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Unified stellar models and convection in cool stars

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Abstract. The formulation of *boundary conditions* can have a significant influence on the solution of a system of differential equations. It is therefore important to apply a most realistic representation of the surface boundary conditions to the equations of stellar structure and evolution. With respect to previous models that usually employ some estimate of the surface temperature drawn from the Eddington approximation, a significant improvement of the outer boundary conditions is achieved by connecting models of stellar atmospheres to stellar structure models.

Up to now stellar evolution calculations for late-type stars are calibrated using the well-observed properties of the *present* Sun. Including the physics of a plane-parallel atmospheric stratification it is necessary to account for a consistent description of the convective energy transfer in the outer layers of a cool star. At this step an apparent contradiction of the observations must be resolved: spectroscopic analyses of the Balmer lines emerging from solar-type stars using line-blanketed model atmospheres are usually carried out with reference to the Böhm-Vitense convection theory. To fit simultaneously the profiles of Hα and Hβ as well as higher series members a small mixing-length parameter $\alpha = l/H_p = 0.5$ is required. Models calibrated to the present Sun instead imply that the internal structure of the Sun follows a substantially higher value of $\alpha = 1.5$.

This discrepancy cannot be removed in the context of Böhm-Vitense's convection theory. It is shown that the convection model of Canuto & Mazzitelli fits both the observed *present* Sun and the Balmer lines with a single common mixing-length parameter. The convection theory of Canuto & Mazzitelli thus offers for the first time a unified physical model of the Sun that is valid from the center to the upper photosphere.

Key words: Convection - sun: atmosphere - evolution - stars: atmospheres - evolution - late-type

1. Introduction

The differential equations of stellar structure respond to boundary conditions both at the center and at the surface. In structure calculations the center conditions are easy to treat whereas an accurate solution taking account of the surface boundary conditions is more complicated. Approaching the outer layers of a star the gas is gradually becoming optically thin, and as a result the diffusion approximation of radiative energy transport will no longer be valid. Furthermore, the location of the stellar 'surface' is not as simple as it is for the center. Whereas the theory of stellar atmospheres provides the tools for accurately computing stellar envelopes such calculations are time-consuming. It is therefore customary to use some simplifying approximations.

The method described by Kippenhahn et al. (1967, 1990) is widely used. The treatment of the atmosphere is separated from that of the interior structure at a certain cut or fit point. Integration of the hydrostatic equation

$$\frac{\mathrm{d}\bar{\tau}}{\mathrm{d}p} = \bar{\kappa}\,(\rho, T)\,\frac{R^2}{GM} \tag{1}$$

along the Rosseland optical depth $\bar{\tau}$ using the well-known Eddington temperature law Eq.(2) and the ideal gas equation results in radius and pressure at the atmospheric fit point (see Sect.2.1). The Henyey algorithm needs two outer boundary conditions which are now available. Generally, the fit point is situated at optical depth $\bar{\tau} = 2/3$, where the diffusion approximation is assumed to be a valid description of radiative energy transfer. But this method has two important disadvantages. First the diffusion approximation is definitely not valid at a fit point at $\bar{\tau} = 2/3$. Second the Eddington temperature law, which is used for the integration of Eq.(1), is also derived using the diffusion approximation. The influence of these approximations on the solution of the stellar differential equations cannot be predicted in a simple way.

Morel et al. (1994) postulate that only with a reference star model well constrained by observations the influence can be revealed, and only for the Sun one has such accurate values for age, radius, mass, effective temperature and luminosity. In their careful investigation of the Sun on her way from the zero-age main sequence to present age they conclude that the diffusion approximation is not valid outside Rosseland optical depth $\bar{\tau} \approx 10$. They extract several temperature stratification laws from different model atmospheres and restore them in their stellar structure calculations. Finally, they compare the temperature stratifications from Eq.(2), Kurucz' (1992) ODF model atmosphere program ATLAS9, and the empirical

Harvard-Smithsonian Reference Atmosphere (HSRA) of Gingerich et al. (1971). The latter one suffers from the difficulty to establish the temperature distribution empirically at great optical depths. Gingerich et al. admit that their temperature is possibly 200 K too hot at $\bar{\tau} = 10$. The disadvantage of the Eddington stratification is already mentioned above, and therefore only the temperature stratifications of ODF models are sufficiently realistic. However, Morel et al. did not use one important piece of information in their investigation which seems to be well constrained by observation.

The spectrum of a star is formed in the stellar atmosphere and therefore any model atmosphere connected to the stellar structure model should represent the observed spectrum. From spectroscopic observations of the Sun, Procyon, and the two metal-poor stars HD 140283 and G41-41 Fuhrmann et al. (1993) demonstrated that a precise determination of a cool star's effective temperature is determined by fitting the observed Balmer lines to the theoretical line profiles. A consistent fit to *all* Balmer lines requires an adjustment of the temperature stratification in the inner photosphere between $\bar{\tau} = 1$ and 10, which in turn forces the convective mixing-length parameter to $\alpha = 0.5 \pm 0.3$ using the convection model of Vitense (1953) and Böhm-Vitense (1958). This has recently been confirmed by Steffen et al. (1995), who carried out 2D hydrodynamic calculations and van't Veer-Menneret & Mégessier (1996) who used the Kurucz ATLAS9 program and derived a mixing-length parameter of $\alpha = 0.5$ for the Sun and for Procyon in full agreement with Fuhrmann et al. (1993).

