Radiative processes, stellar atmospheres and winds (WS 2017)

Problem set 5

Problem 1 [8 points] *Flux conservation by scattering, and Thomson acceleration*

a) Prove that coherent scattering conserves the monochromatic flux, both in planar ($F_\nu(z) = \text{const}$) and in spherical symmetric geometry ($r^2 F_\nu(r) = \text{const}$, to account for geometrical dilution).

b) Thomson acceleration is the radiative acceleration due to Thomson scattering. As we have seen above, (Thomson) scattering *conserves the monochromatic radiative flux* (except for the geometric dilution), and thus also the total flux.

Let's return to Problem 4/2 (Solar sail). Instead of the sail, we now assume that a certain volume of completely ionized hydrogen shall be accelerated by the solar irridiance, due to Thomson scattering.

Use the results from the script (page 71/73) to derive an expression for the corresponding radiative acceleration. Show that the result is independent of the mass, density or volume of the hydrogen plasma, namely

$$g_{\mathrm{rad}}(r) = \frac{f_\odot}{c}\frac{\sigma_{\mathrm{e}}}{m_{\mathrm{H}}},$$

or, more generally (for an arbitrary, spherically symmetric stellar configuration),

$$g_{\mathrm{rad}}(r) = \frac{F(r)}{c}\frac{\sigma_{\mathrm{e}}}{m_{\mathrm{H}}},$$

with $F(r)$ the total radiative flux at distance r from the stellar center.

c) When is the acceleration of a black solar sail (configuration as in Problem 4/2) equal to the Thomson acceleration derived in 1b)? From this condition, calculate the corresponding radius of the solar sail for a total weight of 10 tons.

d) Show that the ratio between the Thomson acceleration from problem 1b (outwards) and the stellar gravitational acceleration (inwards) depends basically on the ratio between stellar luminosity and mass, and that it is independent of distance,

$$\frac{g_{\mathrm{rad}}^{\mathrm{Thomson}}(\mathrm{ionized\,H-plasma})}{g_{\mathrm{grav}}} = \mathrm{factor1}\frac{L}{M} = \mathrm{factor2}\frac{L/L_\odot}{M/M_\odot}$$

Derive "factor1", and calculate "factor2".

Problem 2 [4 points] *Plane-parallel slab*

Consider a plane-parallel slab of gas of thickness L and constant temperature T. At the surface of the slab, $\tau_\nu = 0$, and the total optical depth shall be $\tau_\nu^{\max}$. Assume spatially constant opacities and emissivities, and thermodynamic equilibrium when $\tau_\nu^{\max} \gg 1$.

a) No radiation shall enter from the boundaries.

   Show by using the solution of the equation of radiative transfer that when looking at the slab from above, you see black-body radiation if $\tau_\nu^{\max} \gg 1$, and emission lines if $\tau_\nu^{\max} \ll 1$ and the emissivity, η_ν, is large.

b) Now, incident intensity I_ν^{inc} shall enter the bottom of the slab.

   Show that when looking at the slab from above, you again see black-body radiation if $\tau_\nu^{\max} \gg 1$.

   What do you see if $\tau_\nu^{\max} \ll 1$, and (i) $S_\nu < I_\nu^{\mathrm{inc}}$, or (ii) $S_\nu > I_\nu^{\mathrm{inc}}$. Use again the solution of radiative transfer. Case (i) corresponds to a typical stellar photosphere, and case (ii) to a stellar chromosphere.

Have fun, and much success!