Statistical methods – an introduction (SS 2016)

LAST problem set (10)

Problem 1 [2 points] *Exponential distribution: ML-estimator*

Calculate the ML-estimator for the life time τ of an exponential distribution, $f(t) = (1/\tau)\exp(-t/\tau)$, given a set of N observed life times t_i.

Problem 2 [2 points] *Lorentzian distribution*

As already discussed in problem set 6/2, a Lorentzian distribution (sometimes also called Breit-Wigner distribution) can be used, e.g., to describe the profile function of certain spectral lines. With respect to angular frequency ω, it is defined as

$$\Phi(\omega)\mathrm{d}\omega = \frac{\Gamma}{2\pi}\frac{1}{(\omega-\omega_0)^2 + \left(\frac{\Gamma}{2}\right)^2}\,\mathrm{d}\omega,$$

where ω_0 is the central=transition frequency and Γ the so-called 'damping parameter'.

Construct the ML-estimator for the damping parameter, Γ, given a sample of N angular frequencies, $\Delta\omega_i = \omega_i - \omega_0$, drawn from this distribution. Derive only the (non-linear) equation that defines $\tilde{\Gamma}$, since there is no analytic solution.

Problem 3 [4 points] *Information and other expectation values*

a) From some general arguments, it can be shown that $E(d\ln L/d\lambda) = 0$ (see script, Eq. 7.10). Convince yourself about the validity of this relation, by *explicitely* calculating (i) $E(d\ln L/d\mu)$ and (ii) $E(d\ln L/d\sigma)$, for the case of a normal distribution with expectation value μ and standard deviation σ.

b) Calculate the information $I(\sigma)$, using (see script p. 186)

$$I(\sigma) = -E\Big(\frac{d^2\ln L}{d\sigma^2}\Big),$$

for a normal distribution with standard deviation σ.

Problem 4 [4 points] *Minimum variance bound*

Calculate the MVB for the estimator $\mathrm{S}(\sigma^2) = s'^2 = \frac{1}{N}\sum(x_i - \bar{x})^2$ given a Gaussian distribution. Show that this MVB is in agreement with the fact that $Var(s'^2) = 0$ for $N = 1$.

Note: In problem 3b), you should have calculated $I(\sigma)$. Here, however, we are dealing with an estimator for σ^2!

Have fun, and much success!