In contrast Morel et al. worked with ATLAS9 for some temperature stratifications always with a mixing-length parameter $\alpha > 1.6$ which is at variance with the observation of the Balmer lines. Van't Veer-Menneret & Mégessier point at this discrepancy between the low value of $\alpha = 0.5$ compared with a mixing-length parameter of 1.8 derived by Morel et al. for the Sun as calibration star, who found that the parameter $\alpha = 1.8$ is the optimal choice to connect the atmosphere and the thermodynamic quantities associated with the solar convection zone to a satisfying accuracy. This paper will overcome the dichotomy of two mixing-length parameters, and the conclusion will be that the problems cannot be solved with the convection model of Böhm-Vitense. Replacement of convective energy transfer by the model of Canuto & Mazzitelli (1991, 1992) will remove the outer boundary problem and lead to unified model of cool stars.

A basic description of the programs and the method connecting the model atmospheres to the stellar structure model is given in Sect. 2. Section 3 discusses the attempt to produce a consistent convective flux using the convection models of both Böhm-Vitense and Canuto & Mazzitelli. In Sect. 4 the results of the calculations and the conclusions are presented.

2. Model interface between stellar interior and atmosphere

The calculations described in this paper are based on the Henyey algorithm as adapted in the Kippenhahn et al. (1967) code. The program has recently been modified and described by Wagenhuber & Weiss (1994), who used the traditional way of connecting atmospheres to stellar structure models at $\bar{\tau} = 2/3$. In this former version the integration of Eq.(1) starts at $\bar{\tau} = 0$ where the total gas pressure p equals the radiation pressure. The density $\rho(p, T)$ and the opacity $\bar{\kappa}(\rho, T)$ are evaluated with the approximation of the well-known Eddington temperature law,

$$T^4 = \frac{3}{4} T_{\text{eff}}^4 \left(\bar{\tau} + \frac{2}{3} \right) . \tag{2}$$

and the ideal gas equation as equation of state. The integration is followed out to a fit point at optical depth $\bar{\tau} = 2/3$. The local kinetic temperature at that depth is equal to the effective temperature T_{eff} . The radius $R_\star$ of the star is defined with the Stefan-Boltzmann law and the total luminosity of the star $L_\star$.

$$L_\star = 4\pi R_\star^2 \sigma T_{\text{eff}}^4 \tag{3}$$

Now radius and pressure at the atmospheric fit point are available as functions of luminosity and effective temperature, $R_{\text{f}}(L_\star, T_{\text{eff}})$, and $p_{\text{f}}(L_\star, T_{\text{eff}})$. The interior part of the stellar structure can be calculated by the well-known Henyey scheme, and for a consistent connection to the atmosphere the outermost grid point in that scheme must be placed at $\bar{\tau} = 2/3$. Furthermore the physical parameters of the atmosphere and of the inner solution must 'fit' at this point and the total mass of the stellar model should be the sum of the atmospheric and the interior mass. In principle the conservation of total mass can be achieved by integrating the atmospheric mass and varying the inner mass by inserting or deleting the outermost gridpoints in the Henyey scheme. But often the atmospheric mass can be ignored with respect to the total mass. The physical parameters then can be fitted as follows. Luminosity and temperature from the last Henyey iteration are taken at the outermost gridpoint, and thus the temperature is equal to the effectiv temperature. From atmospheric integration R_{f} and p_{f} are now calculated, and they must fit the radius R_{o} and the pressure p_{o} at the outermost gridpoint of the Henyey scheme. The Henyey algorithm needs two outer boundary conditions which are now available ($R_{\text{f}} - R_{\text{o}} = 0, p_{\text{f}} - p_{\text{o}} = 0$) and the fit will be achieved with convergence of the Henyey iteration scheme.

Because of the arguments stated in the introduction it is recommended to replace this boundary condition by a more appropriate one.

2.1. New outer boundary conditions of the Henyey scheme

The principal difference between the boundary condition as described in the part before and those reported here is that in the latter the fit point is moved to $\bar{\tau} = 20$ which, according the examination of Morel et al. , is deep enough to establish the validity of the diffusion approximation.

An optical depth of $\bar{\tau} = 20$ relates to an atmospheric depth of only 100 - 200 km in the Sun. Therefore in *quiet* evolution of cool unevolved stars (i.e. neglecting pulsations) the atmosphere is neither a source nor a sink for the energy, and it follows that the luminosity at the fit point is equal to the luminosity of the atmosphere, $L_{\text{f}} = L_\star$. The same argument of a negligible extension of the atmosphere is used to evaluate the radius of the star at the fit point such that $R_\mathrm{f} = R_\star$. The error of this approximation can be estimated with help of Eq.(3). Varying the radius of the present Sun by 300 km leads to a temperature change of less then 2 K.

