

Combined models of planet formation and evolution:

# *The planetary mass-radius relationship*

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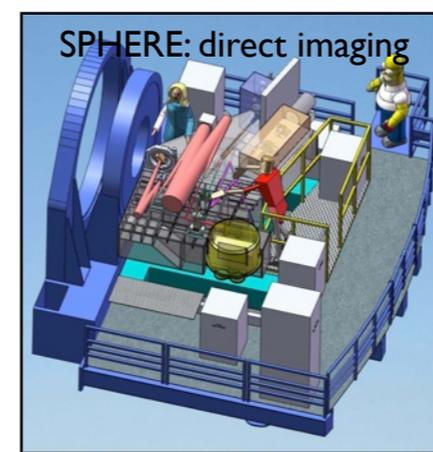
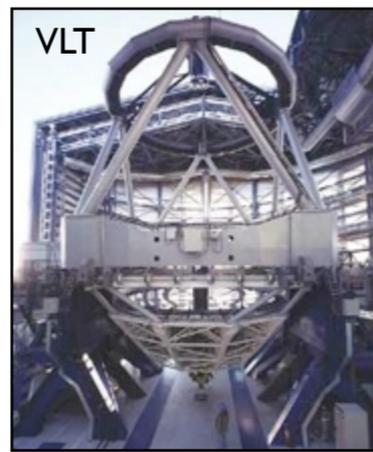
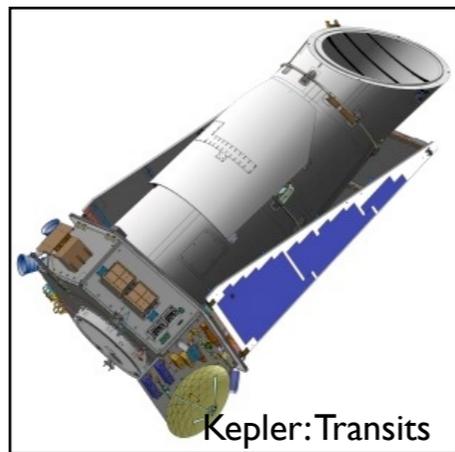


K. M. Dittkrist, P. Molliere, S. Jin, H. Klahr, T. Henning

Y. Alibert, A. Fortier, W. Benz

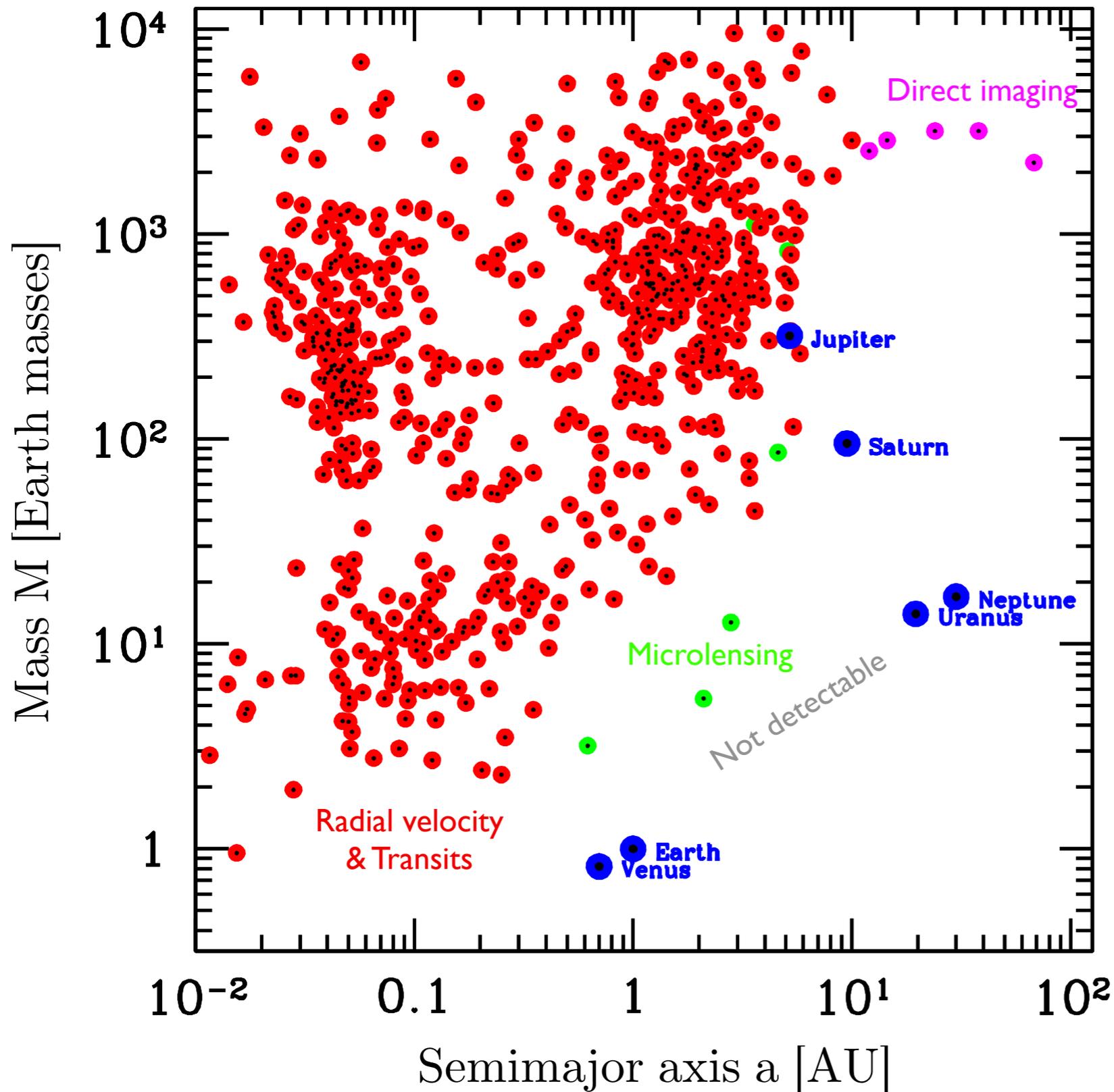
# Linking observations and planet formation

- **Large** number of observations from space mission (transits, spectra) and ground (radial velocity, transits, spectra, direct imaging). More to come (SPHERE, Gaia, ESPRESSO, CHEOPS, EChO..)



