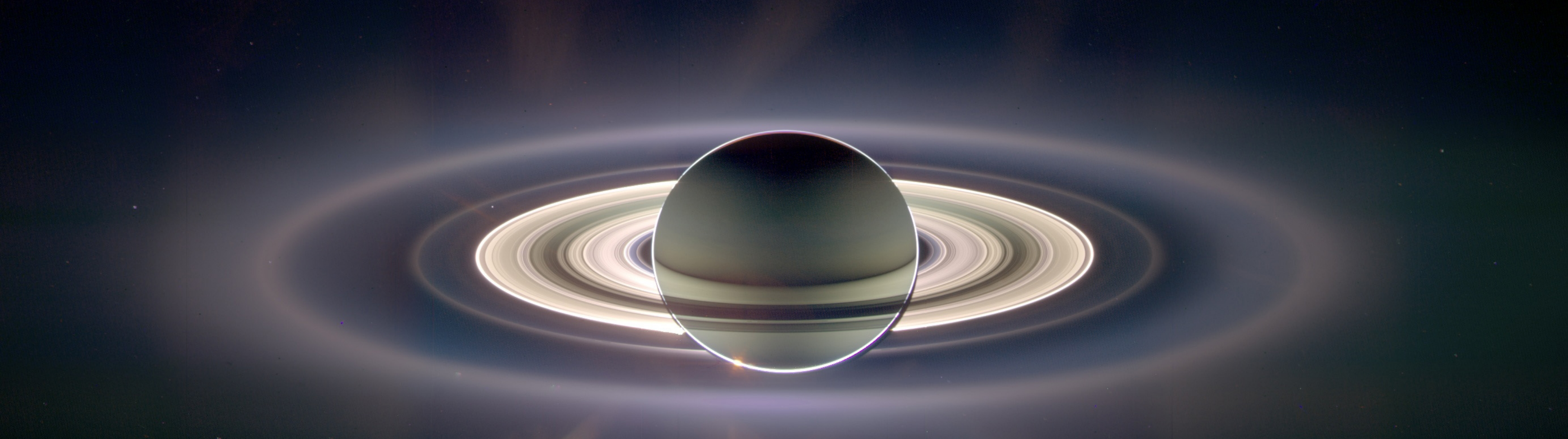
# FORMATION OF SATELLITES from a tidal disk



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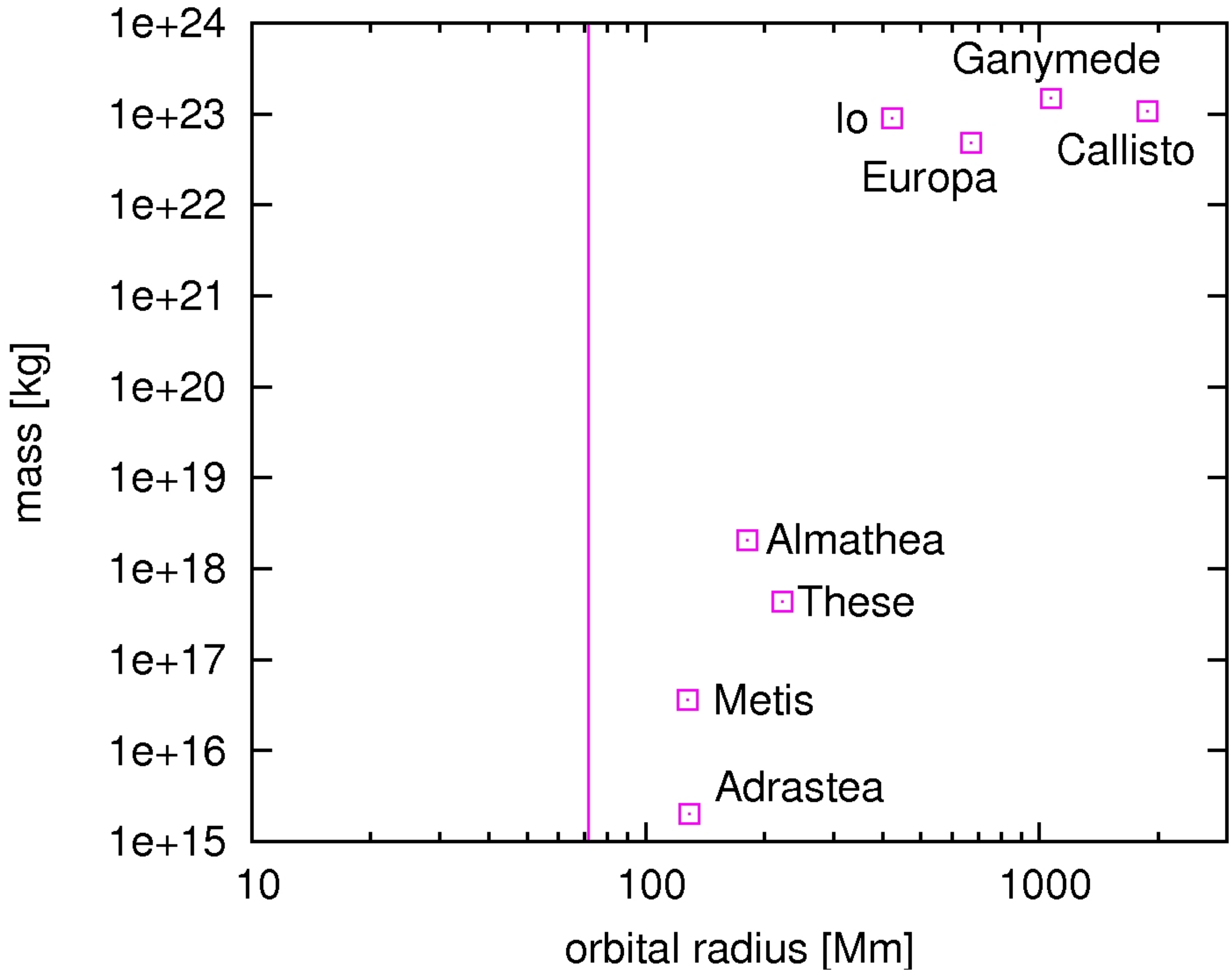
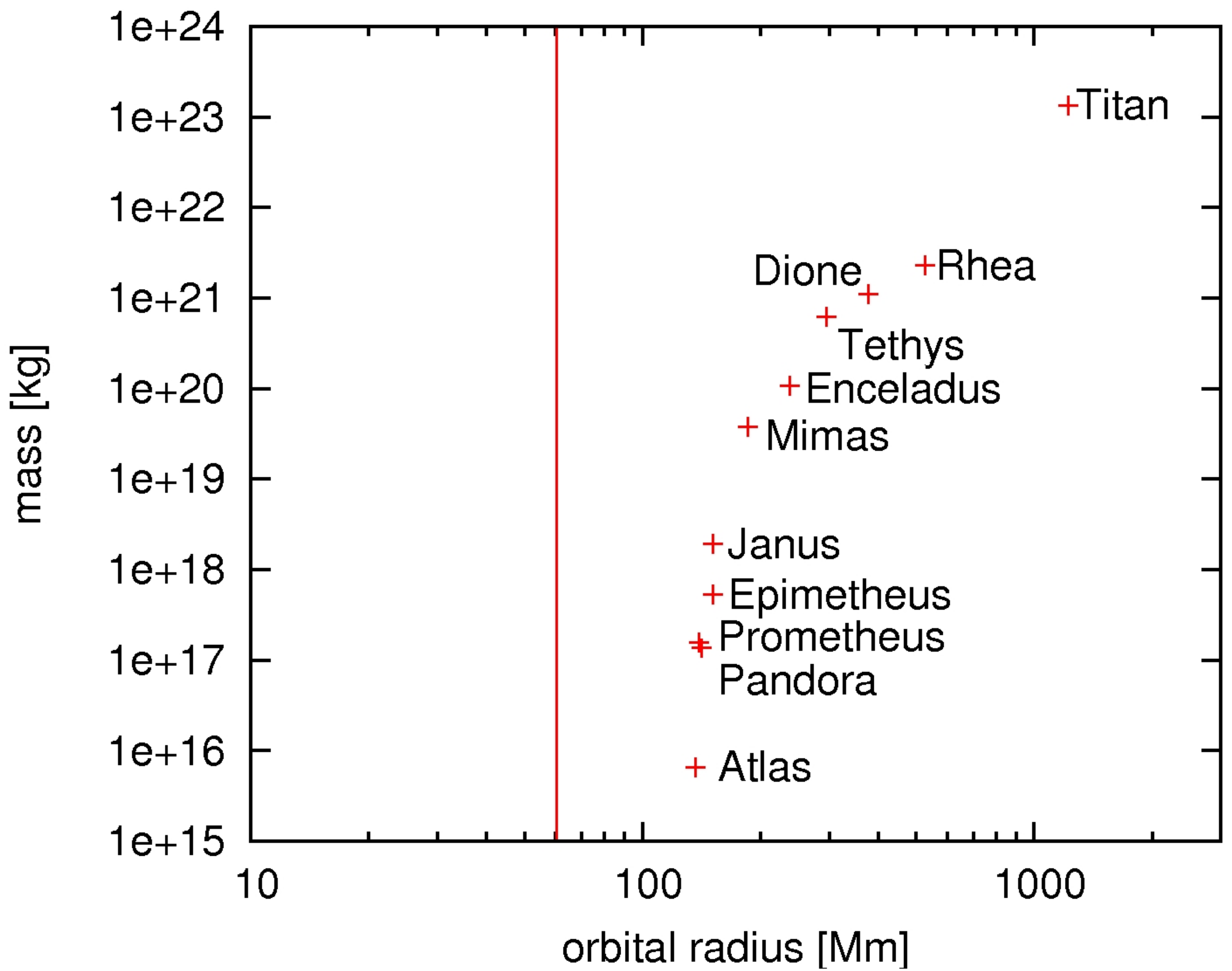
# JUPITER