The modified outer boundary conditions thus require the interesting physical variables of a grid of stellar atmosphere models to be available (see Sect.2.2). These variables are the atmospheric pressure p_f and temperature T_f at the fit point, the effective temperature T_eff , radius $R_\star$, luminosity $L_\star$ and the mass M_a of the stellar atmosphere. The Henyey algorithm provides its own four physical variables at the fit pont: $L_\mathrm{o}, T_\mathrm{o}, p_\mathrm{o}$, R_o. Luminosity and temperature are chosen to be independent, which means that the atmospheric table entry must fit both variables, $L_\mathrm{f} = L_\mathrm{o}$, and $T_\mathrm{f} = T_\mathrm{o}$. If that operation is successful one obtains from the tables the effective temperature of the star and the two atmospheric fit point variables R_f and p_f. Finally, the two outer boundary conditions for the Henyey scheme are available ($R_\mathrm{f} = R_\mathrm{o}$, $p_\mathrm{f} = p_\mathrm{o}$), and the Henyey iterations will converge to these values. A two-dimensional Taylor expansion is used to interpolate the atmospheric tables data with high accuracy.

2.2. Atmospheric models

The models of the stellar atmospheres have been calculated using a plane-parallel stratification with energy transported by radiation and convection and conserved through the atmosphere (Gehren 1977). In addition the program models line-blanketing with opacity distribution functions (ODF) provided by Kurucz (1979, 1995, and references therein). The basic input parameters are the effective temperature T_eff , the gravitational surface acceleration $\log g$, and a set of abundance ratios by number, X/H, where X refers to all *elements* except hydrogen. $\log g$ relates a given stellar mass to its radius, and with T_eff and Eq.(3) one obtains the luminosity. The *atmospheric* mass results from integration of the equation

$$\mathrm{d}m_\mathrm{a} = \frac{\rho}{\bar{\kappa}}\,\mathrm{d}\bar{\tau} \tag{4}$$

where m is the column mass in [$\mathrm{g\,cm^{-2}}$]. The program and the synthesis of the Balmer lines has been described in detail by Fuhrmann et al. (1993,1996).

2.3. The opacity problem

Currently two sets of ODFs are available, an older set (Kurucz (1979) and a more recent one (Kurucz 1992). Compared with the 1979 data the new ODFs differ not only in their substantially extended line list but also in their abundance pattern. In particular the iron abundance is the subject of discussions. Whereas the 1979 data was computed with an iron abundance of $\log(\mathrm{Fe/H}) = 7.55$, with hydrogen number densities normalized to 10^{12}, the new data uses $\log(\mathrm{Fe/H}) = 7.67$ with reference to results published by Blackwell et al. (1984). Holweger et al. (1990) instead derived $\log(\mathrm{Fe/H}) = 7.48$, while the *meteoritic* value is $\log(\mathrm{Fe/H}) = 7.51$.

There is an extended list of publications that deal with the question which of these results should be accepted as the *solar* iron abundance (see Holweger et al. 1991; Grevesse 1991; Hannaford et al. 1992; Grevesse & Noels 1993; Milford et al. 1994; Anstee & O'Mara 1995; Blackwell et al. 1995 and references therein; Biémont et al. 1991; Kostik et al. 1996). Currently the meteoritic iron abundance seems to be preferred. One way to deal with the new ODFs of Kurucz (1992) is to interpolate the opacity tables using a 'metal' abundance about 0.16 dex smaller than that entering his standard solar abundance mixture. This is only an approximation because iron contributes most but not all of the atmospheric line blanketing. Thus for comparison the calculations have also been repeated using the old ODFs.

The opacities at low temperatures in the evolution code provide yet another problem: the OPAL opacities (Rogers & Iglesias 1992) do not include entries below 6000 K. Thus Wagenhuber & Weiss (1994) implemented LAOL opacities (Weiss et al. 1990) for low temperatures. Consequently, differences of results obtained with either a standard treatment of the outer boundary condition or models with stellar atmospheres tied to the Henyey solution at $\bar{\tau} = 20$ may be smeared out due to the different opacities in the model atmosphere program and in the evolution code. To distinguish these effects a table of Rosseland opacities for low temperatures has been generated with the model atmosphere program. This new opacity table was used for temperatures below 10 000 K in both types of boundary conditions, with and without model atmospheres. In the final section it will be shown that at 10 000 K the OPAL opacities and the new opacity table fit smoothly.

3. Consistent formulation of convection

A unified model of stellar evolution including stellar atmospheres as an outer boundary condition requires a consistent formulation of convective energy transfer, because the fit point at $\bar{\tau} = 20$ is well inside the solar hydrogen convection zone, and any differences between formulations used for the model atmosphere and the stellar interior will lead to *different convective fluxes*. Further any description of solar convection has to fulfil *observational* restrictions implied by radius and luminosity of the present Sun, the Balmer line spectrum emerging from the surface of the Sun and by solar oscillations. Two models of convection are considered for this purpose.