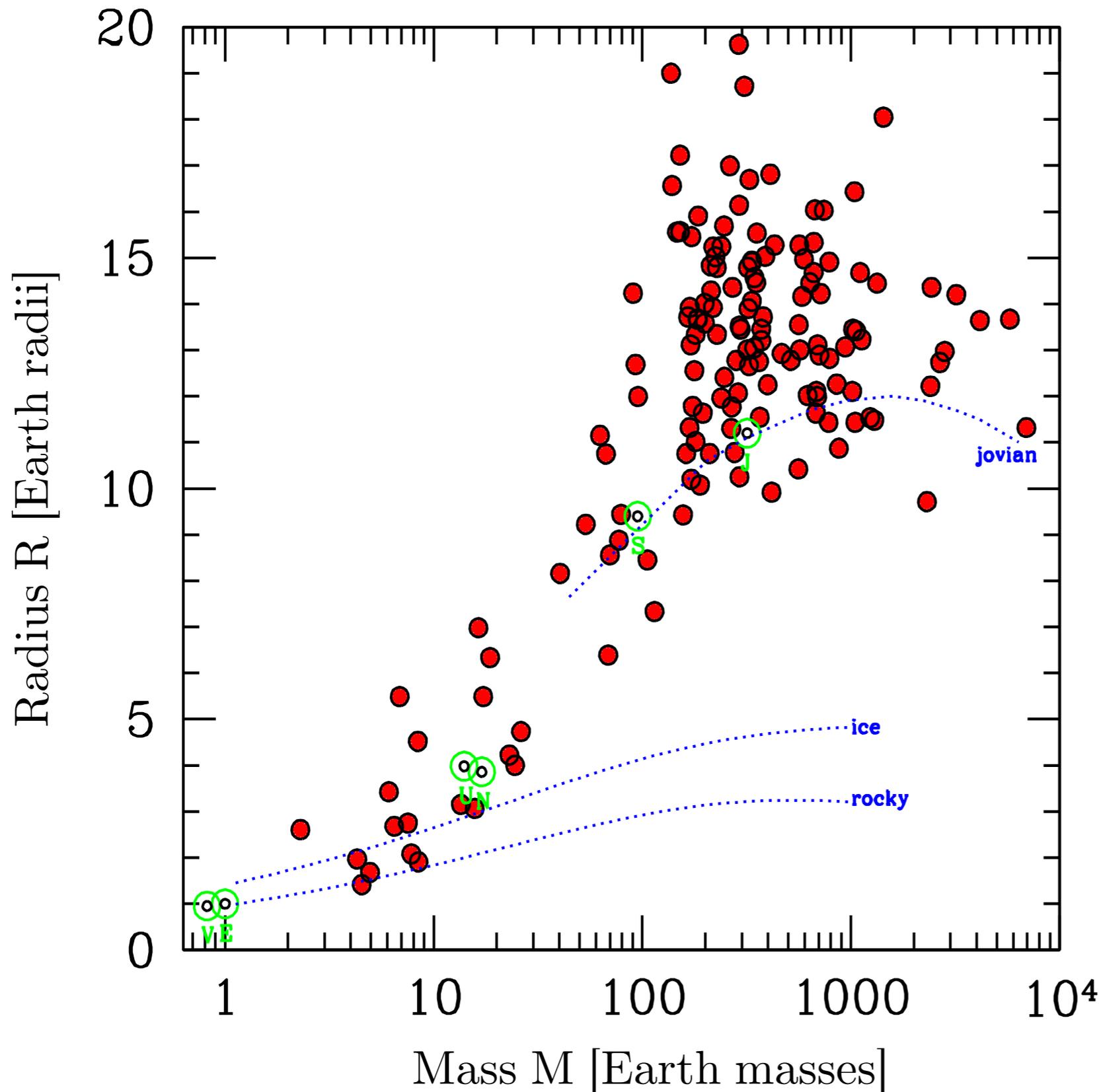
- Improve understanding of **planet formation by comparing theory and observation.**
- For a sufficiently large number of exoplanets: treat as a **statistical** ensemble.
- Planetary population synthesis: **statistical** comparison
- Difficulties: 1) different techniques constrain **different** aspects of the theory.  
2) between formation and observation: **Myrs-Gyrs of evolution.**

# From the $a$ - $M$ ....



- Many fundamental constraints from the  $a$ - $M$  diagram.