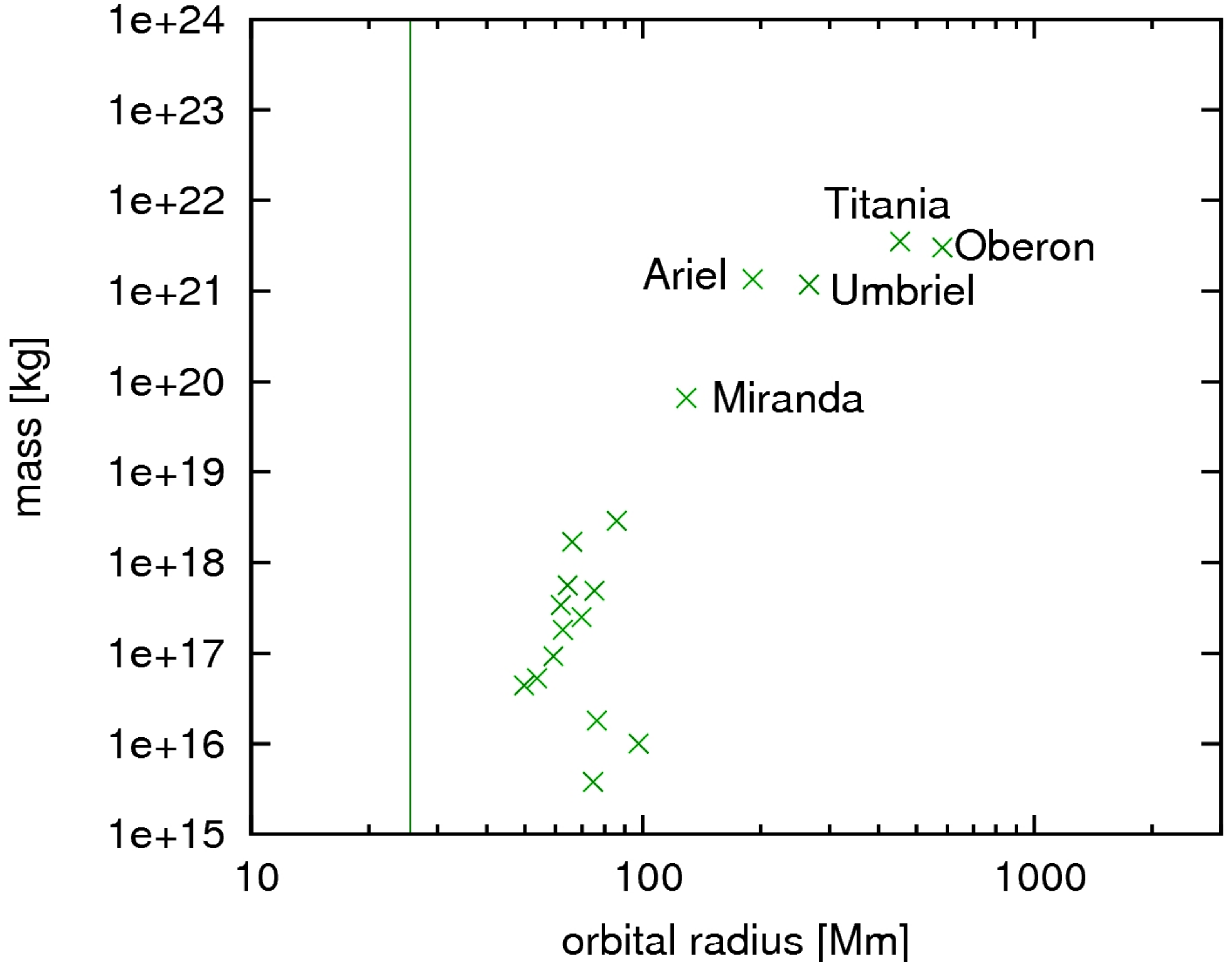
Ganymede
Io
Callisto
Europa
Almathea
These
Metis
Adrastea
1e+24
1e+23
1e+22
1e+21
1e+20
1e+19
1e+18
1e+17
1e+16
1e+15
mass [kg]
10
100
1000
orbital radius [Mm]