3.1. The model of Böhm-Vitense

Mixing-length theory as developed by Vitense (1953, see also Böhm-Vitense 1958) and formulated by Cox & Giuli (1968) seems to fit the properties of the present Sun provided the mixing-length parameter α is greater than 1.5, a value that has been used as a lower limit in many treatments of the interior structure of the Sun. A mixing-length parameter of $\alpha = 0.5$ as it would be required by the observations of the Balmer lines is outside any acceptable error limit imposed by the solar radius. However, the model of Böhm-Vitense has more free parameters than only the mixing-length parameter α, and in principle it could be possible to find a set of parameters that fits the present Sun *and* the Balmer lines.

In the following T, ρ, g, C_p, H_p, Q, v, α, Λ, σ, ∇, ∇' and ∇_{ad} have their usual meaning as given in the refernces (see Vitense 1953, Böhm-Vitense 1958 or Henyey et al. 1965). Besides α a second parameter, ν, is found in the expression for the convective velocity v,

$$v^2 = \frac{gH_pQ\alpha^2}{\nu}(\nabla - \nabla') \tag{5}$$

In 1953 Vitense used $\nu = 4$ whereas in Böhm-Vitense (1958) she changed to $\nu = 8$ to improve the description of the turbulent friction. Henyey et al. (1965) argued that the actual value might even be somewhat greater than $\nu = 8$. The equation determining the convective efficiency factor γ has two more free parameters replacing the volume to surface ratio of a convective element and different expressions for different optical thickness $\bar{\kappa}\Lambda$ of the convective bubble. Vitense (1953, her Eq.8) formulated γ as

$$\gamma = \frac{C_p\rho v}{4\sigma T^3\Lambda} \cdot \frac{\text{volume}}{\text{surface}} \cdot \begin{cases} \bar{\kappa}\Lambda & : \ \bar{\kappa}\Lambda \ll 1 \\ \frac{1}{\bar{\kappa}\Lambda} & : \ \bar{\kappa}\Lambda \gg 1 \end{cases} \tag{6}$$

She represented the unknown geometry by a new parameter y such that the volume to surface ratio can be written as $y\Lambda/2$. The transition from optically thick to thin bubbles is then described by an arithmetic mean with a weighting factor w. With an expression for the optical thickness $\tau_\text{e} = \bar{\kappa}\Lambda$ Eq.(6) becomes

$$\gamma = \frac{C_p\rho v}{8\sigma T^3} \cdot y \cdot \left(\frac{1}{\tau_\text{e}} + w\tau_\text{e}\right) \tag{7}$$

where now y and w are the two additional free parameters.

Equation (7) corresponds to Henyey et al. (1965, their Eqs.39 and 40) if y is set equal to 1. Equation (14.39) in Cox & Giuli (1968) is recovered from Eq.(7) for $y = 1/3$ and the expression in brackets reduced to τ_e. All the formulations discussed above are based on $\nu = 8$. Due to the different versions of the Böhm-Vitense mixing-length theory it must be stressed that the specification of the parameter α is *meaningless* if there is no reference to the corresponding formulation of the theory.

3.2. The model of Canuto & Mazzitelli

While the mixing-length theory in its original formulation (Böhm-Vitense 1958) has been the primary source of coding convective energy transfer for more than three decades, it has also been referred to as being essentially a one-parameter theory (Gough & Weiss 1976). In fact it can be shown that under conditions such as are found in convective envelopes of cool unevolved stars the variation of the additional 'free' parameters ν, y or w is mostly compensated by the mixing-length parameter α. In order to improve the representation of turbulent convective elements in inviscid flows such as encountered in stellar interiors Canuto & Mazzitelli (1991) have developed a different model of convection. In contrast to the standard mixing-length theory they use in their description not only *one* single type of convective elements with fixed geometric proportions but a *spectrum* of eddies. They introduce a distribution function $E(k)$ for the turbulent kinetic energy in the different eddies, in which k is related to the scale size ℓ of an eddy by $\ell = \pi/k$. To calculate $E(k)$ they solve a set of coupled equations that describe their turbulent convection model. Finally, they obtain new expressions for the convective flux, and they propose two different formulations of the mixing-length Λ. One refers to the well known $\Lambda = \alpha H_p$ with H_p as pressure scale height, the other introduces a parameter-free theory where $\Lambda = z$ with z being the distance from the top of the convection zone. Further improvements and discussions are found in Canuto & Mazzitelli (1992) and Canuto (1996) who have thoroughly investigated the differences between standard mixing-length theory and their energy spectrum representation.

4. Results and discussion

The stellar evolution code used for a *reference model* treats the atmospheric boundary condition according to the rules described in the introduction. Convection is included in terms of the standard mixing-length theory ($y = 1/3$, $z = 1$, $\nu = 8$), low-temperature opacities are taken from the data of LAOL (see Sect. 2.3), and the metal abundance is set equal to the meteoritic value Z = 0.01743 (Holweger 1979, Holweger et al. 1995). With this input data the reference model is calibrated to fit the observed parameters of the present Sun. Setting the mixing-length parameter $\alpha = 1.58$ and a helium abundance Y = 0.273, luminosity and effective temperature of the model fit to the values of the Sun after an evolution time that corresponds to its present age. Luminosity and age of the present Sun have been adopted from Bahcall & Pinsonneault (1995) as $L_\odot = (3.844 \pm 0.004) \times 10^{33}\text{erg s}^{-1}$ and $t_\odot = (4.57 \pm 0.02) \times 10^9\,\text{yr}$. The solar radius is taken as $R_\odot = (6.96265 \pm 0.00065) \times 10^{10}$ cm according to Wittmann (1977), and the corresponding effective temperature of the Sun according to (Eq. 3) is $T_\text{eff} = 5777$ K. All evolutionary tracks are calculated for one solar mass; they start at the pre-main sequence and end at an age of 6 Gyr.