# From the $a$ - $M$ to the $M$ - $R$ diagram



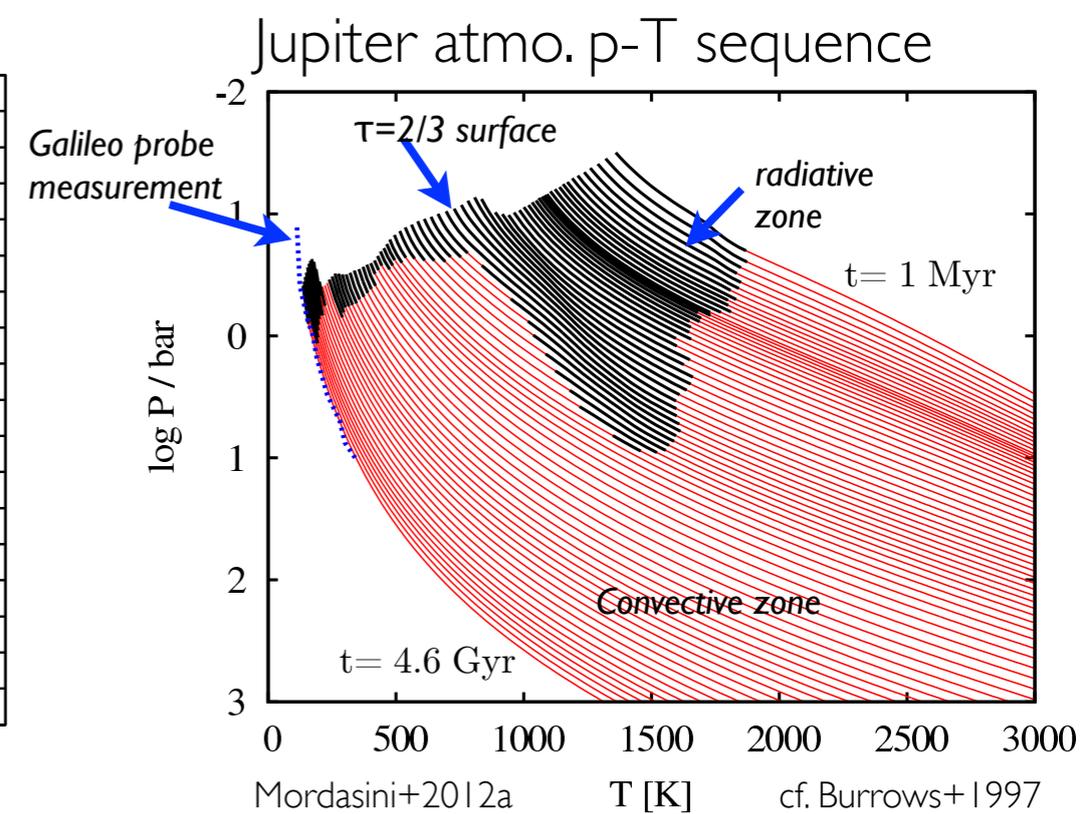
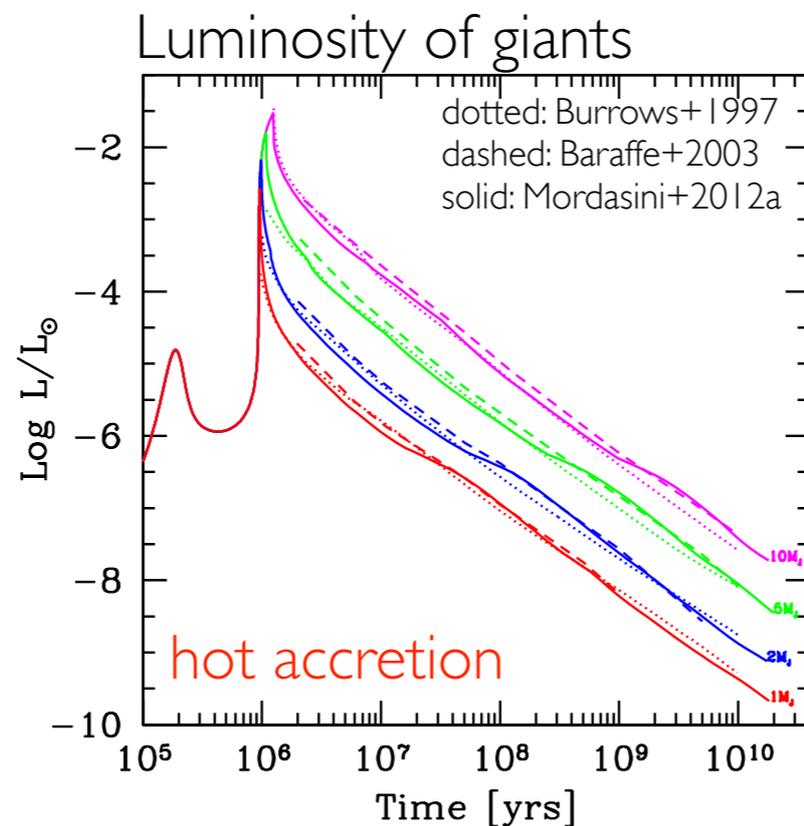
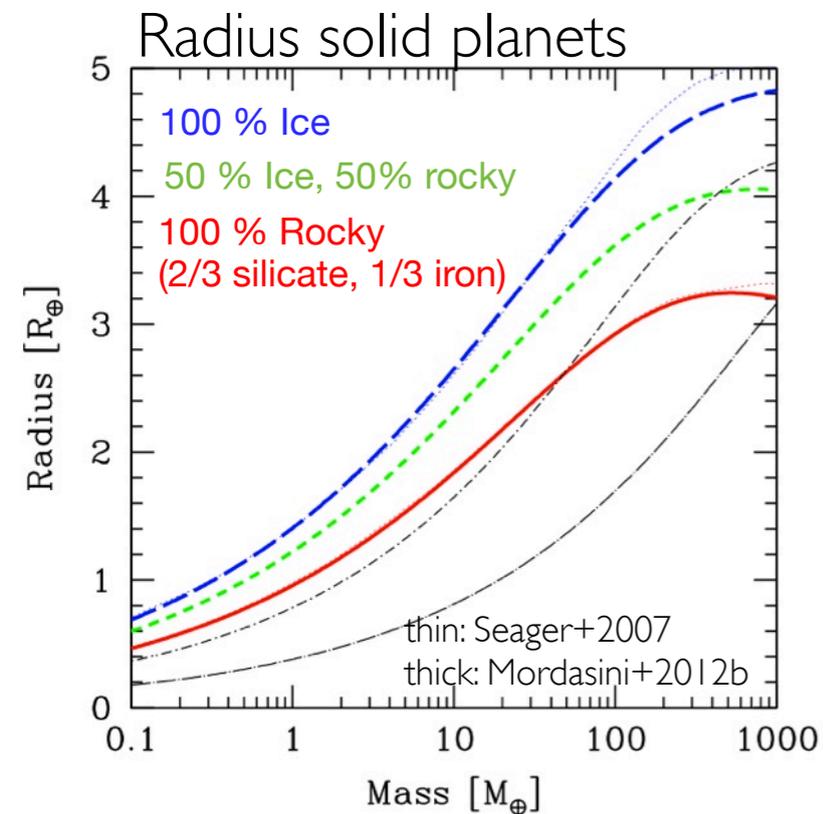
- M-R diagram: **diversity**, too!
- Bulk **composition**
- **Constraints** for formation beyond the  $a$ - $M$ :
  - migration (icy planets close-in?)
  - efficiency of H/He accretion & loss
  - runaway when? opacity?
- **Understandable** with theoretical models?

# Adding planet evolution

**Formation:** Based on [core accretion](#) paradigm, growth of seed embryo accreting gas and planetesimals in an evolving protoplanetary disk, undergoing orbital migration.