# SATURN



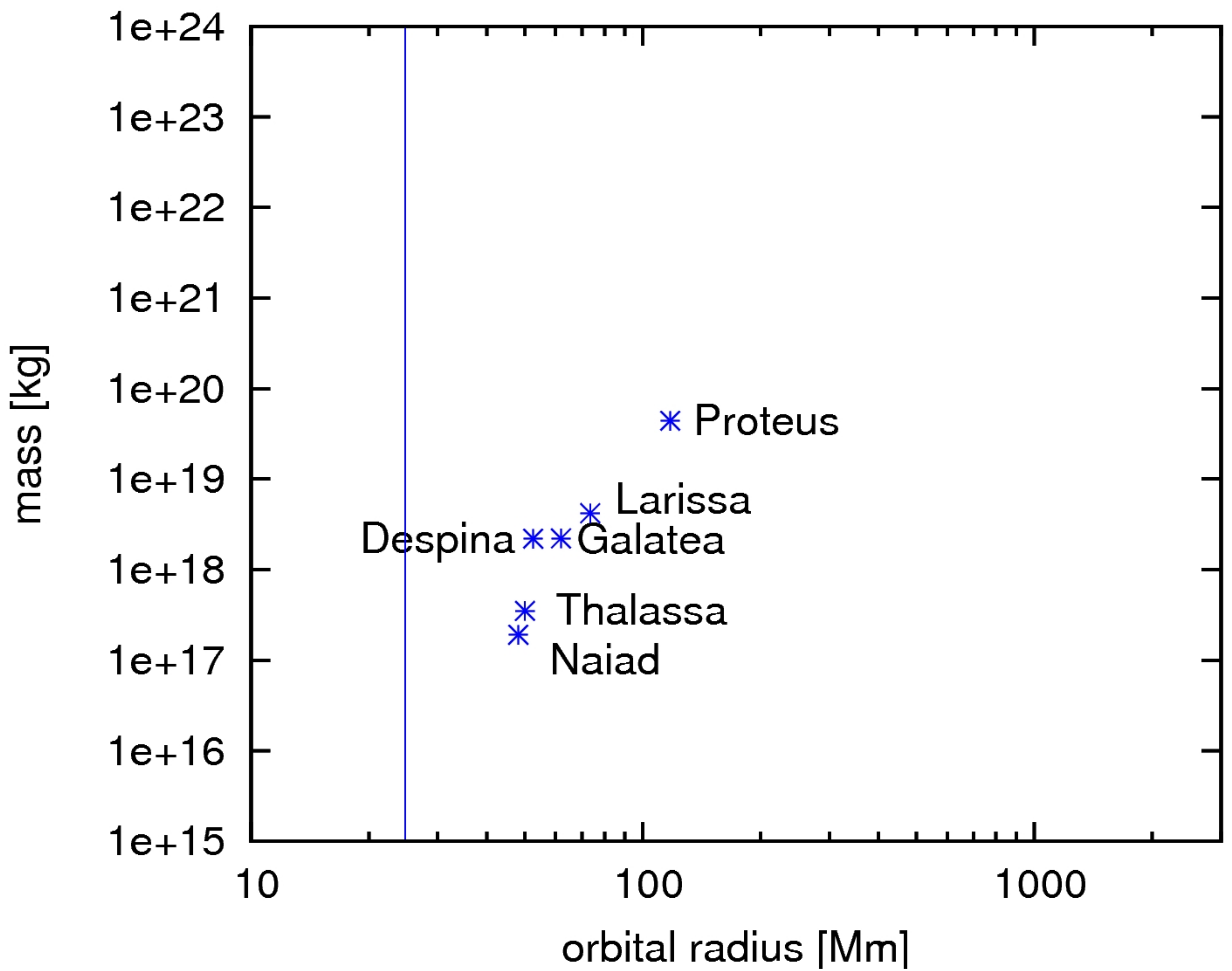
1e+24
1e+23
1e+22
1e+21
1e+20
1e+19
1e+18
1e+17
1e+16
1e+15
mass [kg]
10
100
1000
orbital radius [Mm]
Titan
Dione
Rhea
Tethys
Enceladus
Mimas
Janus
Epimetheus
Prometheus
Pandora
Atlas

# URANUS



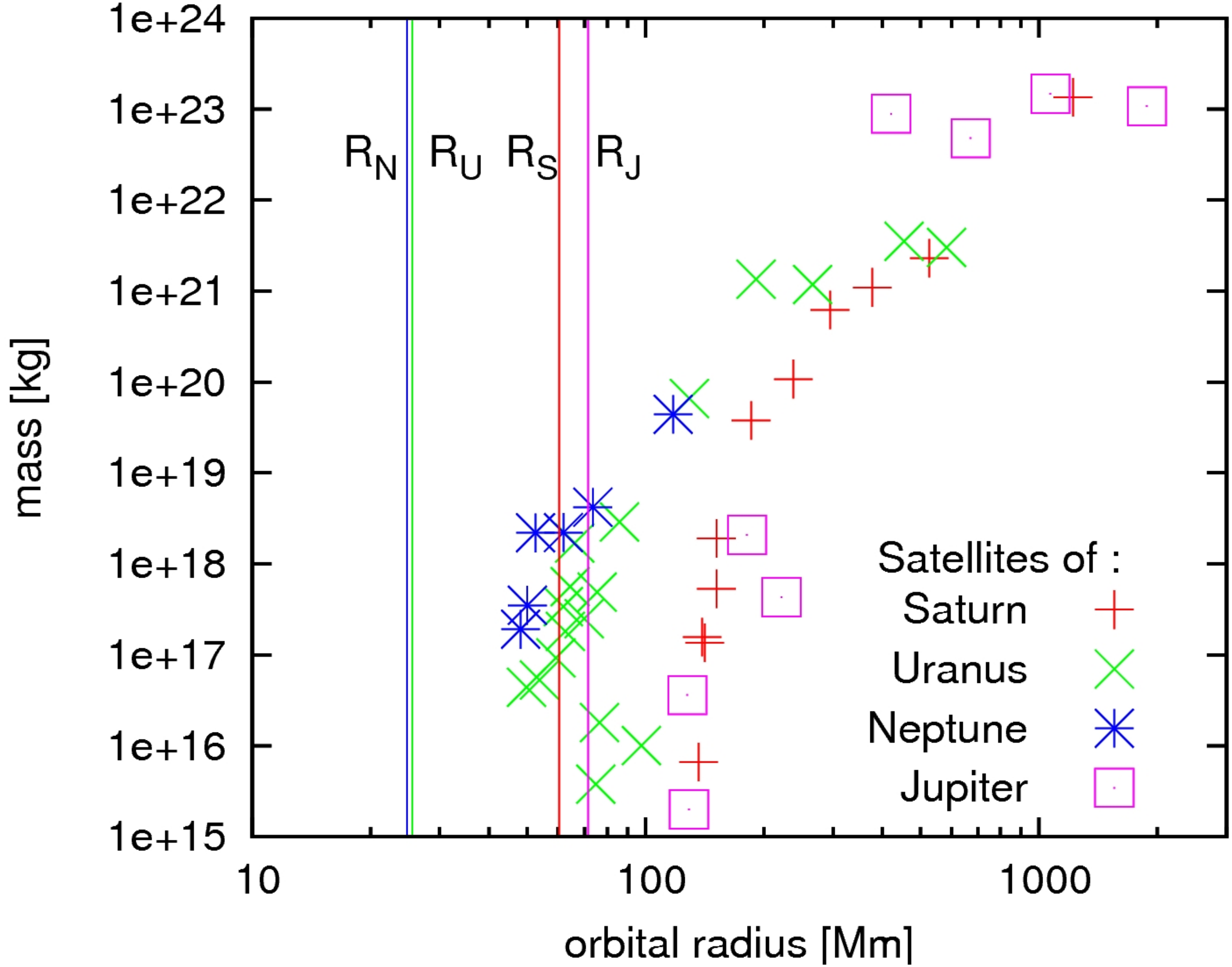
1e+24
1e+23
1e+22
1e+21
1e+20
1e+19
1e+18
1e+17
1e+16
1e+15
mass [kg]
10
100
1000
orbital radius [Mm]
Titania
Oberon
Ariel
Umbriel
Miranda

