Cosmology and Large-Scale Structure

WS 17/18

Problem sheet 8

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Problem 1 [Perturbations from inflation]
Inflation produces curvature and gravitational wave perturbations $\Delta_{\mathcal{R}}$ and Δ_h with

$$\Delta^2_{\mathcal{R}} = \frac{1}{8\pi^2 \varepsilon} \frac{H^2}{M^2_{\rm Pl}} \,, \quad \Delta^2_h = \frac{2}{\pi^2} \frac{H^2}{M^2_{\rm Pl}} \tag{1}$$

These expressions are evaluated at $k = aH$, as the perturbations freeze in at horizon crossing.

i) Derive the scalar and tensor slope of the power spectrum, n_s and n_t, defined via

$$n_s - 1 \equiv \frac{d \ln \Delta^2_{\mathcal{R}}}{d \ln k} \,, \quad n_t \equiv \frac{d \ln \Delta^2_h}{d \ln k} \,. \tag{2}$$

Express them up to linear order in terms of the slow-roll parameters ε and η.

ii) Compute $\Delta^2_{\mathcal{R}}$ and Δ^2_h in the slow-roll approximation with the potential $V \equiv V(\phi)$ and derive n_s and n_t in terms of the potential in this case.

iii) Assume a potential of the form $V(\phi) = A\phi^p$. Express n_s and n_t in terms of p and the number of e-folds N. Evaluate for $p = 2, 4$ and $N = 60$.

iv) Compute the value of A for the same cases as above, assuming the observed value of the curvature perturbation is given by $\Delta^2_{\mathcal{R}} = 10^{-9}$. Express A in units of the Planck mass and in units of GeV.

Problem 2 [Power spectrum]
The correlation function is defined as $\xi(r) \equiv \langle \delta(\mathbf{x})\delta(\mathbf{x}+\mathbf{r})\rangle$, with the relative fluctuation of mass density around its mean $\delta(\mathbf{x}) = \rho(\mathbf{x})/\bar{\rho} - 1$.

i) Show that the equivalent expression in Fourier space yields

$$\langle \delta(\mathbf{k})\delta(\mathbf{k}')\rangle = 2\pi^3 P(k)\delta_D(\mathbf{k}+\mathbf{k}') \,, \tag{3}$$

where the power spectrum $P(k)$ is defined as the Fourier transform of $\xi(r)$.

ii) If $\delta(\mathbf{x})$ is real-valued, what does this imply for its Fourier transform $\delta(\mathbf{k})$?

iii) For $P(k) \propto k^n$, calculate the variance $\sigma^2_R = \langle \delta^2_R(\mathbf{x})\rangle$ of the density field above a cut-off scale R. Associate this scale with a mass M, assuming constant density.