# Stellar Wind Momentum in Galaxies and a New Parametrization of the Radiative Line Force

R.P. Kudritzki[1], U. Springmann, J. Puls, A.W.A. Pauldrach and M. Lennon[1]

*Institut für Astronomie und Astrophysik der Universität München, Scheinerstr.1, D-81679 München, Germany*

**Abstract.**

We demonstrate that the classical concept of "line force multipliers" $\hat{k}$, $\alpha$, $\delta$ to calculate the radiative line acceleration is insufficient. General tables with these force multiplier parameters as function of effective temperature only are bound to fail in predicting accurate wind properties across the HRD in all stages of stellar evolution. As a consequence, we develop a new parametrization method which is more precise and will allow a much better description of radiation driven winds.

## 1. Introduction

One of the triumphs of the theory of radiation driven winds is the prediction of the **Wind Momentum – Luminosity Relationship (WLR)**

$$\dot{\mathbf{M}}\mathbf{v}_\infty \mathbf{R}_*^{0.5} \propto \mathbf{L}^{\mathbf{x}} \quad (1)$$

and its confirmation by observations of supergiants of spectral type O, B, and A (see Fig. 1). This relationship may become the basis for a new extragalactic distance determination method capable of reaching out to the Virgo and Fornax clusters of galaxies. It may also be used to estimate momentum and energy input into the ISM and the strength of mass-loss throughout all phases of hot star evolution by combining the empirical WLR with the empirical relation between terminal and escape velocity. At present, a vigorous observing programme is carried out within a collaboration of colleagues in Caltech, Tenerife, Minnesota and Munich using all large telescopes available to test the WLR-method in Local Group galaxies and to calibrate its metallicity dependence (see the corresponding contributions in this volume and Kudritzki 1998).

Another important aspect is the quantitative theoretical interpretation of the observed WLR. Puls *et al.* 1996 have already shown that for O-stars the theory predicts roughly the correct slope but fails to explain the difference in absolute strength between supergiants and dwarfs. A careful inspection of Fig. 1

[1]Max-Planck-Institut für Astrophysik, D-85740 Garching bei München, Germany

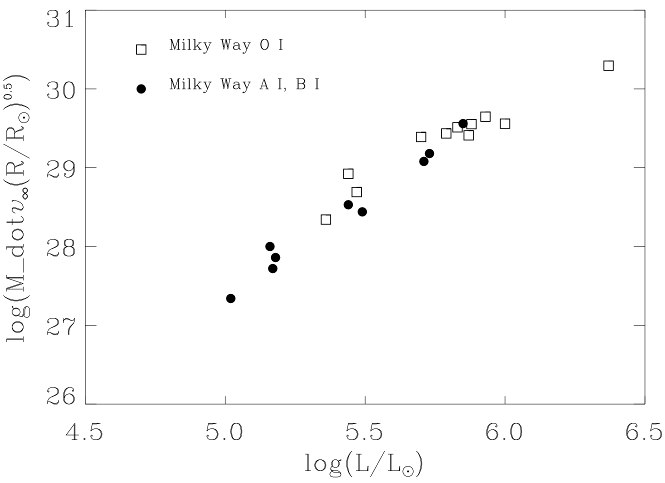


Figure 1. The observed Wind Momentum - Luminosity Relationship of A-, B- and O-supergiants in the Galaxy. All mass-loss rates have been determined from $H_\alpha$. The terminal velocities are derived from the blue edges of UV P-Cygni profiles for the O- and B-supergiants and from $H_\alpha$ for the A-supergiants (data from Puls *et al.* 1996 and Kudritzki *et al.* 1998, in preparation. The results for the A- and B-supergiants are still preliminary and need final refinement).