# NEPTUNE



1e+24
1e+23
1e+22
1e+21
1e+20
1e+19
1e+18
1e+17
1e+16
1e+15
mass [kg]
10
100
1000
orbital radius [Mm]
Proteus
Larissa
Despina
Galatea
Thalassa
Naiad

# ALL GIANT PLANETS

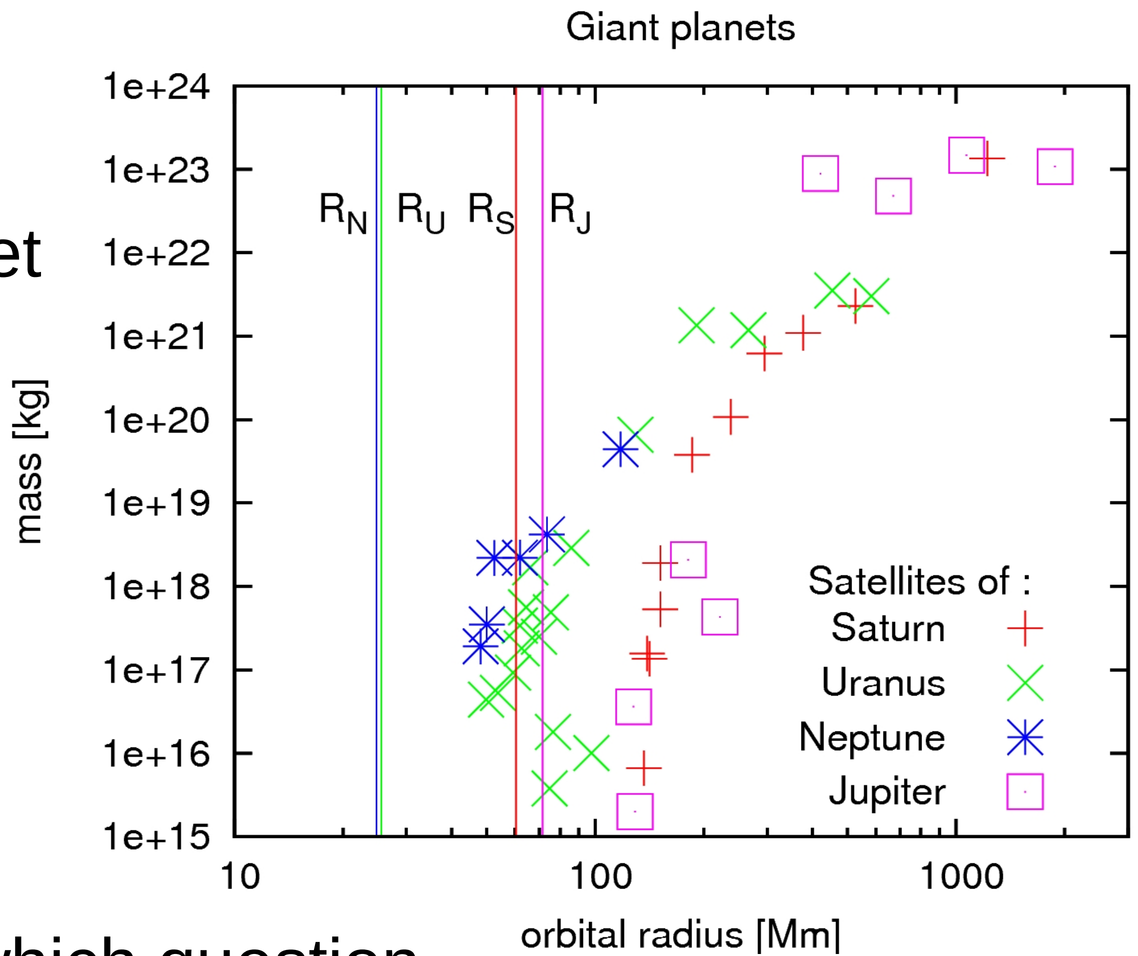


# INTRODUCTION

Distributions of giant planets' regular satellites :

- don't reach the planet
- ranked by mass
- pile-up at a few planetary radii (small bodies)

Why ?

