Cosmology and Large-Scale Structure

WS 17/18

Problem sheet 4

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Problem 1 [Massless scalar field]

Consider a massless scalar field $\phi(\vec{x}, t)$ whose potential energy density is $V(\phi) = 0$. Now suppose that this scalar field is initially rolling, so $\dot{\phi} \neq 0$, and that the kinetic energy density associated with this rolling dominates the energy density of the universe. Show that this implies $\rho \propto a^{-6}$, where a is the scale factor, in two ways:

(i) by recalling how the energy density of matter with an equation of state $p = w\rho$ scales with a;

(ii) by solving the equation of motion for ϕ in an expanding universe.

Problem 2 [Phenomenology of ϕ^4–inflation]

Consider $V(\phi) = \lambda\phi^4$ and assume that the field rolls toward $\phi = 0$ from the positive side.

(i) Calculate the values of ϕ where each of the slow roll conditions break down.

(ii) Assuming that inflation ends when $\epsilon = 1$, calculate the number of e-folds of inflation that occur for an initial value ϕ_i.

(iii) Demonstrate that the slow-roll solutions with $\phi(t = t_i) \equiv \phi_i$ and $a(t = t_i) \equiv a_i$ are

$$\phi(t) = \phi_i \exp\left[-\sqrt{\frac{16\lambda M_{\rm Pl}^2}{3}}(t - t_i)\right], \tag{1}$$

$$a(t) = a_i \exp\left\{\frac{\phi_i^2}{8M_{\rm Pl}^2}\left[1 - \exp\left(-\sqrt{\frac{64\lambda M_{\rm Pl}^2}{3}}(t - t_i)\right)\right]\right\} \tag{2}$$

(iv) Use the solution for $\phi(t)$ to calculate the time when inflation ends.

(v) Determine the number of e-folds using the solution for $a(t)$ and verify that you get the same result as in (ii).