Cosmology and Large-Scale Structure

WS 17/18

Problem sheet 1

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Issued: 25.10.2017
Due: 08.11.2017

Problem 1 [General Relativity basics]

(i) Given is the curvature tensor

$$R_{abcd} = K\left(g_{ac}g_{bd} - g_{ad}g_{bc}\right)$$

in three dimensions, with K constant. Compute the Ricci tensor $R_{bd} = g^{ac}R_{abcd}$ and show that the Ricci scalar $R = g^{ab}R_{ab}$ is constant. Note that the matrix g_{ab} is the inverse of the matrix g^{ab}.

(ii) In three dimensions the line element is given by

$$ds^2 = g_{ab}dx^a dx^b = B(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \,.$$

Compute the metric connection

$$\Gamma^a_{bc} = \frac{1}{2}g^{ad}\left(\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{bc}\right) \,,$$

where the partial derivatives ∂_a are with respect to the coordinates (r, θ, ϕ).

(iii) The Riemann tensor is given by

$$R^a_{bcd} = \partial_c \Gamma^a_{bd} - \partial_d \Gamma^a_{bc} + \Gamma^e_{bd}\Gamma^a_{ec} - \Gamma^e_{bc}\Gamma^a_{ed} \,,$$

where the Einstein sum convention is used, that double indices are summed over. Compute the non-vanishing components of the Ricci tensor

$$R_{ab} = R^c_{acb} \,.$$

By using the relation from above for the Ricci tensor, show that

$$B(r) = \frac{1}{A - Kr^2} \,.$$

Determine the value of the constant A.