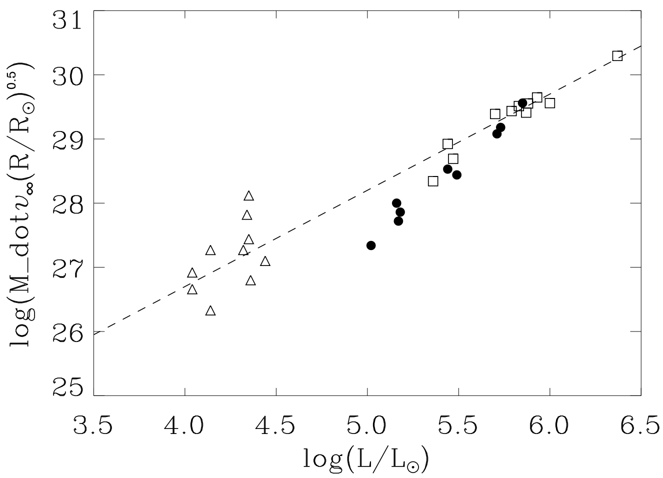


Figure 2. The observed WLR for galactic A-, B- and O-supergiants compared with wind momenta of Central Stars of Planetary Nebula (triangles) also obtained from $H_\alpha$ and UV P-Cygni profiles (see Kudritzki *et al.* 1997). The dashed line is the extrapolation of the WLR for O-stars with slope $x = 1.5$.

now reveals that there may be another challenge for the theory, since there is an indication of a change in slope between O-supergiants and A,B-supergiants. This becomes more evident in Fig. 2, where we have included the observed wind momenta of Central Stars of Planetary Nebulae as determined recently by Kudritzki *et al.* 1997. Strikingly, the data for the CSPN fall on the extrapolation of the WLR for the O-stars. That means that for very hot stars such as O-stars or CSPN the exponent of the WLR is $x = 1.5$, whereas for cooler objects such as A- and B-supergiants we observe $x = 2.5$ (see also McCarthy *et al.* 1997). Obviously, there is a temperature dependence in the slope of the WLR so that it becomes steeper at lower temperatures. The question is why.

The analytical solutions for radiation driven winds developed by Kudritzki *et al.* 1989 yield

$$\dot{M} v_\infty R_*^{0.5} \propto \hat{k}^{1/(\alpha-\delta)} L^{1/(\alpha-\delta)}, \tag{2}$$

where $\hat{k}$,$\alpha$ and $\delta$ are the (in)famous " force multiplier parameters" (fmps) introduced by Castor, Abbott, & Klein 1975 and Abbott 1982 to calculate the radiative line force as a local quantity depending on radius, density, velocity and velocity gradient (see also Pauldrach, Puls, & Kudritzki 1986, Kudritzki *et al.* 1987, Pauldrach 1987 and Kudritzki 1998). These are the crucial numbers needed as function of effective temperature, luminosity, gravity and metallicity to understand the strengths of winds in all stages of stellar evolution. The observed WLR indicates that there is a systematic variation as function of effective temperature. Lamers *et al.* 1995 came to a similar conclusion by analyzing the observed ratios of terminal to escape velocity for O-stars and B-, A-supergiants.

To investigate the systematic behaviour of line driven winds across the HRD we have therefore started a new study of radiative line forces based on the improvements achieved during the last decade with regard to line lists and atomic physics. Here we report first results that came as a complete surprise to us. It turns out that the classical concept of "general" fmps $\hat{k}$, $\alpha$, $\delta$ is insufficient. General tables with these fmps as function of effective temperature only are bound to fail in predicting accurate wind properties. As a consequence, we develop a new parametrization method which is more precise and will in future allow a much better description of radiation driven winds.

The paper is organized as follows. In the next section we describe the physical assumptions leading to the classical parametrization. Then, in section 3, we describe our new calculations of radiative line forces and show how badly they are reproduced by the classical approach. In section 4 we introduce the new parametrization, which gives a much more precise representation of radiative line forces. In section 5 we discuss the next steps of future work.

