Radiative processes, stellar atmospheres and winds (WS 2017)

Problem set 7

Problem 1 [3 points] *Eddington limit*

a) Show that the quantity $s_\mathrm{e} = n_\mathrm{e}\sigma_\mathrm{e}/\rho$ (page 86, right panel) equals 0.34 cm^2/g for hot stars with a solar helium content $Y_\mathrm{He} = n_\mathrm{He}/n_\mathrm{H} = 0.1$, neglecting metals. Hint: How many free electrons are there per Helium atom?

b) Provide a compact expression for the Eddington factor, Γ_e, in dependence of effective temperature T_eff, gravitational acceleration g, Helium content Y_He and number of free electrons per Helium atom, I_He (neglect metals).

c) Provide a rough estimate for the ZAMS-luminosity (in $L_\odot$) where the Eddington limit is (almost) reached. Hint: Use the global scaling relation $L \propto M^3$ which is valid for not too massive objects close to the ZAMS.

   (Note that in reality it is quite difficult to *hit* the Eddington limit, since for very massive stars $L \propto M$).

Problem 2 [3 points] *Pressure scale height*

a) Carry out the excercise from page 86, bottom of left panel. With respect to the figure, assume that $\rho(m = 1\mathrm{g/cm}^2) = 10^{-9.4}\mathrm{g/cm}^3$. Note that the Helium content is solar.

   What is the corresponding mean molecular weight, μ? Hint: Don't forget the contribution by electrons.

b) Verify that the influence of the Thomson acceleration mitigates the discrepancy between the pressure scale height derived from the stellar parameters and the one implied by the figure.

Problem 3 [3 points] *Limb darkening revisited* Solar limb darkening provides an ideal test bed to check the accuracy of plane parallel, grey atmospheres using the Eddington approximation. Remember first that a linear source function, $S_\nu = a + b\tau_\nu$, gives rise to $I_\nu^\mathrm{em} = a + b\mu$, i.e., provides linear 'limb darkening coefficients' a and b.

Our plane-parallel, grey atmosphere in Eddington approximation just predicts these coefficients, from $S = J$ (radiative equilibrium) and the run of $J(\tau)$. Use this information to derive a and b, and calculate the ratio $I(\mu)/I(\mu = 1)$ for frequency-integrated intensities.

Show that the predicted ratio is independent from any stellar parameter. Note also that the limb-darkening coefficients do not depend on the LTE assumption! Which limb-darkening coefficients would be obtained in a purely scattering atmosphere? What can be said about the frequency dependence in the latter case?

For the sun, the ratio $I(\mu)/I(\mu = 1)$ has been measured by different observational campaigns, both in integrated light (remember, that all quantities derived by the grey model are integrated quantities), as well as at various wavelengths.

Examples for such solar measurements are provided in the following table. For observations at 3000, 5500 and 10000 Å see 'Allen's Astrophysical Quantities', ed. A.N. Cox, p. 356, whilst the info on the observed limb darkening in integrated light has been drawn from Aller, 'The atmospheres of the sun and the stars', p. 221, which base on observations by Abbot, Aldrich & Fowle and Moll, Burger & van der Bilt.

μ	1.	.9	.8	.7	.6	.5	.4	.3	.2	.1	.05	.02
$I(\mu)/I(1)$ (integrated)	1.	.944	.898	.842	.788	.73	.67	.602	.485			
$I_\lambda(\mu)/I_\lambda(1)$ (3000 Å)	1.	-	.77	-	.57	.48	.39	.30	.22	.14		
$I_\lambda(\mu)/I_\lambda(1)$ (5500 Å)	1.	-	.89	-	.769	.703	.633	.556	.468	.371	.31	.24
$I_\lambda(\mu)/I_\lambda(1)$ (10000 Å)	1.	-	.941	-	.87	.828	.783	.731	.675	.59	.54	

Plot your result for the plane parallel, grey atmosphere in Eddington approximation, together with the observed values. What do you conclude, and why is there a wavelength dependence?

Problem 4 [3 points] *Grey atmosphere*

a) Show that in a plane-parallel, grey atmosphere in Eddington approximation the (spatially constant!) frequency integrated radiative flux is given by

$$F = \pi S(\tau = 2/3).$$

This is a specific variant of the usual Eddington-Barbier relation.

b) Use the results for a plane-parallel, grey atmosphere in Eddington approximation to calculate $I^+(\tau)$ and $I^-(\tau)$ as a function of τ, when assuming that these frequency-integrated intensities are angle-independent (cf. Problem 6/3a, remembering that this approximation is consistent with the Eddington approximation).

At which optical depth is the (frequency integrated) radiation isotropic to within 1%? Measure the degree of anisotropy by the ratio

$$\frac{I^+ - I^-}{0.5(I^+ + I^-)}$$

Note again that for 4a) and 4b) the LTE assumption is not required!

Merry Christmas, and a Happy New Year!