Cosmology and Large-Scale Structure

WS 17/18

Problem sheet 5

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Problem 1 [Particle decay]
Suppose the universe contained only a comoving density n of particles with mass m which have a lifetime τ and which decay to relativistic products. Calculate the entropy density – $s = (\rho + P)/T$ – produced when the particles have finished decaying and compare with the naive answer obtained from assuming instantaneous decay at $t = \tau$.

Hint: Consider that the radiation density today is given by $\rho_{rad,0} = -\int a(t) \mathrm{d}\rho_{particle}$, and the particles comoving number density evolves as $n(t) = (n/\tau) \exp(-t/\tau)$.

Problem 2 [Scale factor ratio]
Assume at $T_1 = 150$ MeV the scale factor was a_1 and the relativistic particles were $\mu^{\pm}, e^{\pm}$, neutrinos and photons. At $T_2 = 10$ MeV muons have annihilated or decayed and the scale factor is a_2. Compute a_2/a_1.

Problem 3 [Helium recombination]
The electric charge of the helium nucleus is 2 (2 protons), so it has to capture 2 electrons to become neutral. This process occurs in 2 steps. In the first one the helium nucleus captures one electron and becomes a single charged hydrogen-like ion $^{+}$He. The binding energy of this ion is 4 times larger than the binding energy of H: $B_{+} = m_e + m_{2+} - m_{+} = 54.4$eV (where m_{2+} and m_{+} are the masses of He^{2+} and He^{+} respectively). After most of the helium is converted into He^{+} ions, the charged ion captures a second electron and becomes neutral. The electron-electron interaction reduces the binding energy that is only $B_{\mathrm{He}} = m_e + m_{+} - m_{He} = 24.62$eV for the second electron. Assuming chemical equilibrium, derive the expression for the ratio

of the number densities of He^+ and the neutral He. Verify that for $\eta_{10} \simeq 5$ this ratio is equal to unity at $T \simeq 6800$K and is about 10^{-4} at $T \simeq 5600$K.

Hint:You can use the approximation for the number density of free electrons:

$$n_e \approx n_B \simeq 2\zeta(3)10^{-10}\eta_{10}T^3/\pi^2.$$