## 2. The classical parametrization

The crucial term in the hydrodynamics of radiation driven winds is the radiative line acceleration $g_{\rm RAD}^{lines}$, which can be expressed in units of $g_{\rm RAD}^{\rm TH}$, the radiative acceleration provided by Thomson scattering.

$$g_{\rm RAD}^{lines} = g_{\rm RAD}^{\rm TH} CF(r, v, \frac{dv}{dr}) \mathrm{M}(t). \tag{3}$$

$CF(r, v, \frac{dv}{dr})$ is the finite cone angle correction factor, which takes into account that a volume element in the stellar wind is irradiated by a stellar disk of finite angular diameter rather than a point source. M(t) is the **line force multiplier** which gives the line acceleration in units of Thomson scattering. In the Sobolev approximation the contribution of all spectral lines $i$ at frequencies $\nu_i$ and at spectral luminosities $L_{\nu_i}$ to the line force multiplier is given by

$$\mathrm{M}(t) = \frac{v_{\rm THERM}}{c} \frac{1}{t} \sum_i \frac{\nu_i L_{\nu_i}}{L} (1 - e^{-\tau_i}), \tag{4}$$

where $v_{\rm THERM}$ is the thermal velocity of hydrogen, $c$ the speed of light and

$$\tau_i = k_i t(r) \tag{5}$$

is the local (Sobolev) optical depth of line $i$ computed as product of two factors, the **line strength** $k_i$

$$k_i \propto \frac{n_l}{n_e} f_{lu} \lambda_i \tag{6}$$

and the **Thomson optical depth parameter** $t(r)$

$$t(r) = n_e \sigma_e \frac{v_{\rm THERM}}{dv/dr} \tag{7}$$

(for details, see Kudritzki 1998).

In a realistic hydrodynamic stellar wind code the contributions of hundred thousands of lines are added up to calculate the radiative acceleration at every depth point. However , for numerical reasons these numbers are not used directly to solve the hydrodynamical problem. Instead, following the pioneering work of Castor, Abbott, & Klein 1975 and Abbott 1982, $\mathrm{M}(t)$ is fitted by the parametrization ($W(r)$ is the geometrical dilution factor of the radiation field)

$$\mathrm{M}(t) = \hat{k} t^{-\alpha} \{ \frac{n_e(r)}{W(r)} / 10^{11} cm^{-3} \}^{\delta}. \tag{8}$$

$\hat{k}$, $\alpha$, $\delta$ are the **force multiplier parameters (fmps)**. This parametrization has the advantage that it allows very fast numerical solutions (see Pauldrach, Puls, & Kudritzki 1986) and very precise analytical approximations of the complex hydrodynamical problem of line driven winds (see Kudritzki *et al.* 1989), if one assumes that the fmps are constant in the atmosphere.

Why is it possible to approximate the sum over hundred thousands of spectral lines at different wavelengths and of entirely different strengths by such a simple parametrization ? The answer is that to some approximation the distribution function of line strengths obeys a power law

$$n(k,\nu)d\nu dk = (1-\alpha)g(\nu)d\nu k^{\alpha-2}dk, \tag{9}$$

at all frequencies $\nu$ (for a more detailed discussion see, for instance Kudritzki 1998) . The exponent $\alpha$, which physically describes the steepness of the line strengths distribution function – or in simpler words the relative fraction of strong to weak lines –, is mostly determined by the atomic physics of the dominant ionization stages and basically reflects the distribution function of the oscillator strengths. Typical values vary between

$$\alpha = 0.5\ldots0.7. \tag{10}$$

If the sum in M($t$) is replaced by a double integral (in frequency and line strength) using the line strength distribution function, then the first two factors of eq. (8) are obtained. The fmp $\hat{k}$ is then proportional to

$$N_{eff} = \int_0^\infty \frac{\nu L_\nu}{L} g(\nu) d\nu \tag{11}$$

the total number of lines effectively driving the wind. Since $N_{eff}$ changes if the ionization changes in the stellar wind and since in NLTE the ionization balance to first order is determined by the ratio of electron density $n_e$ to geometrical dilution $W(r)$ of the radiation field, the third factor is introduced. It is important to note that neglecting the ionization dependence of the force multiplier described by the third factor leads to unrealistic stellar wind stratifications, in particular for the cooler supergiants of spectral type B and A. On the other hand, we also realize that accounting for ionization effects in the form of eq. (8) assumes that the exponent $\alpha$ of the line strength distribution function does not depend on ionization. We will see later on that this is not true.