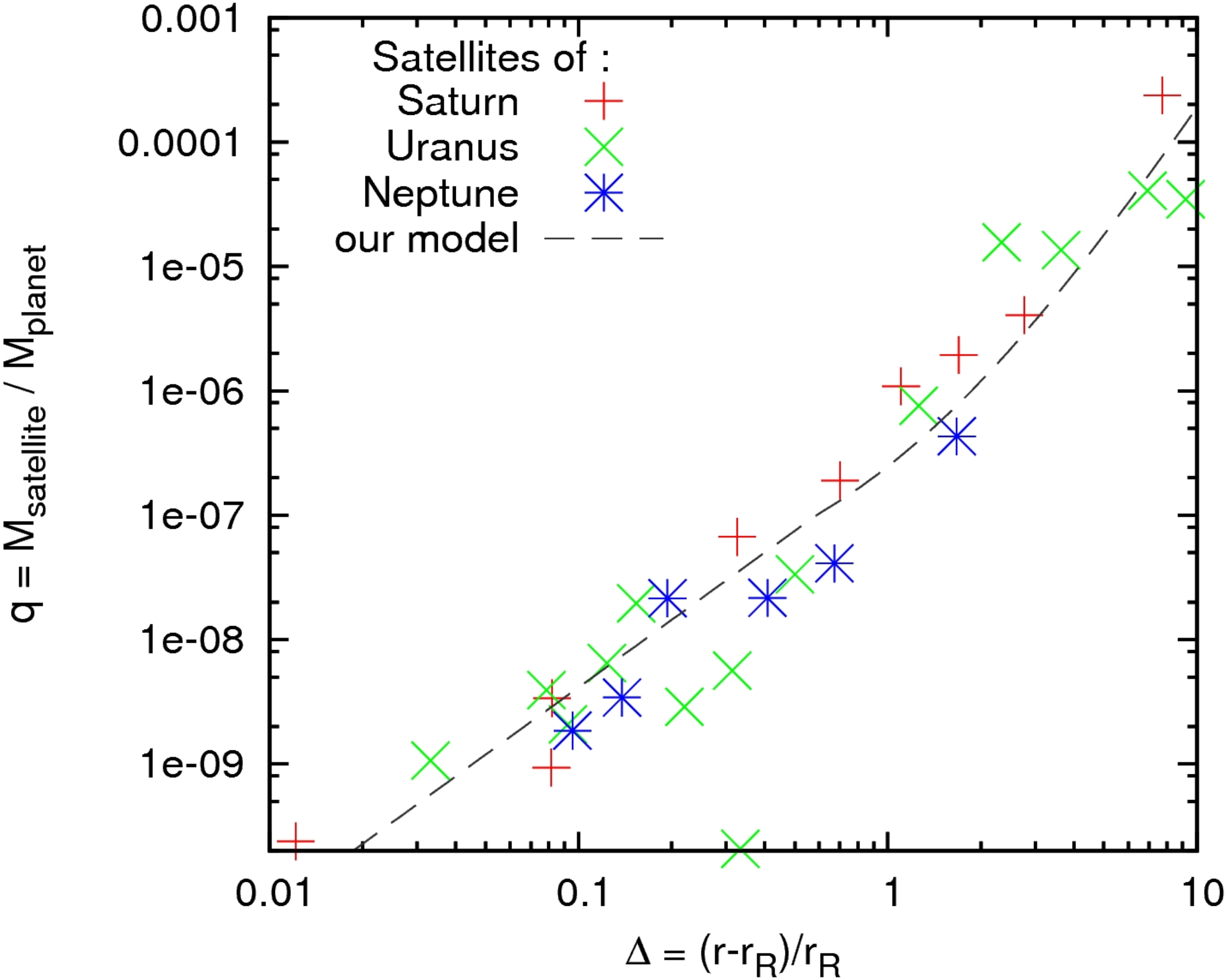



It's not a power law, which question the Circum-Planetary Disc model...



Crida & Charnoz 2012, Science

# CONCLUSION

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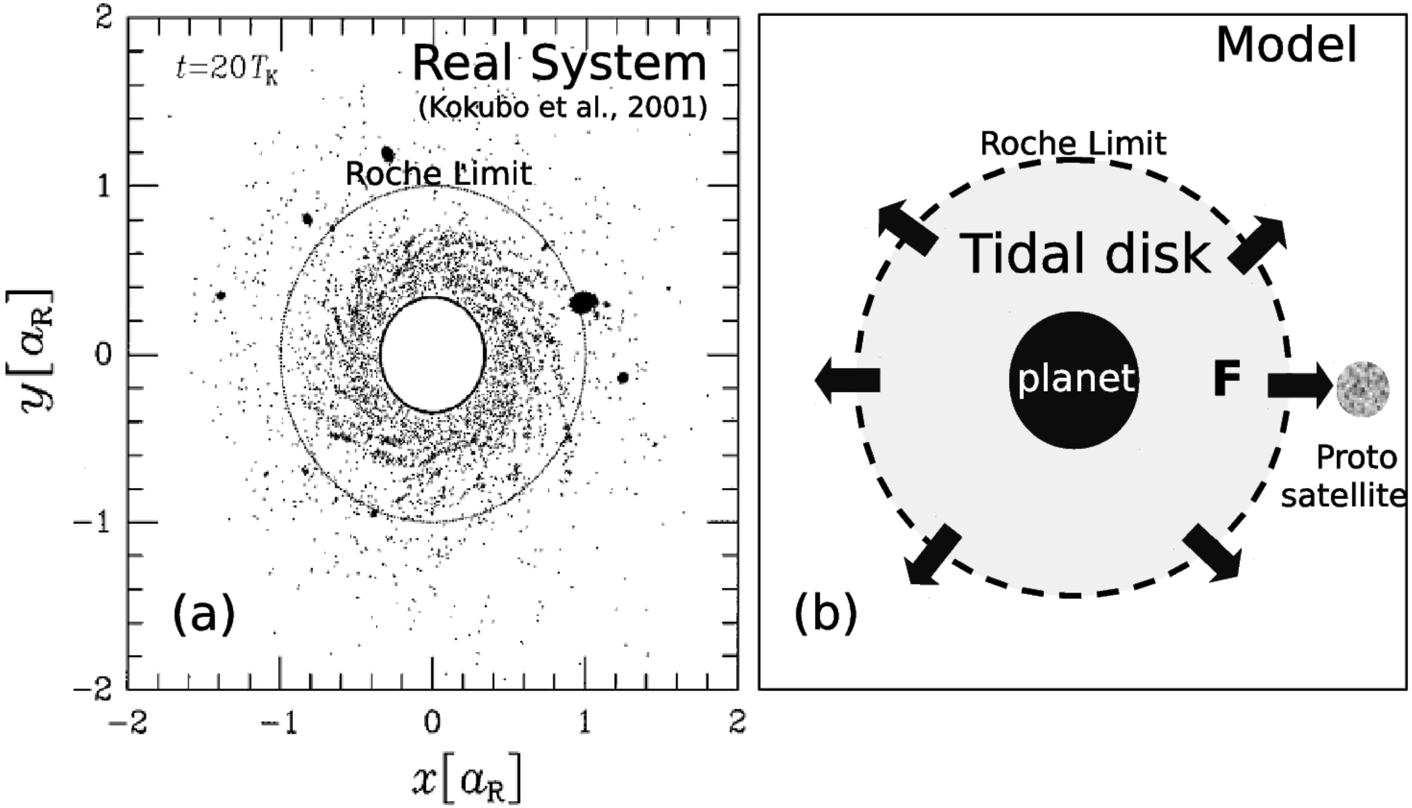
# Spreading of a tidal disk

1D model.

Inside the Roche radius $r_R$, there is a « tidal disk », that spreads with a mass flow **F** (assumed constant).



# Notations

Be $\mathbf{T_R}$ the orbital period at $r_R$ , and

$\tau_{disk}$ **=** $\mathbf{M_{disk} / FT_R}$ , the normalized life-time of the disk.

The disk spreads with a viscous time $t_\nu = r_R^2/\nu$.

Using Daisaka et al. (2001)'s prescription for $\nu$,
we find $\tau_{disk} = t_\nu / T_R = 0.0425\ D^{-2}$ where $\mathbf{D = M_{disk}/M_p}$ ,

and $F = 23\ D^3\ M_p / T_R$ .

# Continuous regime

Say 1 satellite forms. Its mass is : $\boxed{M = F\,t}$ (1)

It feels a torque from the tidal disk : $\Gamma = \frac{8}{27}\left(\frac{M}{M_p}\right)^2 \Sigma r^4 \Omega^2 \Delta^{-3}$

where $\Delta = (r-r_R)/r_R$ (Lin & Papaloizou 1979).

