# Cosmology and Large-Scale Structure

WS 17/18

## Problem sheet 6

Prof. T. Giannantonio
`www.usm.uni-muenchen.de/people/tommaso/pages`

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**Problem 1** [Neutron freeze-out]

Consider the reactions $n + \nu \longleftrightarrow p + e^-$ and $n + e^+ \longleftrightarrow p + \bar{\nu}$. The reaction rates for the conversion of protons and neutrons determine the evolution of the relative fraction of protons and neutrons

$$\frac{dX_n}{dt} = -\Gamma(n \to p) X_n + \Gamma(p \to n) X_p \, .$$

Assuming $T_\nu(t) = T$, derive the balance equation

$$\frac{dX_n}{dt} = -\Gamma(n \to p) \left[1 + \exp(-Q/T)\right] \left(X_n - X_n^{\mathrm{eq}}\right) ,$$

where $X_n = n_n/(n_p + n_n)$, $X_n^{\mathrm{eq}} = 1/\left[1 + \exp(Q/T)\right]$, $\Gamma(n \to p)$ is the total reaction rate for the conversion of neutrons to protons and $Q = m_n - m_p$. Verify that $dX_n^{\mathrm{eq}}/dt = 0$. The neutron freeze-out temperature $T_n$ is determined via $\Gamma(n \to p, T_n) = H(T_n)$. Derive $H(T)$ for a flat radiation-dominated universe in natural units ($\hbar = c = G = k_{\mathrm{B}} \equiv 1$).

**Problem 2** [Helium abundance]

Show that the relative abundance of $^4$He in mass, $Y_{\mathrm{He}}$, can be estimated as

$$Y_{\mathrm{He}} \simeq \frac{2}{\exp\left[Q/T_n + (t_D - t_n)/\tau_n\right] + 1},$$

where $Q = m_n - m_p$, $T_n$ is the freeze-out temperature for neutrons at time $t_n$, and $\tau_n$ is the lifetime of neutrons. Note that the number of neutrons stays constant once the synthesis of deuterium sets in at time $t_D$. Calculate $Y_{\mathrm{He}}$ for $Q \simeq 1.3$MeV, $T_n \simeq 0.7$MeV, $t_n \simeq 1.5$s, $t_D \simeq 100$s, and $\tau_n \simeq 886$s.

Estimate the relative change in the $^4$He abundance, if

i) the difference between neutron and proton mass is increased by 10%,

ii) the neutron lifetime increases by 10%,

iii) the number of neutrino species is increased from 3 to 4.