The exciting perspective of the concept of fmps is that, if $\hat{k}$, $\alpha$, $\delta$ would be a priori known as function of effective temperature (and abundance), then mass-loss rates and terminal velocities could be immediately computed as function of temperature, luminosity and mass in all evolutionary stages of hot stars (massive and post-AGB) using the analytical formulae of Kudritzki *et al.* 1989. This could be coupled with stellar evolutionary codes and provide important input to understanding stellar and galactic evolution.

In the past, we have therefore tried desperately to use our hydrodynamic NLTE radiation driven wind code (see Pauldrach 1990, Pauldrach *et al.* 1994) to construct such a table of "general" fmps as function of effective temperature and abundance. Unfortunately, we have failed badly. We learned that the approximation of fmps being constant within the atmosphere is too inaccurate. They are definitely depth dependent, although it is always possible a posteriori after the NLTE hydro calculation to define a triple of "representative mean values" that would reproduce the calculated mass-loss rates and terminal velocities. However, these representative mean values could not be predicted a priori as function of stellar parameters before the detailed NLTE hydro calculation. Thus, at the end, we were doubting as to whether the whole concept of "general" fmps is useful at all.

## 3. A reinvestigation

Motivated by the exciting new results of section 1 concerning the WLR we have very recently started a reinvestigation of the fmp-problem. We have used the new line list of atomic data for 2.5 millions spectral lines of the NLTE wind code used in Munich (see also contribution by Pauldrach *et al.* in this volume). In a first step, we were more interested in an investigation of the validity of the concept of "general" fmps rather than in a quantitatively fully correct description. We have, therefore, made several simplifying approximations to calculate M($t$) as a sum over all spectral lines in eq. (4) . We have used analytical approximations for the calculation of NLTE occupation numbers. For the ionization we have adopted

$$\frac{n_1^{atom}}{n_1^{ion}} \propto \frac{n_e}{W} f(T_{rad}, T_e), \tag{12}$$

where $T_{rad,e}$ are the radiation and electron temperature, respectively and f is a function obtained from solving the rate equations including ionization from and recombination to all excited levels. The excitation of subordinate levels is calculated by

$$\frac{n_u}{n_1} \propto W \{\frac{n_u}{n_1}(T_{rad})\}_{Boltzmann}. \tag{13}$$

For metastable levels we use the same formula as eq. (13) but set the dilution factor W equal to unity. Justifications for these approximations are given by Abbot and Lucy 1985 and Springmann 1997.

For the the spectral energy distribution we have adopted a Planck function with $T_{rad} = T_{eff}$. In addition, $T_e = 0.8T_{eff}$ was assumed. It is clear that later on, after the basics of the problem with the "general" fmps would be understood, these simplifications would have to be dropped. It is also clear that this would be straightforward.

With these simplifications it is easy to calculate a very extensive grid of force multipliers M($t$) using equation (4) for

- $T_{eff} = 8000K \dots 55000K$ in small steps of $\Delta T_{eff}$,
- in the relevant range of log t and log $n_e/W$ estimated from observed stellar wind properties of massive stars and post AGB-stars.

Then we investigated whether at each effective temperature the force multipliers can be well represented by the classical fit formula

$$\log \mathrm{M}(t) = \log \hat{k} - \alpha \log t + \delta \log \hat{n}, \tag{14}$$

where

$$\log \hat{n} = \log\{\frac{n_e}{W}\} - 11. \tag{15}$$

The result was devastating. The classical fit is nowhere sufficient. There is an obvious non-linear dependence on $\log t$ and $\log n_e/W$.This is demonstrated