To evaluate the effects due to changes of the low-temperature opacities in the atmospheric models (see Sect. 2.3) it is necessary to investigate at which temperature the opacities of the new table can be fitted smoothly to OPAL opacities. As demonstrated in Fig. 1 the resulting temperature is around 10 000 K. This condition has been tested for different densities and thus a switch to the new opacity table is installed for temperatures lower than 10 000 K. Note that the new opacity tables have been calculated with the 1979 ODFs and with the *scaled* 1992 ODFs. The comparison between both data sets reveals that at temperatures lower then 5 000 K only differences of less then 2% must be expected. The calculation of evolutionary tracks with the new opacity tables need a mixing-length parameter of $\alpha = 1.52$ for modelling the present Sun.

The connection of atmospheric models to the stellar structure calculations using the convection theory of Böhm-Vitense while simultaneously fitting the present Sun and the Balmer

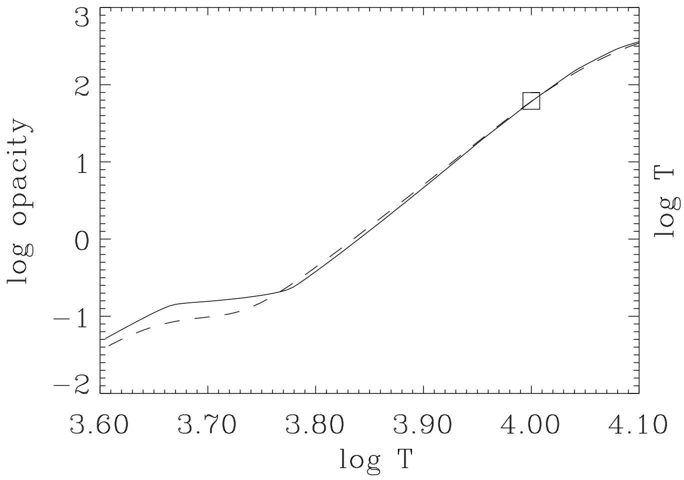

Fig. 1. The temperature dependence of different opacity tables at density $\log \rho \, (\mathrm{g\,cm^{-3}}) = -7.0$. *Full line*: OPAL with old low-temperature opacities according to LAOL. *Dashed line*: New low-temperature opacities based on Kurucz' ODF statistics

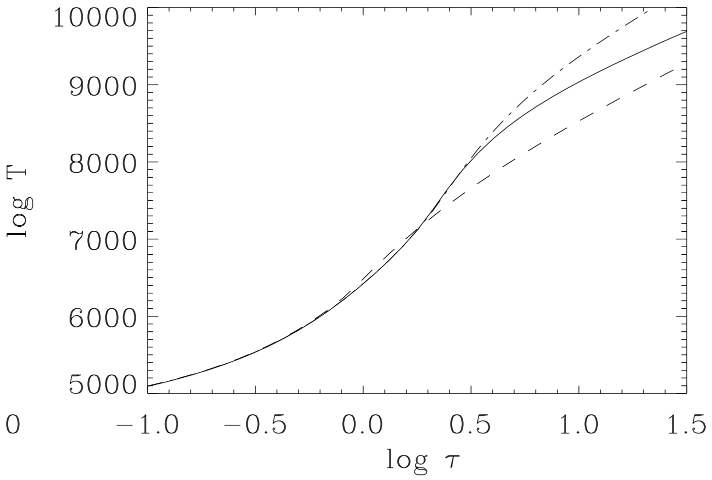

Fig. 2. The temperature stratifications of different convection models. *Full line*: Canuto & Mazzitelli $\alpha_{\mathrm{CM}} = 0.82$. *Dashed line*: Böhm-Vitense $y = 1$, $w = 0.5$, $\nu = 8$ and $\alpha_{\mathrm{BV}} = 1.58$. *Dot-dashed line*: Böhm-Vitense $y = 1$, $w = 0.5$, $\nu = 8$ and $\alpha_{\mathrm{BV}} = 0.5$ (used by Fuhrmann et al. (1993))