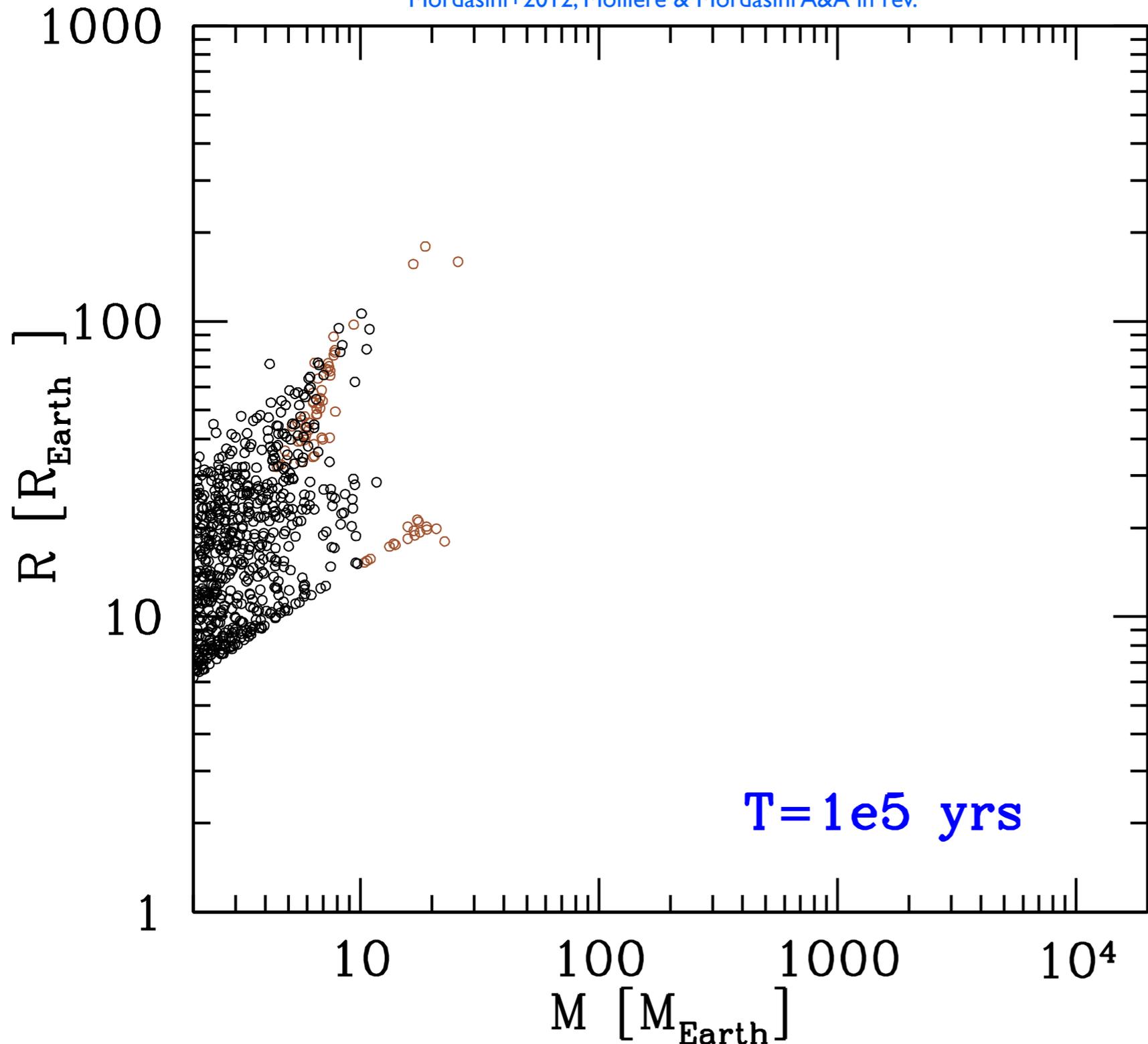
**Evolution (after disk is gone):** couple self consistently

- ◆ Solve 1D (radial) structure equations for the thermal evolution of the H/He envelope on Gyrs (cooling & contraction), including effects of stellar irradiation and radiogenic heating. Gray atmosphere.
- ◆ Solve 1D internal structure equations for the solid core, assuming a differentiated interior.



# Population synthesis: Formation of M-R

Mordasini+2012, Molliere & Mordasini A&A in rev.



Fraction  $Z$  of solids (rest H/He)

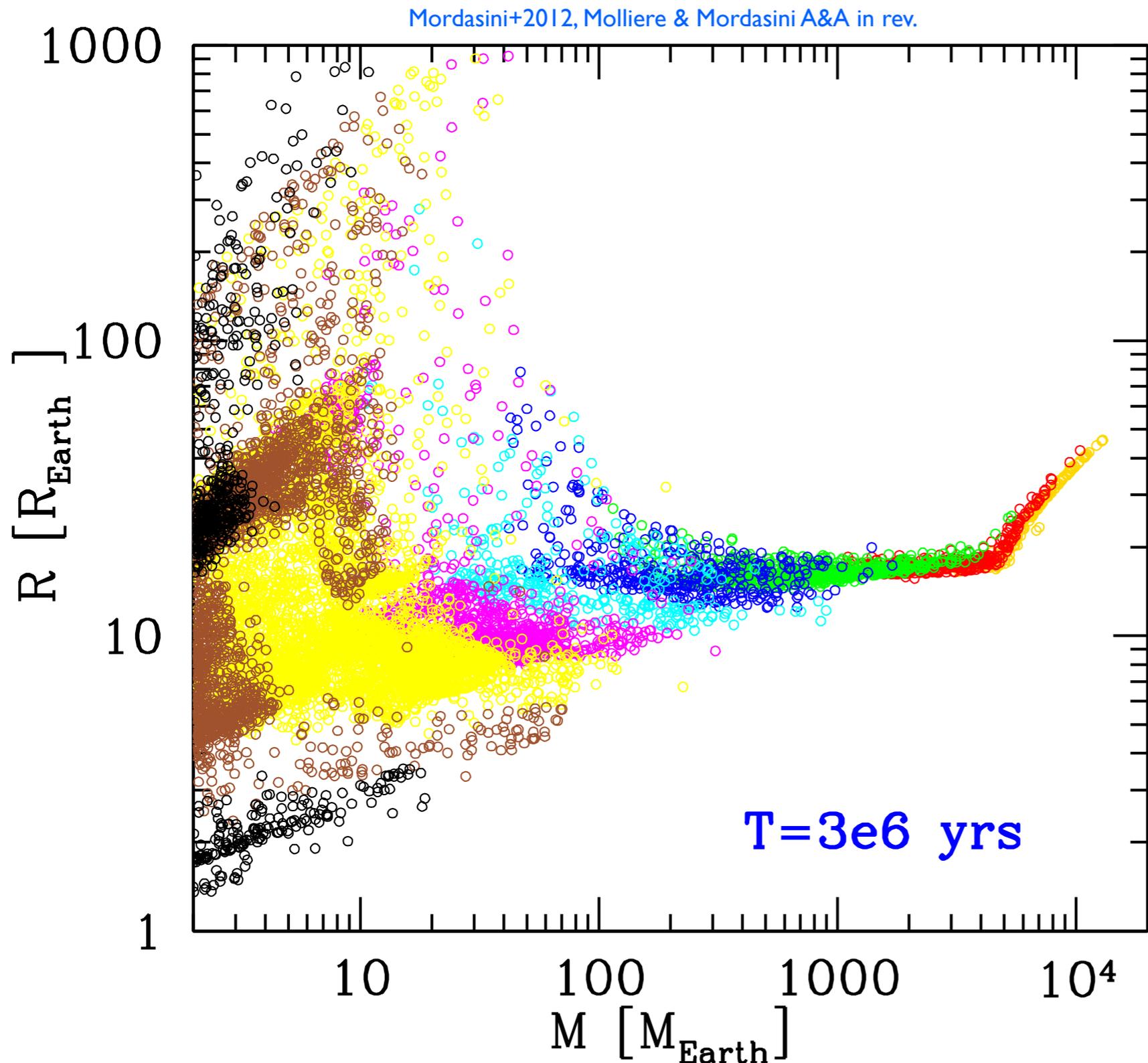
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- Rapid collapse at  $\sim 0.2 M_J$  when  $Z \approx 0.5$  (runaway gas accretion)
- After disk dispersal ( $T > 10$  Myrs), slow contraction.

