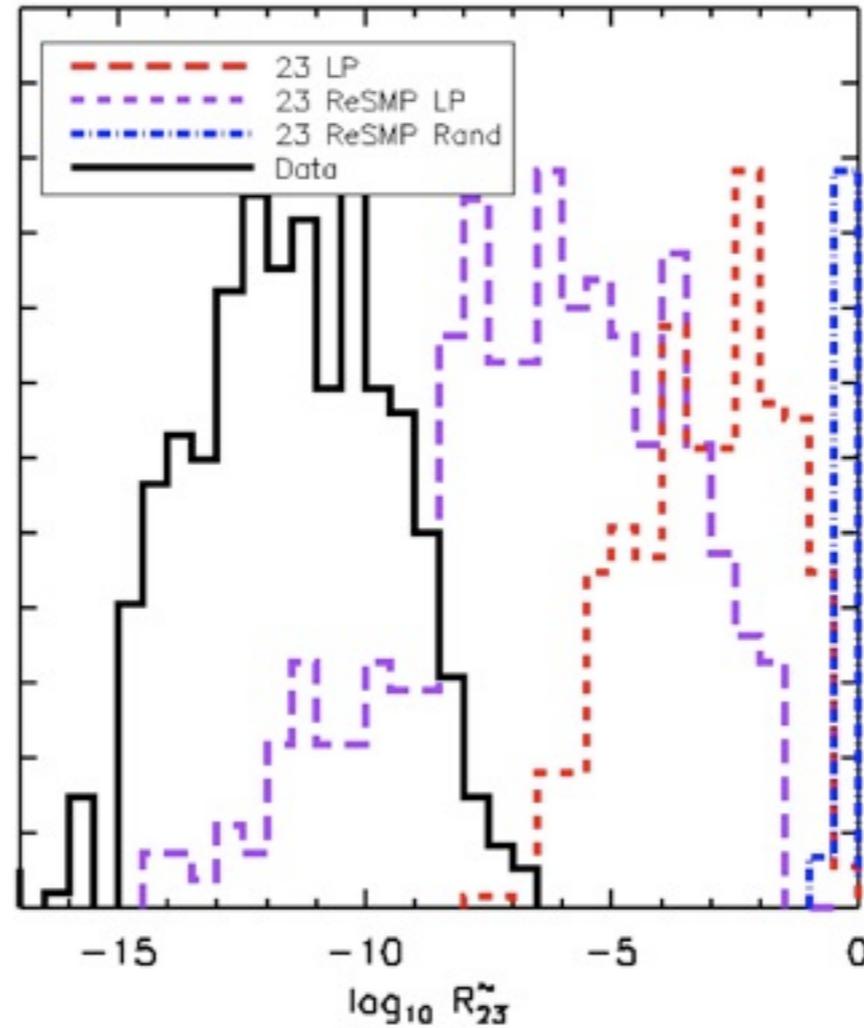
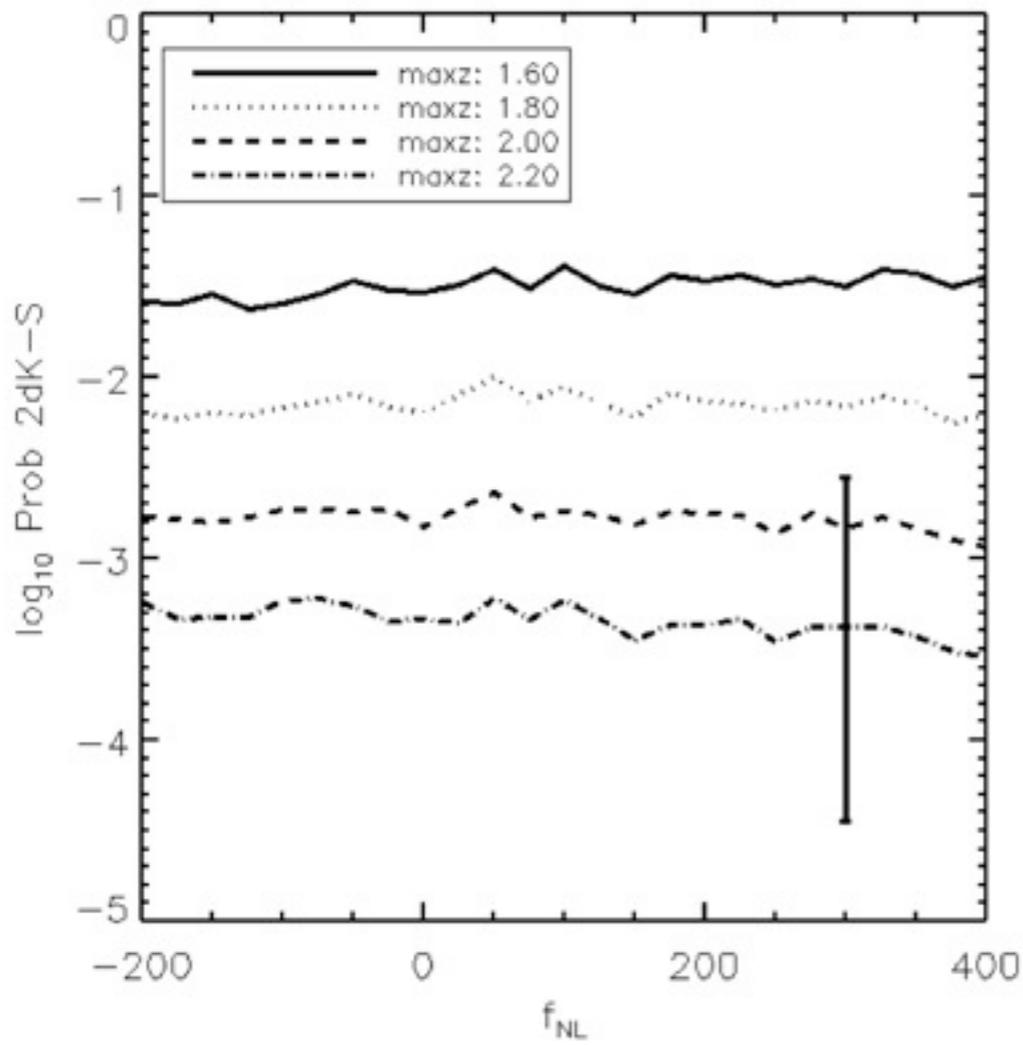
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**But recall, $z=2.07$, $M \sim 5-8 \cdot 10^{13}$
SolMass, Gobat et al arXiv:1011.1837**

Summary

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 - **Addressed the biases, and suggested fixes for the common exclusion curves.**
 - **Built a list of high-redshift ($z > 1$) massive ($M > 10^{14}$ solar mass) clusters.**
 - **Used a realistic footprint/survey geometry.**
 - **Compared observed clusters with distributions of simulated clusters.**
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- Showed that fnl cannot reduce the tension when properly compared to simulations.

These clusters may still be causing some tension with LCDM assuming WMAP priors on cosmological parameters, but more investigation into the selection functions are needed.

- More high-redshift, massive clusters are being found ~weekly. Apex/Planck/XCS. We have built a statistical framework to understand what they tell us about LCDM.

Follow up work: To use samples of clusters with an unknown selection function to bound cosmological parameters (Hoyle et al, in prep.)