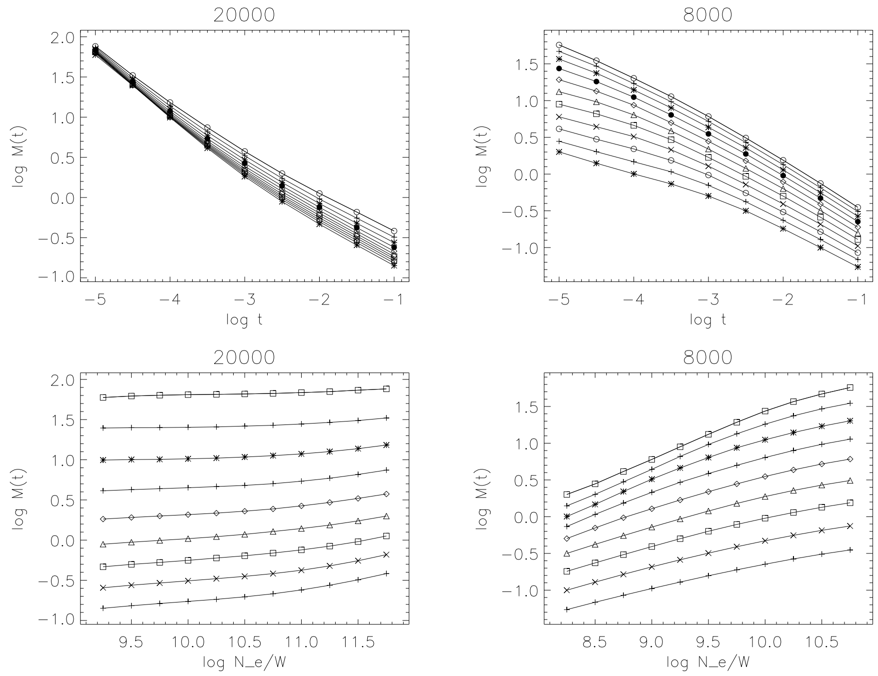


Figure 3. Force multipliers $\log \mathrm{M}(t)$ for $T_{eff} = 20000K$ (left) and $8000K$ (right) plotted as function of $\log t$ and different constant values of $\log n_e/W$ (upper panels) and as function of $\log n_e/W$ for different constant values of $\log t$ (lower panels). The non-linear dependence is striking and also present at all other effective temperatures.

by Fig. 3 for two examples of $T_{eff}$. This means that one either has to work with $\alpha$ and $\delta$ as quantities dependent on $\log t$ and $\log n_e/W$ (see Fig. 4) or one would have to introduce higher order fits for $\mathrm{M}(t)$.

There are two physical reasons for the non-linear dependence of $\log \mathrm{M}(t)$, as the analysis of the line strengths distribution functions shows. First, the simple power law approximation with only one exponent is insufficient. A more detailed inspection reveals that the exponent $\alpha$ varies systematically as function of line strength, which introduces the curvature in the plots of $\log \mathrm{M}(t)$ as function of $\log t$. Second, the exponent $\alpha$ varies as function of ionization, which introduces curvature in the plots of $\log \mathrm{M}(t)$ as function of $\log n_e/W$.