Nominal Model.  $M_{\text{star}} = 1 M_{\text{sun}}$ .  $a > 0.1 \text{ AU}$ .

Non-isothermal Type I. Cold accretion. 1 embryo/disk

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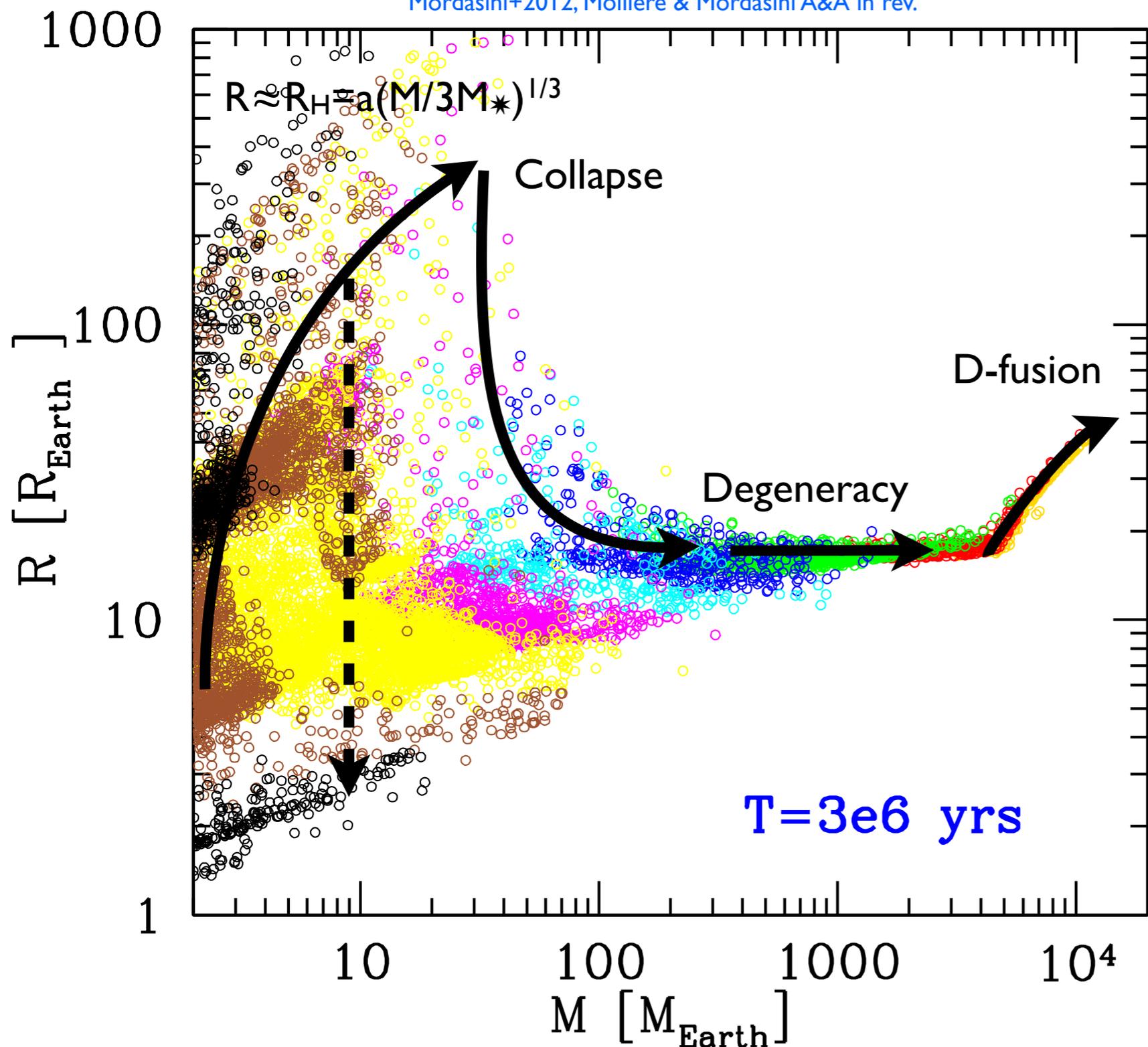
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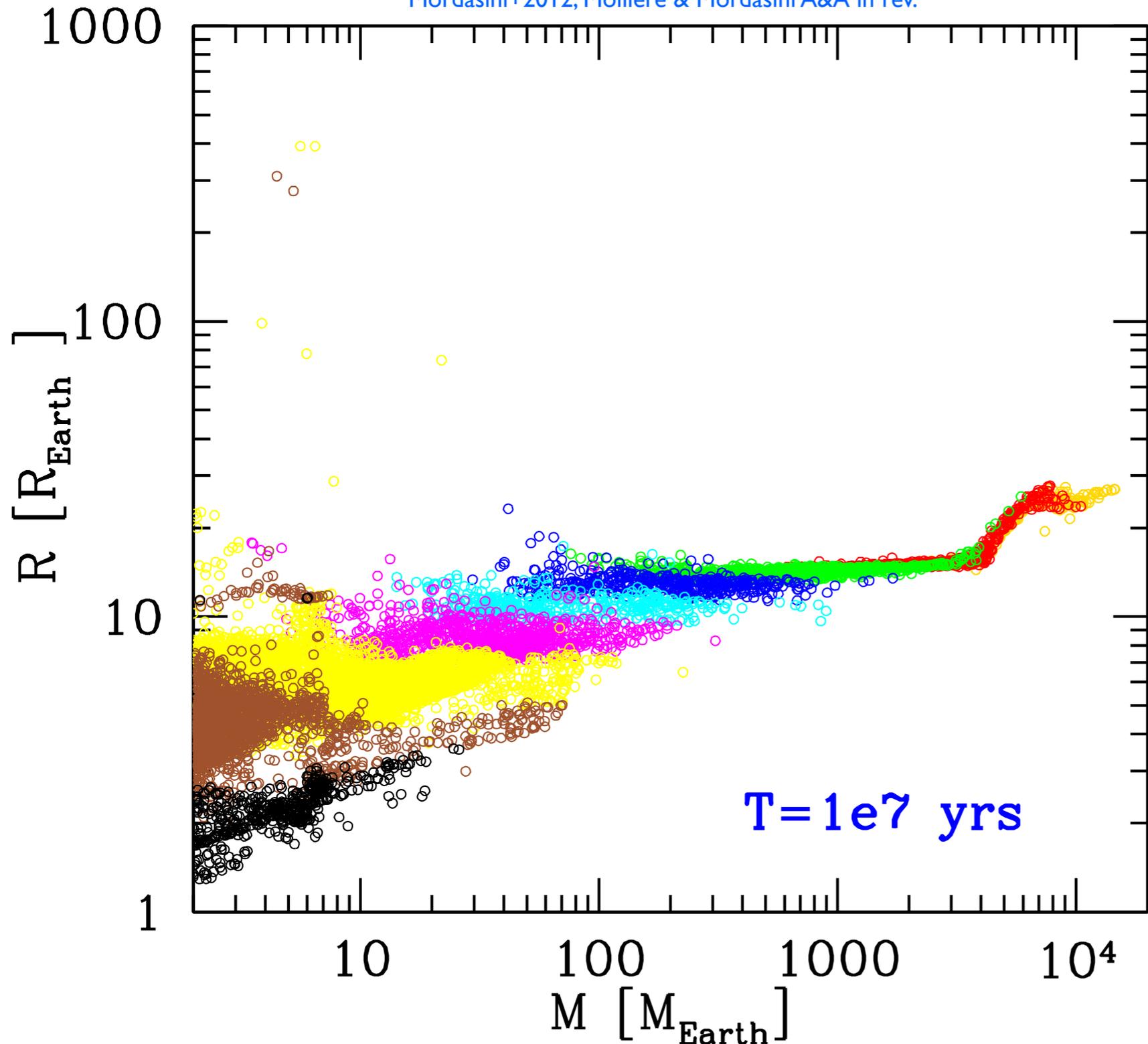
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