Problem set #4

Problem 1 *Interior structure of a star*

The density structure of a star as a function of the radius r can be approximated by

$$\rho(r) = \frac{\rho_0}{R^2}(R-r)^2,$$

where R is the radius of the star and ρ_0 is the central density. Given this density distribution derive:

1) The central density ρ_0 as a function of total mass,
2) The mass as a function of radius, $M(r)$,
3) The pressure as a function of radius, $P(r)$.

Use the equation for hydrostatic equilibrium:

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\frac{GM(r)\rho(r)}{r^2}$$

and the mass-density relation:

$$\frac{\mathrm{d}M(r)}{\mathrm{d}r} = 4\pi r^2 \rho(r).$$

Problem 2 *Timescales*

(a) Hydrogen fusion has an efficiency of about 0.7% for converting mass into energy. Assume that the Sun will use 10% of its hydrogen for fusion (and that it is mostly hydrogen). Given the Sun's luminosity, how long will it shine? How many kg of mass is the Sun losing each second by converting mass into energy?

(b) We know that the Sun is not "on fire" because chemical reactions are not nearly efficient enough to keep the Sun shining at its current luminosity for anything like the amount of time we know it's been around. The efficiency of chemical burning of hydrogen in the reaction: $2H_2 + O_2 \rightarrow 2H_2O$ is about 2×10^{-10}: that is the fraction of its mass that gets converted into energy. Assuming that the Sun were made up of oxygen and hydrogen in just the right proportions, and that it was able to burn all of its mass, how long (for how many years) would it be able to shine at its current luminosity using chemical reactions?

(c) Another possible energy source is the energy released from gravitational contraction. When you drop something from a height, energy is released; you may use that energy to make a sound, break something, etc. Suppose that you consider all the mass of the Sun to have been "dropped" from a great distance on to the Sun. The total energy released is approximately GM^2/R, where M is the mass of the Sun and R is the radius of the Sun. If this were where the Sun got its energy, for how long would it have been able to shine at its current luminosity?

Problem 3 *The Gamow Energy and the sun*

Nuclear burning in the sun would be impossible without the tunneling effect. The probability that a particle A penetrates the Coulomb barrier of a particle B is given by

$$P_{pen} \simeq \exp\left[-\left(\frac{E_G}{E}\right)^{1/2}\right] \tag{1}$$

where the Gamow Energy E_G is given by

$$E_G \simeq 2m_r c^2(\pi\alpha Z_A Z_B)^2. \tag{2}$$

Here α is the fine structure constant $\simeq 1/137$ and m_r is the reduced mass of the two particles.

- Calculate the Gamow energy (in SI units and in electron volts) for the following reactions:
  - the collision of two protons;
  - the collision of two ${}^3_2He$ nuclei. (For simplicity, assume the mass of a ${}^3_2He$ nucleus, denoted by m_3, is $3m_p$.
- The core of the Sun has a temperature of $15.6 \times 10^6 K$. Calculate the penetration probabilities for the two interactions.
- Not all particles within the core of the Sun have the same energy of course. Assume that the fusion probability is actually given by

$$\exp\left[-(E_G/E)^{1/2} - \frac{E}{kT}\right] \tag{3}$$

  and derive an expression for the location of the Gamow peak. Convert the corresponding energy to temperature.

Problem 4 *Stellar Modeling*

For this assignment you will need the program StatStar, which can downloaded from the website

`http://wps.aw.com/aw_carroll_ostlie_astro_2e/48/12319/3153834.cw/index.html`

- Download the code for this assignment. You can use a computer running Windows and open the executable. Linux users: Compile from the code, `C++` and `Fortran95` available.
- Use StatStar to create valid zero-age main sequence (ZAMS) models for the following stars. You can use the masses and temperatures below, and Figure 1 can help you determine the luminosity, but these will just be starting points. You will need to try a few values of the luminosity to find a valid model. A valid model means no negative values for pressure, density and temperature, as well as realistic values for luminosity and radius. For each model, use $X = 0.7$ for the mass fraction of hydrogen, $Y = 0.292$ for the mass fraction of helium, and 0.008 for the mass fraction of metals.