lines failed. With the new low-temperature opacity tables and connected atmospheric models a mixing-length parameter of $\alpha = 1.65$ will reproduce the solar radius, whereas for the representation of the Balmer lines a very low mixing-length parameter of $\alpha = 0.5$ is required according to Fuhrmann et al. (1993). The stellar evolution calculations with the theory of Böhm-Vitense have been carried out with $y = 1/3$, $w = 1$ and $\nu = 8$ (*cf.* Eq. (5), Eq. (7)) in contrast to Fuhrmann et al. , who worked with the parameters of Mihalas (1978) $y = 1$, $w = 0.5$ and $\nu = 8$. In principle a variation of y, w and ν could lead to a solution of this problem but the following characteristics of the Böhm-Vitense convection model are verified by calculations. The optical thickness τ_{e} in Eq. (7) takes values higher than 10, and in case of $w \gtrsim 0.5$ the expression $1/\tau_{\mathrm{e}}$ can be omitted and $w \cdot y$ becomes a single combined parameter. The enhancement of this combined parameter can be compensated by reduction of the mixing-length parameter. Consequently the mixing-length parameter has to be reduced for stellar evolution if the calculations are repeated with the parameters $y = 1$, $w = 0.5$ and $\nu = 8$ of Fuhrmann et al. . If the boundary condition of fitting the present Sun is to be conserved, such a reduction can never compensate a mixing-length as small as $\alpha = 0.5$. Therefore, stellar evolution calculations with parameters that fit the Balmer lines always show too low effective temperatures for the present Sun. Reducing or enhancing ν is inversely equivalent to a change of α^2 (see Eq. (5)); thus it is evident that the variations of the different parameters nearly *cancel*, and the reproduction of the observed Balmer lines and of the present Sun cannot be obtained simultaneously using the convection model of Böhm-Vitense.

A view at the temperature structure of the layers where the Balmer lines are formed (Fig. 2) reveals that the Balmer lines require a steeper temperature gradient such as is obtained by low mixing-length parameter of $\alpha = 0.5$ in contrast to $\alpha = 1.58$. But through the steeper convective gradient the *whole* convection zone is more extended, and therefore the properties of the present Sun cannot be fitted.

In order to escape this dilemma a high degree of overadiabaticity should only occur at the stellar surface. Such a characteristic is provided by the convection model of Canuto & Mazzitelli (see Fig. 8 in Canuto & Mazzitelli 1991). Figure 2 presents the resulting temperature stratification between $\bar{\tau} = 1$ and 10 for a convection model of Canuto & Mazzitelli that fits the present Sun *and* the Balmer lines. Note that for deeper layers the stratification is close to the one with the high mixing-length parameter of Böhm-Vitense, which is a result of the calibration to the present Sun. To distinguish between the mixing-length parameters of Böhm-Vitense and Canuto & Mazzitelli an index CM is introduced. Table 1 gives an overview of the models calculated with the theory of Canuto & Mazzitelli.

Table 1. Results of the stellar evolution calculations with the convection theory of Canuto & Mazzitelli. T_{eff} is the effective temperature found at the age of the present Sun; old and new opacities refer to the use of low-temperature opacities, and in the second column the connection of model atmospheres is indicated

Opacities	Atmosphere	α_{CM}	T_{eff}
old	no	0.80	5773 K
new	no	0.80	5804 K
new	yes	0.80	5748 K
old	no	0.77	5777 K
new	yes	0.82	5776 K

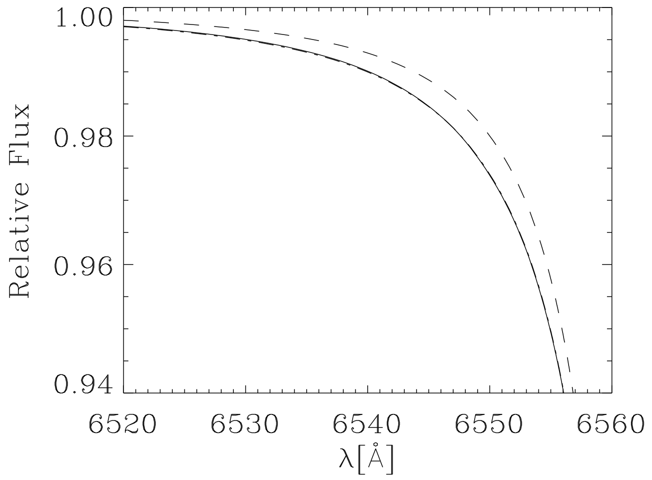

Fig. 3. Blue wing of theoretical Hα profiles. *Full line*: Canuto & Mazzitelli $\alpha_{\rm CM} = 0.82$. *Dashed line*: Böhm-Vitense $y = 1$, $w = 0.5$, $\nu = 8$ and $\alpha_{\rm BV} = 1.58$. *Dot-dashed line*: Böhm-Vitense $y = 1$, $w = 0.5$, $\nu = 8$ and $\alpha_{\rm bv} = 0.5$ (used by Fuhrmann et al. 1993)