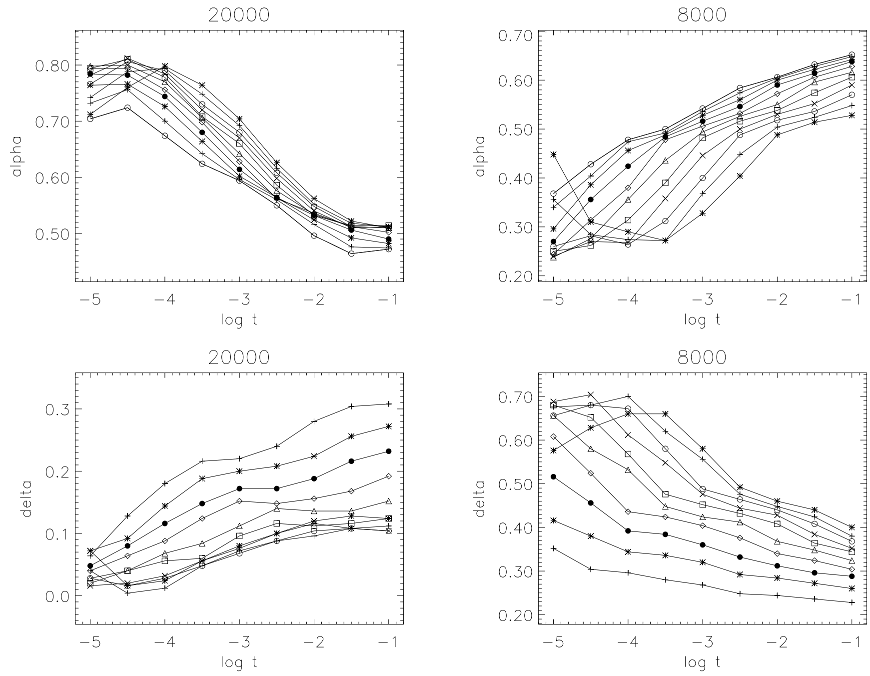


Figure 4. Fmps $\alpha$ (upper panels) and $\delta$ (lower panels) for $T_{eff} = 20000K$ (left) and $8000K$ (right) plotted as function of $\log t$ and different constant values of $\log n_e/W$. The values are obtained from a pointwise differentiation of Fig. 3.

## 4. The new parametrization

The simplest higher order approach of a fit formula for the force multiplier is to assume that both $\alpha$ and $\delta$ depend linearily on $\log t$ and $\log n_e/W$. With this assumption one obtains a new parametrization of the form

$$\log \mathrm{M}(t) = \log \hat{k} - \alpha_o(1+\alpha_1 \log t) \log t + \delta_o(1+\delta_1 \log \hat{n}) \log \hat{n} + \gamma \log t \log \hat{n} \quad (16)$$

This new parametrization gives a very accurate representation of $\mathrm{M}(t)$ at every effective temperature over the full range in $\log t$ and $\log n_e/W$. An example is given in Fig. 5. This means that we have now, in principle, found a way to describe a priori the radiative acceleration for every effective temperature at all densities before we enter a hydrodynamic calculation, which will then yield stellar wind structures, mass-loss rates and terminal velocities. Or – in other words – with the six force multipliers $\hat{k}, \alpha_o, \alpha_1, \delta_o, \delta_1, \gamma$ as function of effective temperature we can now calculate stellar wind properties as function of luminosity, mass etc. in all stages of stellar evolution.

