Cosmology and Large-Scale Structure

WS 17/18

Problem sheet 7

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Issued: 13.12.2017
Due: 20.12.2017

Problem 1 [Evolution DM perturbations]

(i) Show that the fluctuations in the dark matter density in a flat radiation dominated universe grow logarithmic with time.

(ii) Show that the fluctuations in the dark matter denisity in a flat Universe, with a dominant background component, with $w = -1$, grow like $\delta_m \propto H = const.$, where H is the Hubble parameter

Problem 2 [Density Pertubations: Euler's Equation in the expanding Universe]
Starting from Euler's equation

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\vec{\nabla})\vec{v} = -\frac{\vec{\nabla} p}{\rho} - \vec{\nabla}\phi. \tag{1}$$

show that in the perturbed expanding universe it can be written as

$$\frac{\partial \mathbf{v}_p}{\partial t} + H(t)\mathbf{v}_p + \frac{c_s^2}{a}\nabla\delta + \frac{1}{a}\nabla\delta\phi \ = 0, \tag{2}$$

where δ is the density pertubation, $\mathbf{v}_p$ the peculiar velocity, H is the Hubble parameter, c_s the sound speed of the fluid, and $\delta\phi$ is the pertubed gravitational potential.

Hint: Consider $\delta p \simeq c_s^2 \delta\rho$ and $\vec{\nabla}_r \delta p = \vec{\nabla}_r c_s^2 \delta\rho_0 = c_s^2 \delta \nabla_r \rho_0 + c_s^2 \rho_0 \vec{\nabla}_r \delta$. Neglect 2nd order terms of the pertubed quantities.

Problem 3 [Solutions in the presence of dark energy]
Show that the Ansatz

$$\delta_1(x) = \sqrt{1 + x^{-3}} \tag{3}$$

solves the equation

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$$x^2(1+x^{-3w})\frac{d^2\delta}{dx^2}+\frac{3}{2}x(1+(1-w)x^{-3w})\frac{d\delta}{dx}-\frac{3}{2}\delta=0. \tag{4}$$

for the cosmological constant ($w=-1$). Furthermore show that

$$\delta_2(x)=\delta_1(x)\int_0^x\frac{dy}{y^{3/2}(1+y^{-3w})^{1/2}\delta_1^2(y)}. \tag{5}$$

Compute the complete solution of eq. (4), $\delta(x)=C_1\delta_1(x)+C_2\delta_2(x)$ ($C_i = const.$), for $w=-1$ in the limit of dark energy domination ($x\gg 1$) and show that perturbations stop growing.

Hint:

$$I(x)=\int_0^x\frac{x'^{3/2}}{(1+x'^3)^{3/2}}dx' \qquad I(\infty)\simeq 0.57 \tag{6}$$