Problem set #3

Problem 1 *Critical density for forbidden lines*

Forbidden transitions are spontaneous radiative transitions from a meta-stable state which occur in low denisty envirorments.

The spectra of gaseous nebulae show prominent emission from forbidden transitions, whereas these forbidden lines are absent from the spectra of stars.

- What is the critical density (order of magnitude) above which these forbidden lines will not appear?

Make a rough approximation regarding the velocity of the electrons and the interaction cross-section between electrons and ions. The Einstein "A" coefficient of the forbidden O III transition at 4959, 5007 Å is $A = 2.6 \times 10^{-2}/\mathrm{s}$. Assume the nebular kinetic temperature to be $T = 8000$ K.

(*Hint:* Emissions in the forbidden lines are observed if the collosion rate is smaller than the spontaneous transition rate)

Problem 2 *H II regions around hot stars*

Hot stars emit enough UV photons to ionize the surrounding interstellar medium, forming a sharply-bounded "Strömgren sphere", inside of which the material is practically fully ionized, and outside of which it is essentially neutral.

- How many hydrogen-ionizing UV photons does a typical 40000 K main-sequence star emit per second?

Approximate the star by a blackbody with a radius of $10\,R_\odot$.

(*Hint:* The number of Photons in the range $[\nu, \nu + \mathrm{d}\nu]$ emitted per second per unit area by a blackbody is $\frac{\pi B_\nu}{h\nu}\,\mathrm{d}\nu$. What is the ionization energy of hydrogen? Is the Wien approximation applicable?)

How large is the H II region around the star described in the previous problem, if the interstellar medium consists of pure hydrogen with a number density of $n_\mathrm{H} = 10^2/\mathrm{cm}^3$? What is its mass?

Assume that the ionized region has a temperature of $T = 8000$ K. At this temperature, the recombination rate coefficient for hydrogen is about $\alpha_\mathrm{B} = 3.5 \times 10^{-13}$ cm^3/s. (Consult the lecture for details on what the recombination rate coefficient describes.)

(*Hint:* The Strömgren radius is the region where the recombination rate equals the ionization rate. It is related to the number of ionizing photons per unit time by the relation:

$$R_\mathrm{S} = \left(\frac{3}{4\pi}\frac{L_\gamma}{n_\mathrm{H}^2 \alpha_\mathrm{B}}\right)^{1/3}$$

In equilibrium, the number of ionizing photons emitted by the star per second must equal the number of recombinations per second in the H II region.)

Problem 3 *Radiation through an isothermal layer*

Electromagnetic radiation can be described by the Planck law and its relatives (the Wien displacement law and the Stefan-Boltzmann law). These equations hold strictly in TE ("Thermodynamic Equilibrium") and reasonably well in most stellar photospheres. The Planck function specifies the radiation intensity emitted by a gas or a body in TE (a black body, cf. Figure 1).

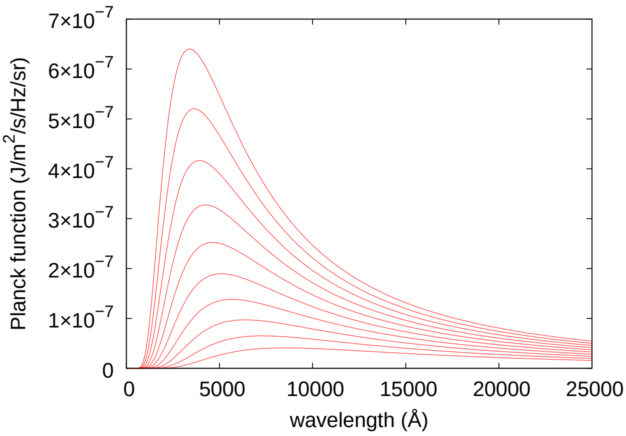

Figure 1: Planck function B_ν for temperatures from 6000 K to 15000 K.

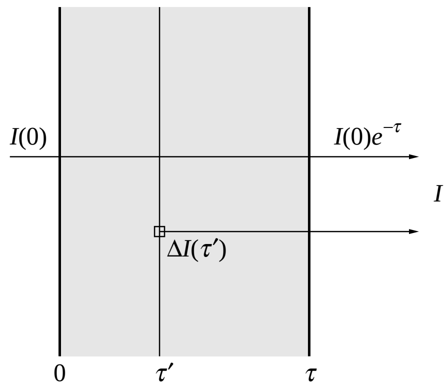

Figure 2: Emergent radiation from a layer consists of transmitted radiation and radiation produced in the layer itself.

Another quantity next to the radiation produced by a gas of temperature T is the amount of absorption. Take the situation sketched in Figure 2.

A beam of radiation with intensity $I(0)$ passes through a layer in which it is attenuated. The weakened intensity that emerges on the right is given by $I = I(0)e^{-\tau}$ in which the decay parameter τ specifies the attenuation by absorption in the layer. It is a dimensionless measure of the opaqueness of the layer that is usually called the "optical thickness" because it measures how thick the layer is, not in cm but in terms of its effect on the passing radiation. Nothing comes through if $\tau \gg 1$ and (almost) everything comes through if $\tau \ll 1$.

The next step in computing the total emergent radiation is to add the radiation that originates within the layer itself. The amount ΔI of radiation that is generated locally at position τ' within the layer is equal to ________ (assume the layer has a fixed temperature $T(\tau')$). This radiation is subsequently attenuated by the remainder of the layer to the right, so that its contribution to the emergent beam is given by ________. The total emergent intensity (containing all contributions) is therefore ________, which for an isothermal layer (one in which T and thus also $B(T)$ is independent of τ') simplifies to ________.

(a) Derive the 4 equations which are marked by "________".

(b) Make plots of the emergent intensity I for given values B and $I(0)$ against the total layer thickness τ. Use $B = 2$, $I(0) = 0$, 1, 2, 3, 4, and $\tau = 0 \ldots 10$.

(c) How does I depend on τ for $\tau \ll 1$ when $I(0) = 0$? And when $I(0) > B$? Such a layer is called "optically thin". Why?

(d) A layer is called "optically thick" when it has $\tau \gg 1$. Why? The emergent intensity becomes independent of τ for large τ. Explain why this is so in physical terms.

Problem 4 *Key questions*

Answer the following questions summarising your Lectures, Matter and radiation in Stars:

- What is a distribution function? What are the characteristic of the distribution functions of fermions and bosons? What is the Planck function?
- What is the Fermi's degeneracy? Which kind of astronomical objects are supported by that?
- Give the equation of state for normal star, a white dwarf (non -relativistic and relativistic case), photons?
- What do the Boltzmann and Saha equations describe?
- How would you define the effective temperature of a star?
- Which kind of interactions are possible between photons and electrons?
- What are 3 main mechanisms responsible for the line broadening?
- Describe the following scattering processes: Rayleigh scattering, Thomson scattering, and Compton scattering.
- What is the synchrotron radiation?
- Define the equation of radiative transfer, the absorption coefficient (opacity), the emission coefficient and their relation.
- What is the optical depth?
- Define the local thermodynamical equilibrium.
- By using the equation of the radiative transfer derive when you can have emission line and absorption line spectra
- What is the equivalent width?