Problem set #5

Problem 1 *Supernova energy*

A supernova typically releases something like 10^{44} Joules of energy. That's a lot; it's similar to the total energy that the Sun will emit throughout its lifetime.

(a) In a core-collapse (type II) supernova, where does all of this energy come from?

(b) Do the calculations to show that roughly the right amount of energy is released by the mechanisms you identified in (a).

Problem 2 *Variable stars*

The luminosities of a solar-type star:

A solar-type main sequence star has a temperature of $T_{\text{eff}} = 5500$ K and a radius of $R = 0.9R_{\odot}$. At the end of its life the star expands to become a red giant star with its temperature dropping to $T_{\text{eff}} = 2000$ K and its radius expanding to $R = 120R_{\odot}$. At the very end, after ejecting its outer parts, only a white dwarf remains with a temperature of $T_{\text{eff}} = 12000$ K and a radius of $R = 10000$ km. Calculate the luminosity of the star in each of the three evolutionary phases and estimate in which phase the star radiates most of its energy.

The change in T_{eff} and R for a variable star:

A long-period variable star is 0.8 magnitudes brighter during its maximum phase compared to the minimum phase. The effective temperature of the star is $T_{\text{eff}} = 5200$ K during the maximum phase. What is T_{eff} during the minimum phase if the change of brightness is only due to the change in effective temperature? If the temperature does not change, how large does the relative change in the radius need to be in order to explain the observed change in brightness?

Problem 3 *Initial mass function of an open cluster*

When stars form, the distribution of their masses follows an *initial mass function (IMF)* dN/dm, which gives the differential number of stars dN in a mass interval $(m \ldots m + dm)$ as

$$dN = \frac{dN}{dm}\, dm \, .$$

Salpeter (1955) determined the initial mass function as a power-law in mass,

$$\frac{dN}{dm} = a \times m^{-\alpha} \, , \tag{1}$$

where a is an amplitude for normalization to the total stellar mass inside a volume and the exponent is $\alpha = 2.35$.

Consider an open cluster, in which a total stellar mass $10^3 M_\odot$ has formed instantaneously following a Salpeter initial mass function in the mass range $0.1 - 20 M_\odot$ (with no stars outside that mass range; why is that a reasonable assumption?).

- Find the normalization constant a.
- Find the initial total luminosity of the cluster, assuming that all its stars are on the main sequence, and a mass-luminosity relation $L \propto M^4$. What fraction of the initial luminosity is contributed by stars more massive than $5 M_\odot$? What fraction of the number of stars is in this high mass end?
- Find the initial mean mass of stars in the cluster.
- Assume that the main sequence lifetime of a $1 M_\odot$ star is 10 Gyr, and main sequence lifetime scales with mass as M^{-2}. From your observations of the cluster, you know that the brightest main sequence stars that still exist are about $100\times$ more luminous than the sun. How long ago did the cluster form its stars?

Problem 4 *The giants of a globular cluster*

You observe a globular cluster. You are able to measure that the largest number of bright stars in this globular cluster each individually have a V-band magnitude of 15.5.

Assume that the total V-band light observed from a globular cluster is dominated by the red giants.

(a) If the absolute V-band magnitude of a red giant star is 0.8, how far away is this globular cluster?

(b) If the globular cluster has an integrated V-band magnitude of 4.5, how many red giant stars are there in the globular cluster?

(c) Make an educated guess at the total number of stars in this cluster given the number of red giants you calculated. Explain your reasoning.

Problem 5 *The giant sun*

Right now the Sun is a main-sequence star. Later in its life, it will become a red giant: it's luminosity will go up hundreds or thousands of times, it will be many (between ten and a hundred) times larger, and it will be a bit cooler.

(a) Will the rate at which the Sun converts some of its mass to energy be higher or lower when it is a red giant as compared to now? Why?

(b) Do you expect that the pressure near the surface, but inside the Sun, will be higher when the Sun is a red giant than it is now? Why?

(c) The "solar wind" is a stream of particles coming off of the surface of the Sun. Do you expect the solar wind to be more intense (i.e. more particles coming off of the Sun per second) when it is a red giant or now? Why?

