Problem set #7

Problem 1 *Radial velocity variations caused by planets*

(a) Derive the amplitude of the radial velocity variation of the host star of a transiting extra-solar planet (inclination $i = 90°$) as a function of period and planetary and stellar mass. Use Kepler's third law in the following approximations:

$$a_{\text{star}} \ll a_{\text{planet}}, \tag{1}$$
$$M_{\text{planet}} \ll M_{\text{star}}. \tag{2}$$

(Hint: use the center of mass theorem and the above approximations to derive a_{planet} as a function of M_{star} and p.)

(b) Calculate the amplitude of the Sun's radial velocity variation caused by Earth, Jupiter, and Saturn ($M_{\text{Earth}} = 5.97 \times 10^{24}$ kg, $M_{\text{Jupiter}} = 1.90 \times 10^{27}$ kg, $M_{\text{Saturn}} = 5.68 \times 10^{26}$ kg, $M_{\text{Sun}} = 1.99 \times 10^{30}$ kg, $p_{\text{Jupiter}} = 11.86$ yr, $p_{\text{Saturn}} = 29.46$ yr).

Problem 2 *Transit detection*

What fraction of sunlight is blocked when Earth passes in front of the Sun and how large is the decrease in brightness expressed in magnitudes? Here it shall be assumed that the observer resides far outside the solar system and that the Sun and the planets appear as uniform disks. How large is the effect for Mercury, Jupiter, and Neptune ($R_{\text{Mercury}} = 0.38 R_{\text{Earth}}$, $R_{\text{Earth}} = 0.00915 R_{\odot}$, $R_{\text{Jupiter}} = 11.2 R_{\text{Earth}}$, $R_{\text{Neptune}} = 3.89 R_{\text{Earth}}$)?

Problem 3 *Transit probability*

Show that the probability p for a suitable orientation of a planet's orbital plane to allow observing a transit is given by the simple formula

$$p = \frac{R_{\star}}{a},$$

where $R_{\star}$ is the radius of the star and a is the planet's orbital radius. How large are the probabilities for an alien observer to be in a position allowing the observation of transits of Mercury, Earth, Jupiter, Neptune ($a_{\text{Mercury}} = 0.387 a_{\text{Earth}}$, $a_{\text{Earth}} = 215 R_{\odot}$, $a_{\text{Jupiter}} = 5.205 a_{\text{Earth}}$, $a_{\text{Neptune}} = 30.14 a_{\text{Earth}}$)?

Problem 4 *Transit duration*

Derive a general expression for the duration of a transit across the center of a star by using Kepler's third law. Assume that the observer is at a very large distance and that the orbits are circular. The formula should give the transit duration t in hours, when the mass $M_{\star}$ of the star is given in $M_{\odot}$, the radius $R_{\star}$ of the star in $R_{\odot}$, and the planet's orbital radius a_{p} in AU. How long does a central transit of Mercury, Earth, Jupiter, and Neptune last? How long does the transit last relative to the total orbital period of the planet?

Problem 5 *Astrometric precision*

How large is the the astrometric wobble (in arcseconds) of a sun-like star at a distance of 10 pc that harbors an earth-like planet? How large is a structure on the moon that has the same angular extent when seen from earth? Why can only interferometers making use of *differential techniques* achieve the required precision to detect such planets? (What is the resolution of a conventional optical interferometer with a baseline of 1 km?)

Problem 6 *Habitability of a planet*

Suppose that ξ Hydrae ($M \simeq 3.3 M_\odot$, $L \simeq 60 L_\odot$) has a planet with orbital period equal to 4 years. What is its semi-major axis a? Can this planet or any of its moons be inhabited by life as we know it (using liquid water)? Assume albedo $A = 0.5$, i.e., that one-half of starlight is scattered and the rest absorbed by the planet. The blackbody temperature is given by $T \simeq 280\text{K}\ (a/\text{AU})^{-1/2}(L/L_\odot)^{1/4}$, as can be shown equating the incoming and radiated fluxes and evaluating constants.

Problem 7 *Key questions*

Answer the following questions related to planets:

- Which criteria define a planet? What distinguishes them from brown dwarfs?
- How do the characteristics of planets change with the distance to their host star?
- What are the possible detection methods for exoplanets? Briefly describe them.