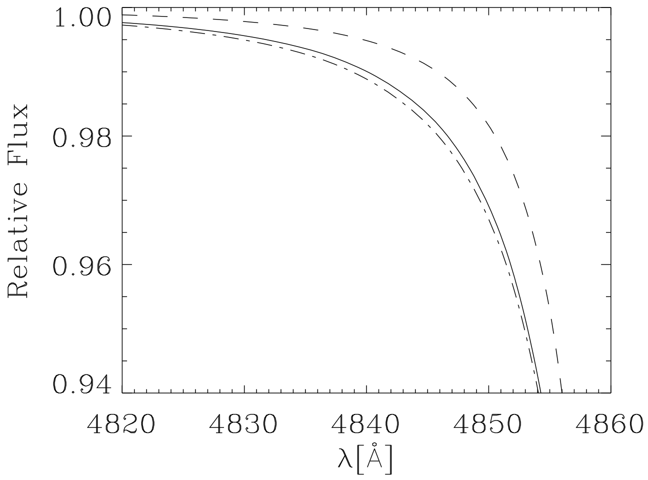

Fig. 4. Blue wing of theoretical Hβ profiles. *Full line*: Canuto & Mazzitelli $\alpha_{\rm CM} = 0.82$. *Dashed line*: Böhm-Vitense $y = 1$, $w = 0.5$, $\nu = 8$ and $\alpha_{\rm BV} = 1.58$. *Dot-dashed line*: Böhm-Vitense $y = 1$, $w = 0.5$, $\nu = 8$ and $\alpha_{\rm BV} = 0.5$ (used by Fuhrmann et al. (1993)).

All stellar structure models with convection theory of Canuto & Mazzitelli have the same hydrogen, helium and metal abundances as the models with Böhm-Vitense's theory. Calculating stellar evolutionary tracks with the old low-temperature opacities and the simplified treatment of the atmosphere as described in Sect. 1 requires a mixing-length parameter of $\alpha_{\rm CM} = 0.80$ to fit the present Sun. Using the new low-temperature opacities and the grey standard outer boundary condition recalibration to the present Sun can be achieved with $\alpha_{\rm CM} = 0.77$. In contrast connecting *model atmospheres* and recalibrating the stellar structure models to the present solar effective temperature requires a mixing-length parameter of $\alpha_{\rm CM} = 0.82$.

As can be seen in Fig. 3, 4 and 5 the last model fits not only the position of the present Sun in the Hertzsprung-Russell diagram but also the Balmer lines. This stellar evolution model with connected atmospheres and a common mixing-length parameter of $\alpha_{\rm CM} = 0.82$ for structure and atmosphere is in the following referred to as *unified model*. The Hα profile of the unified model shows no difference to the profile of Fuhrmann et al. (1993), who used Böhm-Vitense's model for convection (see Fig. 3). The small deviation between the Hβ profile of the unified model and Fuhrmann et al. (*cf.* Fig. 4) corresponds to an increase of only 10 K in the spectroscopic determination of the effective temperature, which can be neglected in view of the larger observational error for the Balmer lines. Both figures clearly show that the Balmer line profiles produced by the Böhm-Vitense mixing-length parameter $\alpha = 1.58$, which is required when stellar evolution should fit the present Sun, are too narrow compared with the profiles of Fuhrmann et al. who have fitted their profiles to spectra of the Sun and several cool stars.

In Fig. 5 the evolutionary tracks of the unified model compared to the Böhm-Vitense standard model with the parameters $\alpha = 1.58$, $y = 1/3$, $w = 1.0$ and $\nu = 8$ are presented. The region of the early pre-main sequence shows no differences in the track whereas at the position of the zero-age main sequence the effective temperature of the model using Canuto & Mazzitelli is about 20 K cooler. Larger deviations cannot be discovered since both tracks are calibrated with the position of the present Sun as indicated by the box. This calibration is also the reason for the good agreement of the tracks in Fig. 6 where no significant difference between the evolutionary tracks with and without connected atmospheres can be revealed.

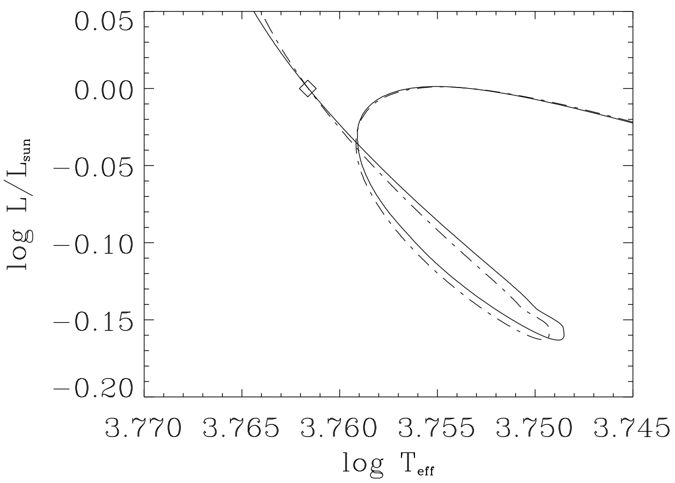

Fig. 5. Evolutionary tracks of stellar models with different convection models. *Full line*: Canuto & Mazzitelli $\alpha_{\rm CM} = 0.82$. *Dot-dashed line*: Böhm-Vitense $y = 1/3$, $w = 1.0$, $\nu = 8$ and $\alpha_{\rm BV} = 1.58$

To work out only the effect of connected atmospheres that use Kurucz' opacity distribution functions, the track with the old atmospheric treatment was recalculated with new low-