→ Migration rate : $$\boxed{\frac{d\Delta}{dt} = \frac{32}{27} q D T_R^{-1} \Delta^{-3}} \quad (2)$$

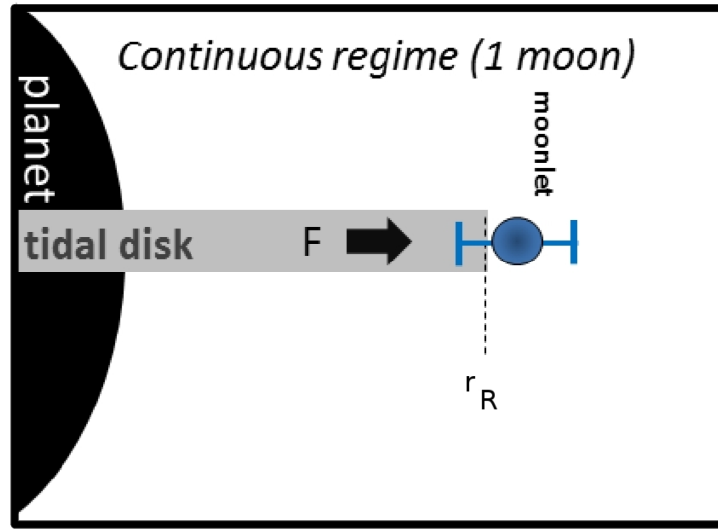
where $q = M / M_p$ .

Solution of (1) & (2) : $$\boxed{q = \left(\frac{\sqrt{3}}{2}\right)^3 \tau_{disk}^{-1/2} \Delta^2} \quad (3)$$

We call this the *continuous regime* .

# Continuous regime

This holds as long as the satellite captures immediately what comes through $r_R$.




That is, as long as $(r-r_R) < 2\ r_{Hill}$ ,
or $\Delta < 2\ (q/3)^{1/3}$ .

Input into Eq.(3), this gives a condition of validity for the continuous regime :

$$\boxed{\begin{array}{ll} \Delta < \Delta_c = \sqrt{\dfrac{3}{\tau_{disk}}} & = \sim 8.4\ D \\[2ex] q < q_c = \dfrac{3^{5/2}}{2^3}\tau_{disk}^{-3/2} & = \sim 222\ D^3 \end{array}}$$

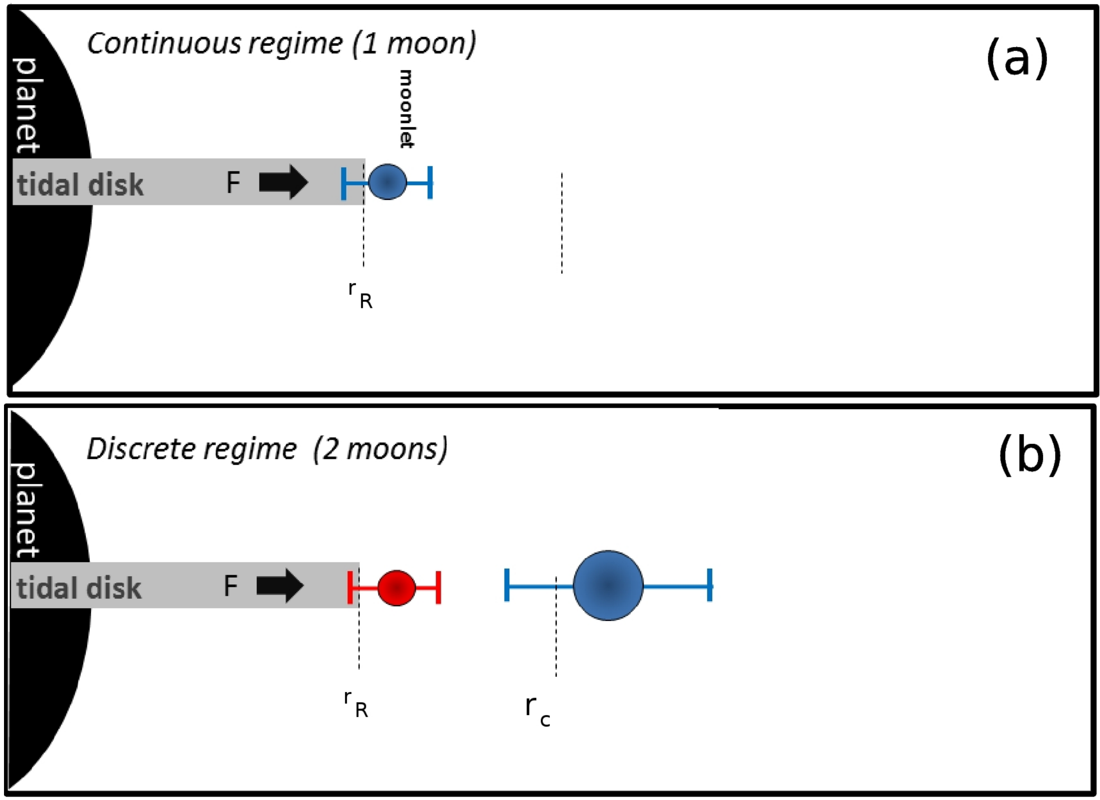
Duration of the continuous regime: $10\ T_R$ .

# Discrete regime

When the satellite is beyond $\Delta_c$ (or $q_c$), the material flowing through $r_R$ forms a new satellite at $r_R$.

This new satellite is immediately accreted by the first one.

And so on...




The first satellite still grows as M=Ft, but by steps : *discrete regime*.

# Discrete regime

This holds as long as $\Delta < \Delta_c + 2(q/3)^{1/3}$ .

It gives the condition :

$$\Delta < \Delta_d = 3.1\Delta_c \quad = \sim 26\ D$$

$$q < q_d = 9.9 q_c \quad = \sim 2200\ D^3$$

The duration of the discrete regime is ~100 $T_R$ .

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$$q < q_d = 9.9\,q_c \qquad = \sim 2200\ \mathrm{D}^3$$

The duration of the discrete regime is ~100 $T_R$ .

Applications :

1) Earth's Moon forming disk : $q_d$ = ~mass of the Moon !
2) Charon never left the continuous regime.
3) Saturn's rings : $q_d = \sim 10^{-18}$ .

# Pyramidal regime

Satellites of mass $q_d$ are produced at $\Delta_d$ every $q_d / F$ .

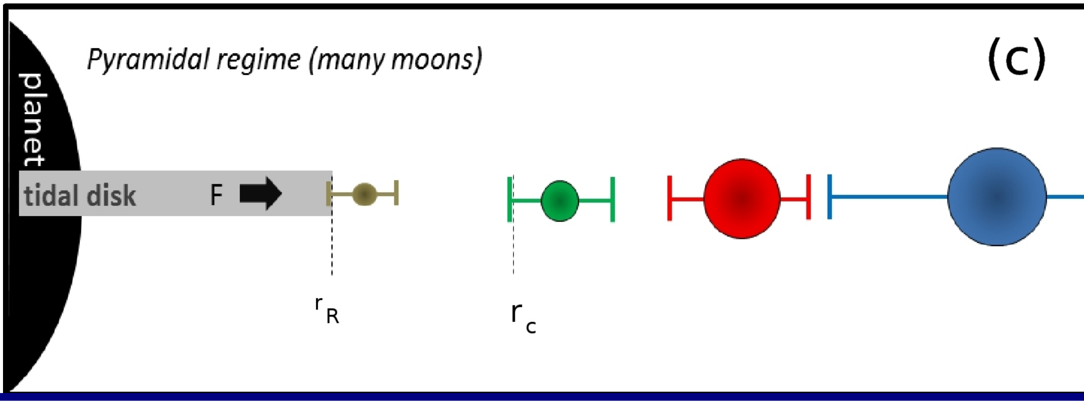
Then, many satellites of constant mass migrate outwards, at decreasing rates. They approach each other.

