Radiative processes, stellar atmospheres and winds (WS 2017)

Problem set 2

Problem 1 [5 points] *Color indices*

[To understand the physical background of this problem, please study p. 60/61 of the script by yourself.]

Apparent magnitudes from stars might be approximated by the relation

$$m_\lambda \approx -2.5 \log_{10}(\pi B_\lambda(T)\Delta_\lambda(R_*/d)^2) + C_\lambda,$$

if one (i) neglects interstellar absorption ("reddening"), (ii) assumes that the star is a black-body radiator, and (iii) approximates the sensitivity function of the photometer by a rectangular shape with amplitude '1' around 'effective wavelength' λ within the 'effective bandwidth' Δ_λ, and '0' elsewhere. m_λ is the magnitude at band λ, B_λ is the Planck-function at effective wavelength λ, R_* is the stellar radius, and d is the distance to the star. C_λ is a constant for the considered band, set from certain conditions (roughly: all magnitudes of the A0V star Vega shall be zero).

In the above approximation for m_λ, the following issues were considered:

- Since the star has been approximated by a black-body radiator, its (outward directed) flux is given by $F_\lambda = \pi B_\lambda(T)$ (p. 61, right).
- The integral over the response function $S(\lambda)$, $\int F(\lambda)S(\lambda)\mathrm{d}\lambda$, collapses to $F_\lambda \Delta_\lambda$ in the above approximation.
- the factor $(R_*/d)^2$ arises because of the quadratic dilution of the radiation field. Since, in the absence of interstellar absorption, the total energy/time remains conserved, i.e., $L = F_* \cdot 4\pi R_*^2 = F_{\mathrm{earth}} \cdot 4\pi d^2$, we find $F_{\mathrm{earth}} = F_*(R_*/d)^2$.

Now to the actual problem. For the blue supergiant ζ Pup (Naos, O4If, surface temperature $\approx$ 40,000 K), the brightest O-star on the southern sky, U = 0.88, B = 1.97, and V = 2.25.

Table 1: Effective wavelengths and bandwidths (in Å) for the UBV system

Magnitude	effective wavelength	bandwidth
U	3640	680
B	4420	980
V	5400	890

a) Approximate, from these numbers, the constants $C_{U-B} = C_U - C_B$ and $C_{B-V} = C_B - C_V$ for the color indices $U - B$ and $B - V$ (for effective wavelengths and bandwidths, see Table 1).

b) Using the constants derived in a), approximate the color indices $U - B$ and $B - V$ for stars with surface temperatures between 10,000 K and 60,000 K, with stepsize 10,000 K.

c) Plot a color-color diagram ($U - B$ vs. $B - V$, with x-axis $B - V$) for these 'stars'. Draw your axes in such a way that the bluest stars appear in the upper left corner. What do you conclude? (Compare Problem 1.3).

d) What could be the reason that the approximate $U - B$ color index at 10,000 K is significantly different from zero as it should be because of the above normalization convention (Vega)? (Hint: consider the actual flux distribution of an A0 star, see lecture script).

Problem 2 [4 points] *Number density of blackbody photons*

a) Show that the number density of blackbody photons, $n_\lambda d\lambda$, is given by

$$n_\lambda d\lambda = \frac{8\pi}{\lambda^4} \frac{1}{\exp(hc/(\lambda kT)) - 1}$$

b) Calculate the total number of (blackbody) photons inside a kitchen oven at 250°C, assuming a volume of 0.5 m^3.
Hint: $\int_0^\infty x^2/(\exp(x) - 1)\mathrm{d}x = 2\mathrm{Zeta}(3) = 2.40411$, with Zeta($x$) the Zeta-function.

Problem 3 [3 points] *Average blackbody photon energy*

a) Use the results from Problem 2b) and the relations from the script to show that the average energy per blackbody photon, u/n, is given by

$$\frac{u}{n} = 2.70kT,$$

where u is the total energy density and n the total number density of black body photons of all wavelengths.

b) Calculate the average energy per blackbody photon at the center of the Sun ($T \approx 1.57 \cdot 10^7$ K) and in the solar photosphere ($T = 5777$ K). Express your results in units of electron Volt (eV).

Have fun, and much success!