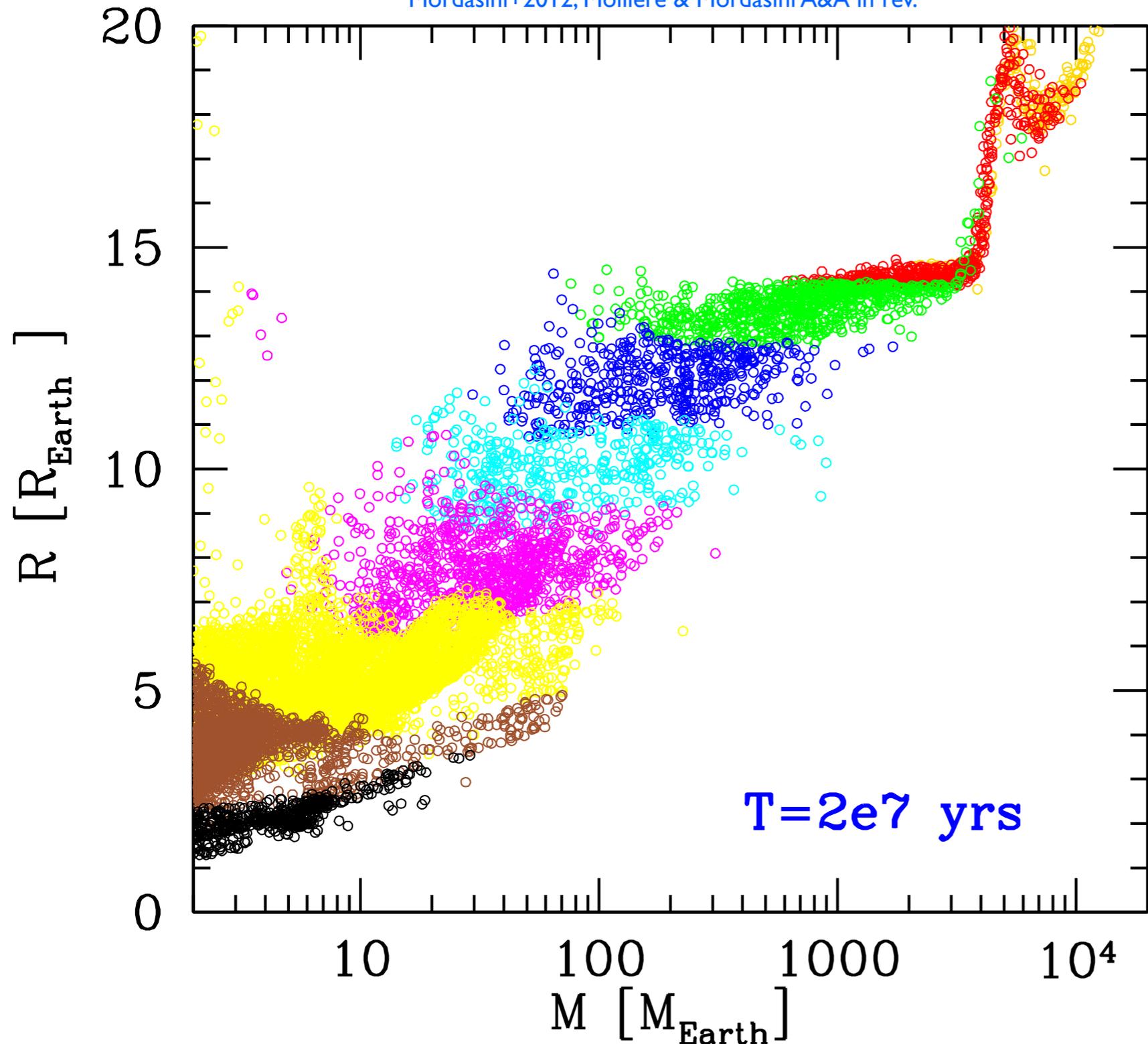
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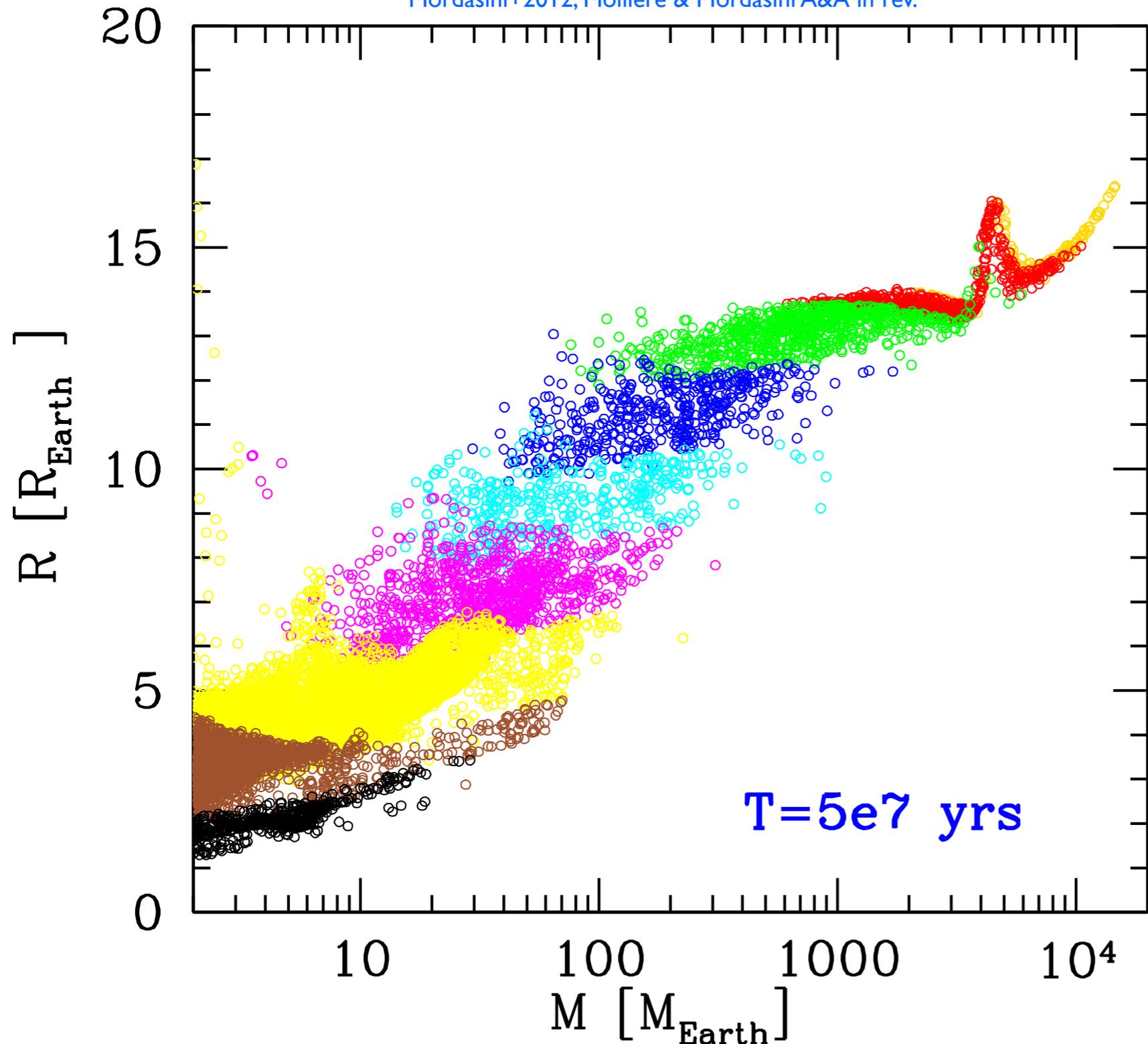
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