If their distance decreases below 2 mutual Hill radii, they merge.

This leads to the formation of satellites of masses $2q_d$ , every $2\,q_d / F$. They migrate away and merge further...

And so on, hierachicaly...

We call this *the pyramidal regime*.



# Pyramidal regime

- Using Eq.(2), we show that in the pyramidal regime, while the mass is doubled, $\Delta$ is multiplied by $2^{5/9}$.

Thus, $\boxed{q \propto \Delta^{9/5}}$ .

In addition, the number density of satellites should be proportionnal to $1/\Delta$ , explaining the pile-up.

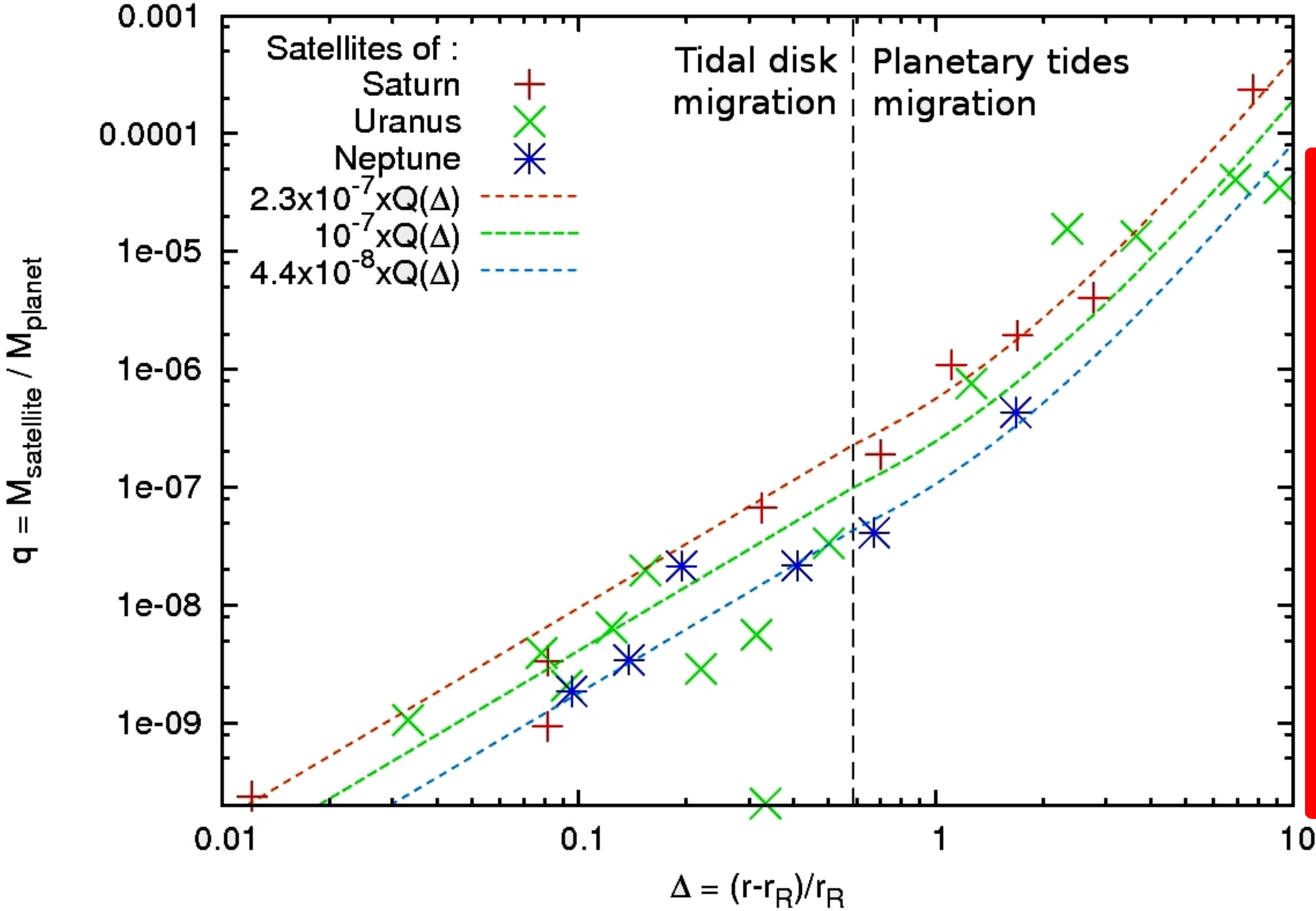
- Beyond the 2:1 Lindblad resonance with $r_R$ ($\Delta$=0.58 ), Eq.(2) doesn't apply. Migration is driven by planetary tides:

$$\frac{dr}{dt} = \frac{3 k_{2p} M \sqrt{G} R_p^5}{Q_p \sqrt{M_p} r^{11/2}} \qquad (4)$$

Using Eq.(4), we find $\boxed{q \propto r^{3.9}}$ .

# Pyramidal regime

The result spectacularly matches the distribution of the Saturnian, Uranian, and Neptunian systems !



