Cosmology and Large-Scale Structure

WS 17/18

Problem sheet 9

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Problem 1 [Spherical collapse]

(a) Verify that the energy conservation equation $\dot{r}^2/2 - GM/r = E$ can be solved in a parametric form by

$$r = A(1-\cos\theta) \quad (1)$$

$$t = B(\theta - \sin\theta) \quad (2)$$

with $A^3 = GMB^2$.

(b) Using the nonlinear evolution of the average density contrast

$$1+\delta = \frac{9}{2}\frac{(\theta-\sin\theta)^2}{(1-\cos\theta)^3}, \quad (3)$$

show that in the limit $\theta \ll 1$, in an Einstein-de Sitter (EdS) universe, it is proportional to the scale factor, i.e.,

$$\delta = \frac{3}{5}\delta_i\left(\frac{t}{t_i}\right)^{2/3} \propto a(t). \quad (4)$$

Hint: Use second-order Taylor expansion. Recall that in terms of initial quantities $B = 3t_i/(4\delta_i^{3/2})$ for $\delta_i \ll 1$ and $\Omega_i = 1$.

(c) Show that virialization ($|U| = 2K$) implies that

$$v_{\rm vir} = \left(\frac{6}{5}\frac{GM}{r_m}\right)^{1/2}, \quad r_{\rm vir} = r_m/2, \quad (5)$$

where r_m is the radius at 'turn-around'.

Problem 2 [Multipoles of the power spectrum]

In the context of galaxy surveys, an alternative to measure the galaxy power spectrum in wedges (μ bins) is to expand the galaxy power spectrum in multipoles:

$$P_g^S(k,\mu) = \sum_\ell P_\ell(k)\mathcal{L}_\ell(\mu) \tag{6}$$

$$P_\ell(k) = \frac{2\ell+1}{2}\int_{-1}^{1} d\mu P_g^S(k,\mu)\mathcal{L}_\ell(\mu) \tag{7}$$

where $\mathcal{L}_\ell(x)$ are the Legendre polynomials.

Given the redshift space galaxy power spectrum:

$$P_g^S(k,\mu) = b^2(1+\beta\mu^2)^2 P_m(k) \tag{8}$$

compute its monopole ($\ell = 0$), quadrupole ($\ell = 2$) and hexadecapole ($\ell = 4$).