M/M_{sol}	$T_{\mathrm{eff}}(K)$
0.50	2287.70
0.75	3788.50
1.00	5402.00
2.00	10,952.60
10.00	27,933.00
15.00	32,873.30

- For each star, use your models to determine its central temperature, central pressure, central density, and radius.
- For two of your models (the highest mass and the lowest mass), generate a plot of temperature vs. radius and a plot of pressure vs. radius.

Problem 5 *Key questions*

Answer the following questions summarizing the Theory of Stellar Structure

- What are the essential equations governing stellar structure?
- How is luminosity generated in stars? Describe the two main processes for the main sequence. In which stars do they set in? Which quantum mechanical effect is important for this to work?
- What is the triple-alpha process? What is special about it and what can you learn about the primordial nucleosynthesis? In which evolutionary state does this process occur? Give other examples for how elements can be produced in stars. What is different for elements heavier than iron and why?
- How is energy transported in stars? Describe the two main processes.
- How does the lifetime of stars depend on their mass? Derive the corresponding scaling relation using a few simple arguments.
- Why can't main sequence stars have arbitrarily low surface temperatures?

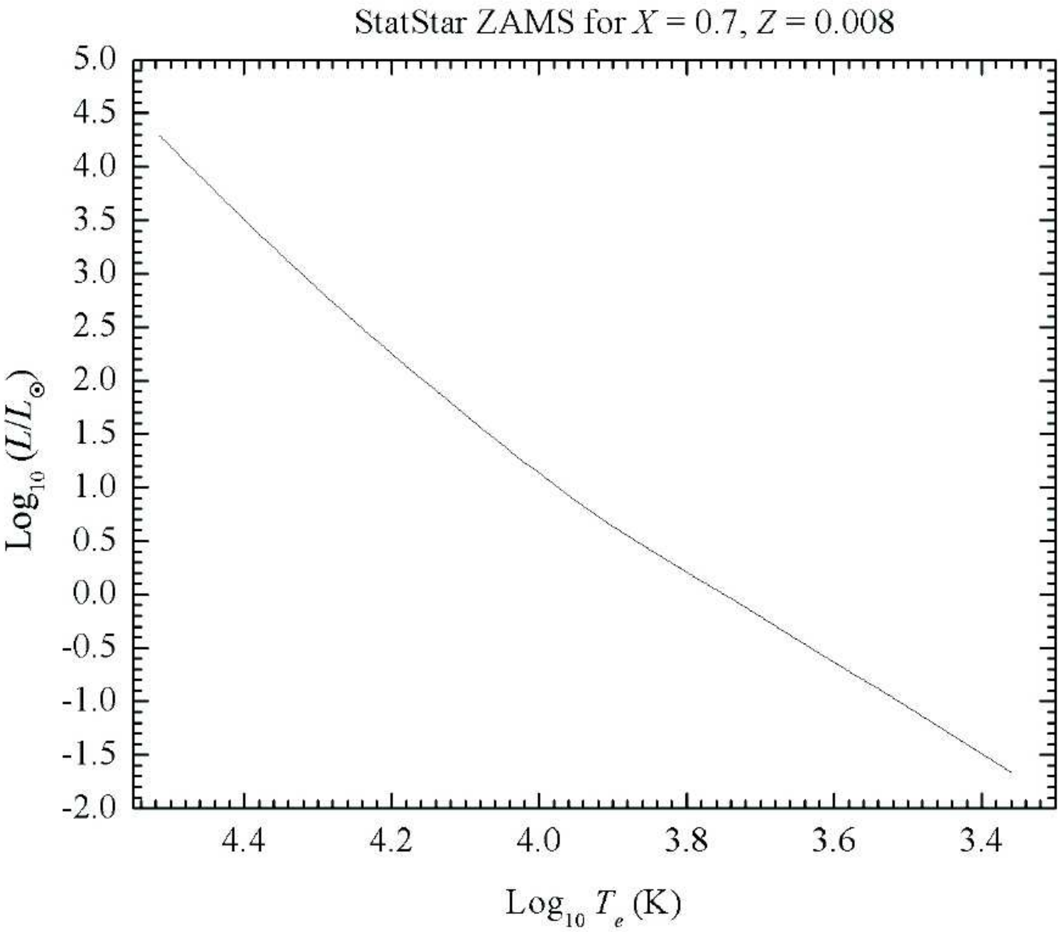

Figure 1: ZAMS Luminosity Estimates