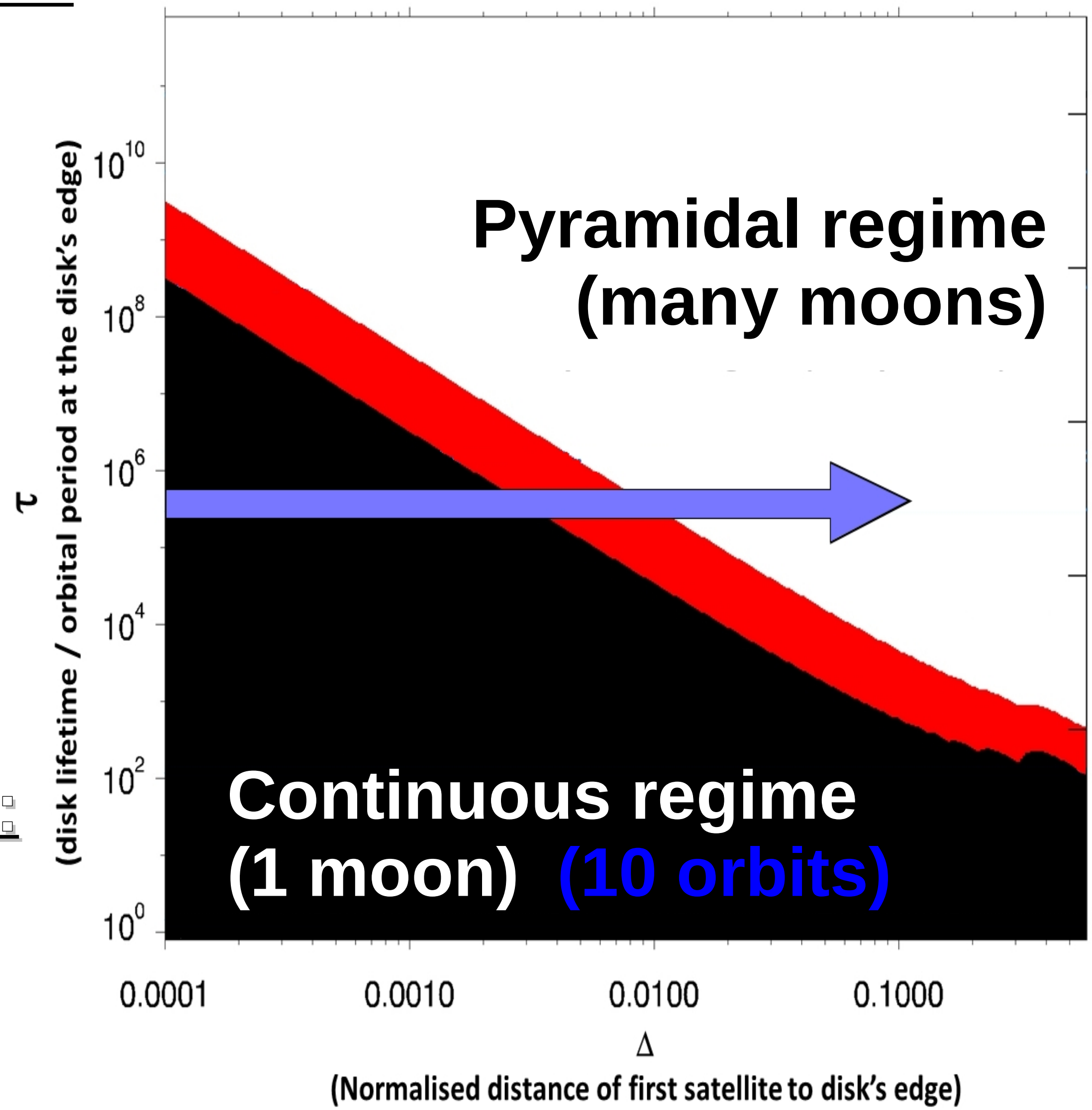

I claim that Uranus and Neptune had massive rings, from which their regular satellites were born.

# Summary

**1) <u>Continuous regime:</u>**
1 moon grows
q α $\Delta^2$
until $\Delta_c$ or $q_c$.

**2) <u>Discrete regime:</u>**
2 moons,
growth by steps
until $\Delta_d$ or $q_d$ .

**3) <u>Pyramidal regime:</u>**
Many moons in the system.
q α $\Delta^{9/5}$ or $r^{3.8}$ .

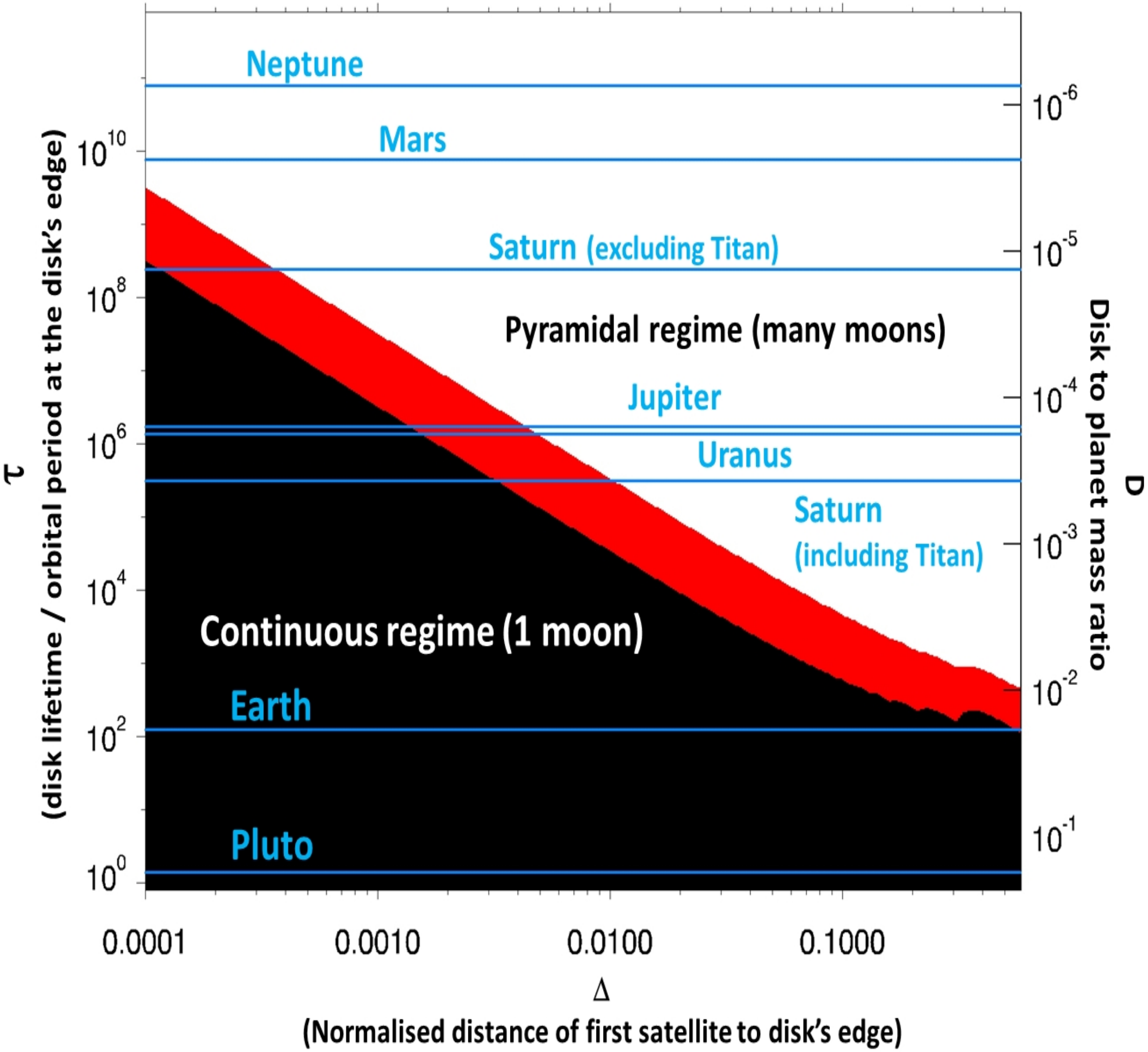


# Summary

Take $M_{disk}$ = 1.5 x the mass of the present satellite system.

Giant planets must be dominated by the pyramidal regime,

while we expect the Earth and Pluto to have 1 large satellite.



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# Conclusion & Discussion

The spreading of a tidal disk beyond the Roche radius

✓ explains the mass-distance distribution of the regular satellites of the giant planets
(observational signature of this process)

✓ unifies terrestrial and giant planets in the same paradigm.

✓ most Solar System regular satellites formed this way.

✗ Jupiter doesn't fit in this picture : probably formed in a circum-planetary disk (Canup & Ward 2002, 2006 ; Mosqueira & Estrada 2003a,b)

• Titan fits very well in this picture, though its « tidal age » is too large... Coincidence ?


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