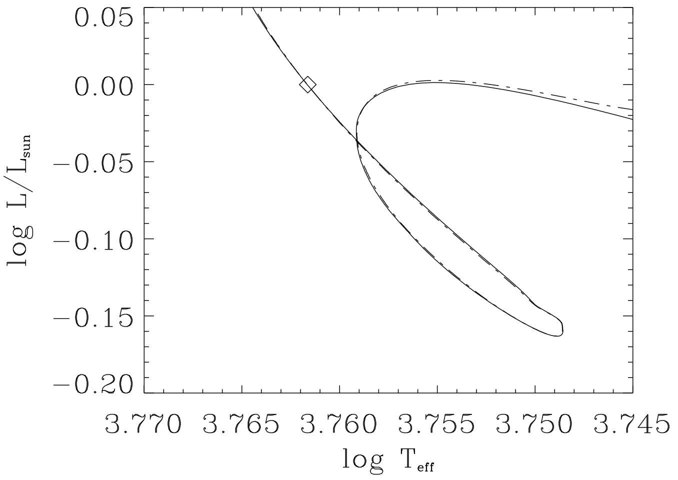

Fig. 6. Evolutionary tracks of solar models using the convection model of Canuto & Mazzitelli. *Full line*: Unified model with atmosphere and $\alpha_{\rm CM}$ = 0.82. *Dot-dashed line*: Model with new low-temperature opacities and $\alpha_{\rm CM}$ = 0.77 but with old atmospheric treatment

temperature opacities based on Kurucz' ODFs. The agreement in Fig. 6 suggests that calculating stellar evolution with unified models provides no significant change compared to the case without atmospheres. However, the tracks have been calibrated with the present Sun, and thus their mixing-length parameters are different. If the models for the evolutionary tracks are calculated both with a common mixing-length parameter one obtains for the present Sun in Table 1 an effective temperature lower by about 50 K for the model with connected atmospheres as compared to the model with low-temperature opacities only.

Evolutionary tracks of different models in the Hertzsprung-Russell diagram with changes in the input physics will not provide enormous differences in the neighbourhood of the present Sun if they have been calibrated to fit the position of the Sun. However, differences between the tracks may become more important in more advanced stages of stellar evolution.

Before drawing the conclusions from the above investigations there are two remarks

1. The formulation $\Lambda = z$ with z being the distance from the top of the convection zone introduced by Canuto & Mazzitelli (1991) was not tested here because in reality Canuto & Mazzitelli did not use a completely parameter-free theory. Instead they worked with a mixing-length parameter $\alpha_{\rm CM} < 0.4$ and a pressure scale height at the top of convection zone $H_{p,\rm top}$ in the equation $\Lambda = z + \alpha_{\rm CM} H_{p,\rm top}$ (Wagenhuber, private communications).
2. A further observational constraint for solar models are the oscillation frequencies of the Sun. Christensen-Dalsgaard et al. (1985) have shown that the observed p-mode frequencies can be inverted to estimate the solar interior soundspeed. The transition between the subadiabatic temperature gradient below the convection zone and the adiabatic gradient in the convection zone can be seen as a distinct feature in the soundspeed profile. The feature has been used for a careful determination of the depth of the convection zone in the Sun (Christensen-Dalsgaard et al. 1991), and the base of the solar convection zone was found at $r_b = 0.713 \pm 0.003 R_\odot$. In contrast to this prediction the stellar models calculated above all show the bottom of the convection zone at $r_b = 0.732 R_\odot$ with no changes between different convection models and the treatment of the atmosphere. The fundamental test of calculating theoretical frequencies emerging from the stellar models and comparing these frequencies with the observed modes was not carried out here. Paterno et al. (1993) investigated the influence of the different convection models formulated by Böhm-Vitense and by Canuto & Mazzitelli on the theoretical oscillation frequencies. Mainly the frequencies are testing the properties of near-surface convection. Paterno et al. found a better representation of the observed frequencies using the model of Canuto & Mazzitelli. Nevertheless the base of their convection zone was in both formulations at $r_b = 0.735 R_\odot$, which is only marginally higher than in the models presented here. The discrepancy to the depth of the convection zone as obtained by inverting the observed p-modes can be removed by incorporating gravitational element diffusion in the stellar evolution code (e.g. Bahcall & Pinsonneault (1995), Proffitt (1994)).

The conclusion of the present investigation is that a simultaneous fit of the observed temperature and luminosity of the present Sun by stellar evolution according to the solar age and the observed Balmer lines with corresponding model atmospheres cannot be achieved using the convection theory of Böhm-Vitense. The replacement of the simple atmospheric boundary condition in stellar evolution by more realistic model atmospheres reveals that unified stellar models can be created which use as a common convection theory the model of Canuto & Mazzitelli, and which fulfil both observational constraints. This can be seen as a strong argument in favour of the use of the Canuto & Mazzitelli convection theory. It will be possible to test the near surface effects of the unified model by comparison with the observed solar oscillation frequencies. In order to remove the discrepancies with the depth of convection zone and for further improvement it will be most probably necessary to implement gravitational diffusion in the stellar evolution code. This last *fine tuning* by diffusion as much as improving the equation of state or the opacities will not affect the fundamental advantages that lie in the description of convection with a unified stellar model that describes both the properties of the stellar atmosphere and its interior.

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