Problem 6 *Globular cluster star population*

When you look at a globular cluster through a small telescope with your eye, the individual stars you can see are largely red giants.

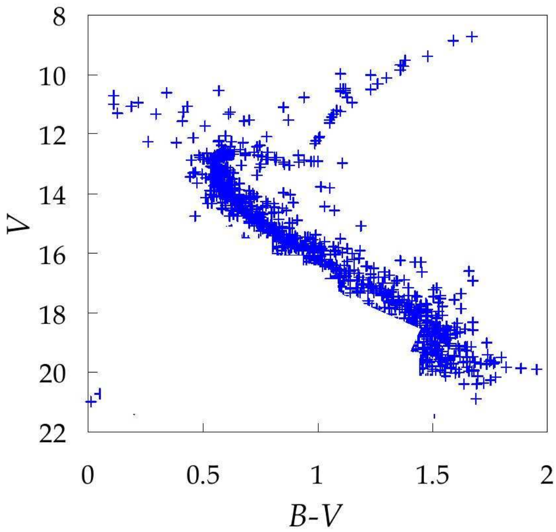
(a) Why is this so?

(b) Why aren't you seeing any high-luminosity massive main sequence stars?

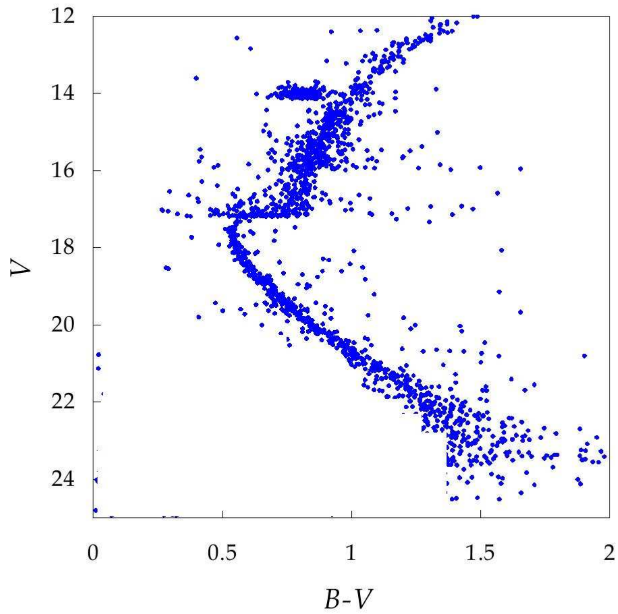
(c) Is what you are seeing representative of the population of stars in the globular cluster? Why or why not?

Problem 7 *Age of stellar clusters*

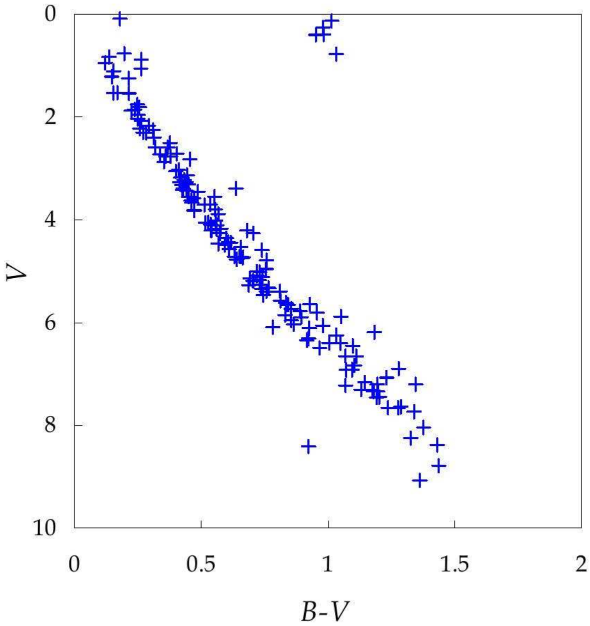
Here are the H-R diagrams of three stellar clusters. Under the assumptions that all the stars formed at the same time in each cluster, rank the stellar clusters according to their ages.)



Cluster A



Cluster B



Cluster C