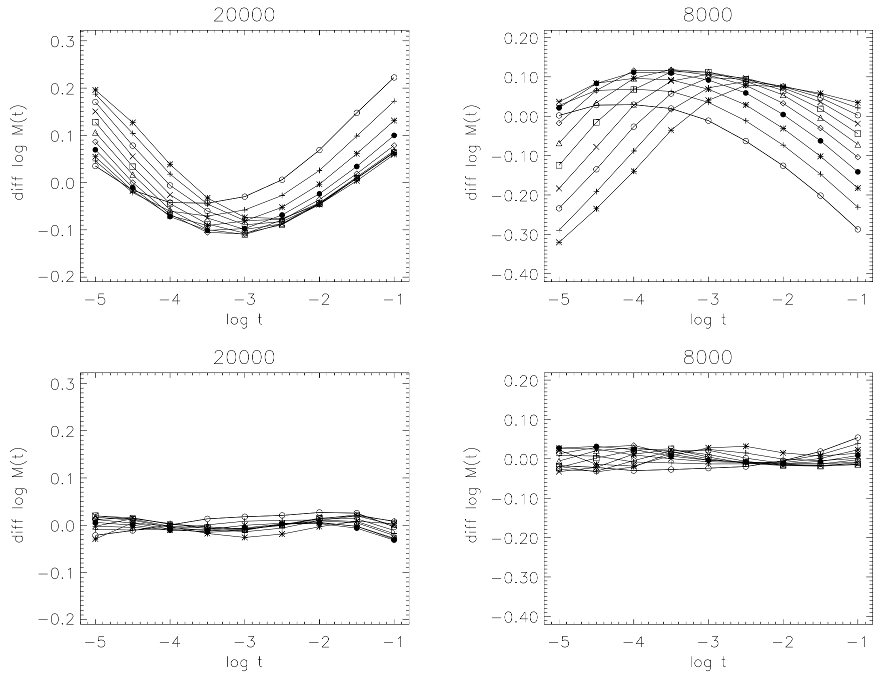


Figure 5. Logarithmic difference between the calculated force multipliers and the fit formulae $\log \mathrm{M}(t) - \log \mathrm{M}_{\mathrm{FIT}}(t)$ as function of $\log t$ and different constant values of $\log n_e/W$. Upper panels use the old force multiplier representation of eq. (14) for $\log \mathrm{M}_{\mathrm{FIT}}(t)$, whereas lower panels use the new one of eq. (16). Again $T_{eff} = 20000K$ (left) and $8000K$ (right) have been selected as examples, but the effects are similar at all other effective temperatures.

## 5. Future work

The obvious next step will be to achieve hydrodynamic stellar wind solutions with the new representation of the radiative line force. This is not easy, but feasible. The complication arises from the fact that the analysis for finding the critical point solutions is vastly complicated by the new form of $\mathrm{M}(t)$. However, equations have already been formulated and we expect to obtain first solutions very soon. Then, we will relax several of the simplifying assumptions such as the Planckian radiation field and the analytical NLTE. We will carefully compare with the full NLTE hydrodynamical stellar wind code (see Pauldrach *et al.* 1994) and will finally try to develop a table for the new force multiplier parameters as function of effective temperature and abundance. We hope that we will then be able to understand the observed WLRs and to give an improved theoretical description of stellar winds across the HRD.

**Acknowledgments.** This work was supported by DARA and the Bundesminister für Forschung und Bildung under grant 50R93040 and the DFG under grant Pa 477/1-2.

## References


ABBOTT D.C. 1979, Proc. IAU Symp. **83**, eds. P.S. Conti & C.W.H. de Loore, p. 237

ABBOTT D.C. 1980, ApJ **242**, 1183

ABBOTT, D.C. 1982, ApJ **259**, 282

ABBOTT, D.C., LUCY, L.B. 1985, ApJ **288**, 671

CASTOR, J.I., ABBOTT, D.C., & KLEIN, R.I. 1975, ApJ **195**, 157

KUDRITZKI, R.P., PAULDRACH, A.W.A., PULS, J. 1987, A&A **173**, 293

KUDRITZKI, R.P., PAULDRACH, A.W.A.,, PULS,J., & ABBOTT, D.C. 1989, A&A **219**, 205

KUDRITZKI, R.P., MENDEZ, R.H., MCCARTHY, J.K., PULS, J. 1997, Proc. IAU Symp. 180 "Planetary Nebulae", invited paper, in press

KUDRITZKI, R.P., PULS, J., LENNON, D.J., VENN, K.A., MCCARTHY, J.K., & HERRERO, A. 1998, A&A , in preparation.

KUDRITZKI, R.P. 1998, Proc. of the 8th Canary Island Winter School "Stellar Astrophysics for the Local Group. A First Step to the Universe", Eds. A. Aparicio and A. Herrero

LAMERS, H., SNOW, T.P., LINDHOLM, D. 1995 ApJ **455**, 269

MCCARTHY, J.K., KUDRITZKI, R.P., LENNON, D.J., VENN, K.A., PULS, J. 1997, ApJ in press

PAULDRACH, A.W.A., PULS, J., & KUDRITZKI, R.P. 1986, A&A **164**, 86

PAULDRACH, A.W.A. 1987, A&A **183**, 295

PAULDRACH, A.W.A., KUDRITZKI, R.P., PULS, J., BUTLER, K. 1990, A&A **228**, 125

PAULDRACH, A.W.A., KUDRITZKI, R.P., PULS, J., BUTLER, K., & HUNSINGER, J. 1994, A&A **283**, 525

PULS, J., KUDRITZKI, R.P., HERRERO, A., PAULDRACH, A., HASER, S.M., LENNON, D.J., GABLER, R., VOELS, S.A., VILCHEZ, J.M., WACHTER, S., & FELDMEIER, A. 1996, A&A **305**, 171

SPRINGMANN, U. 1997, Ph D thesis submitted to the Physics Faculty of Munich University