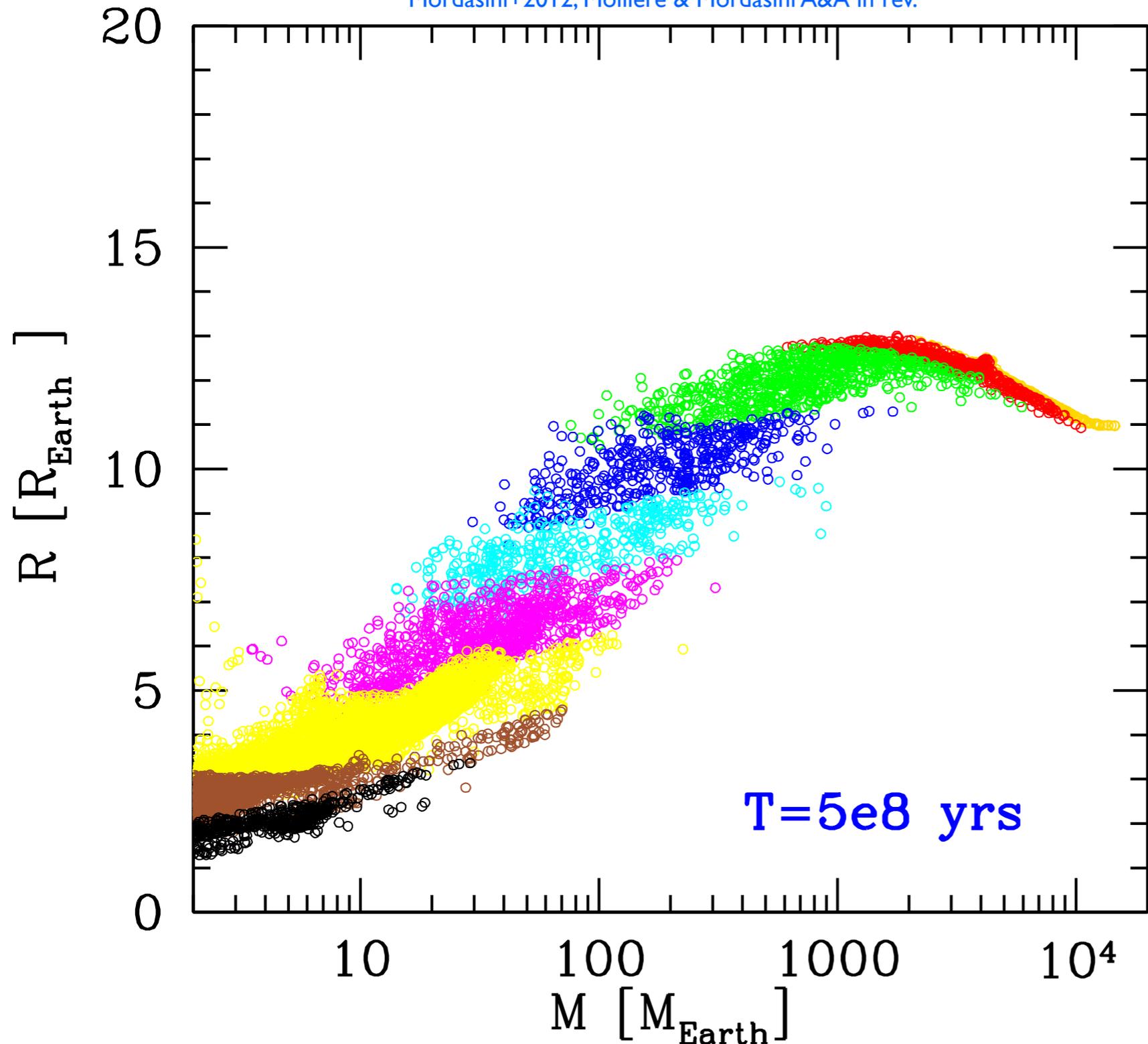
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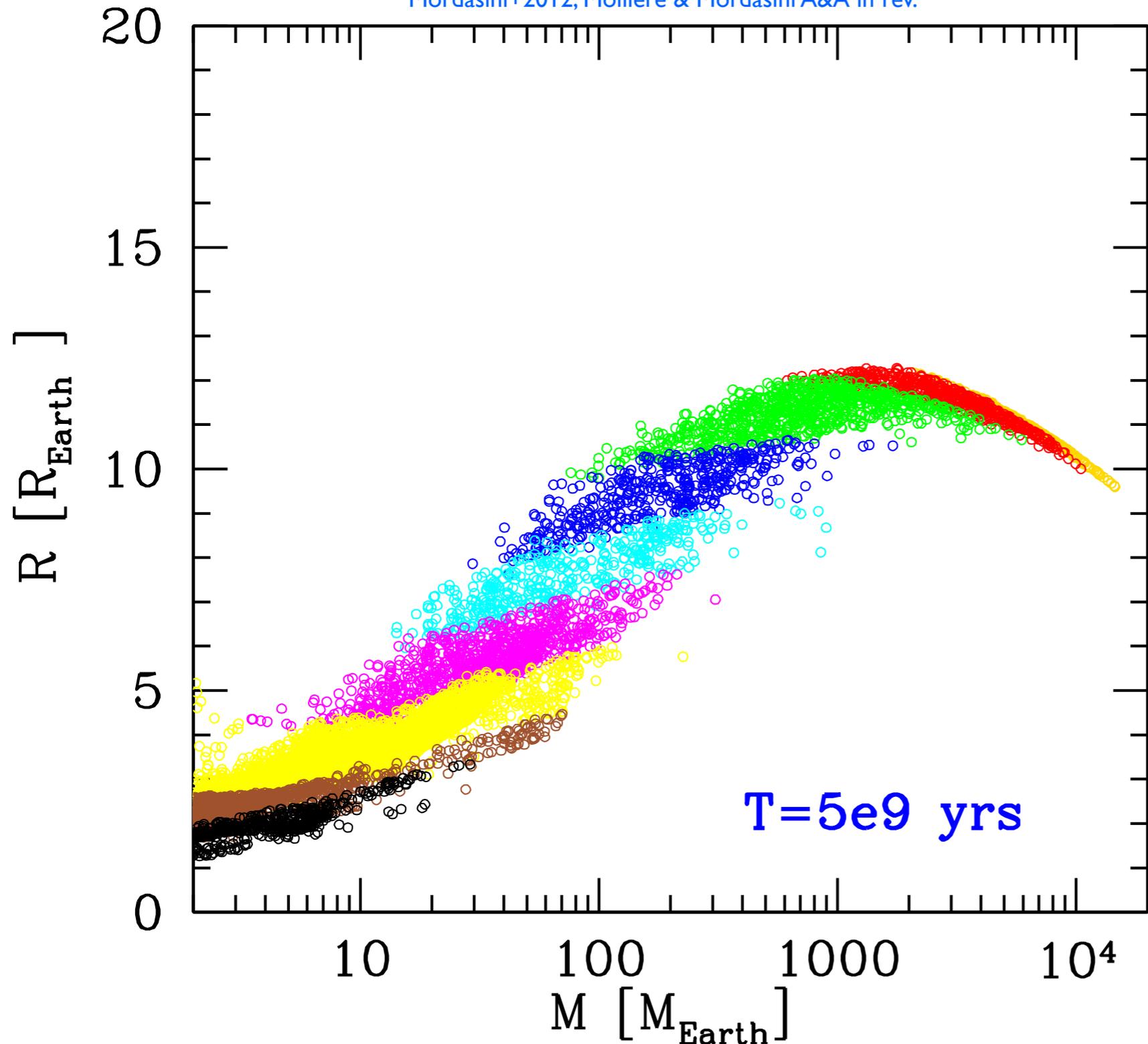
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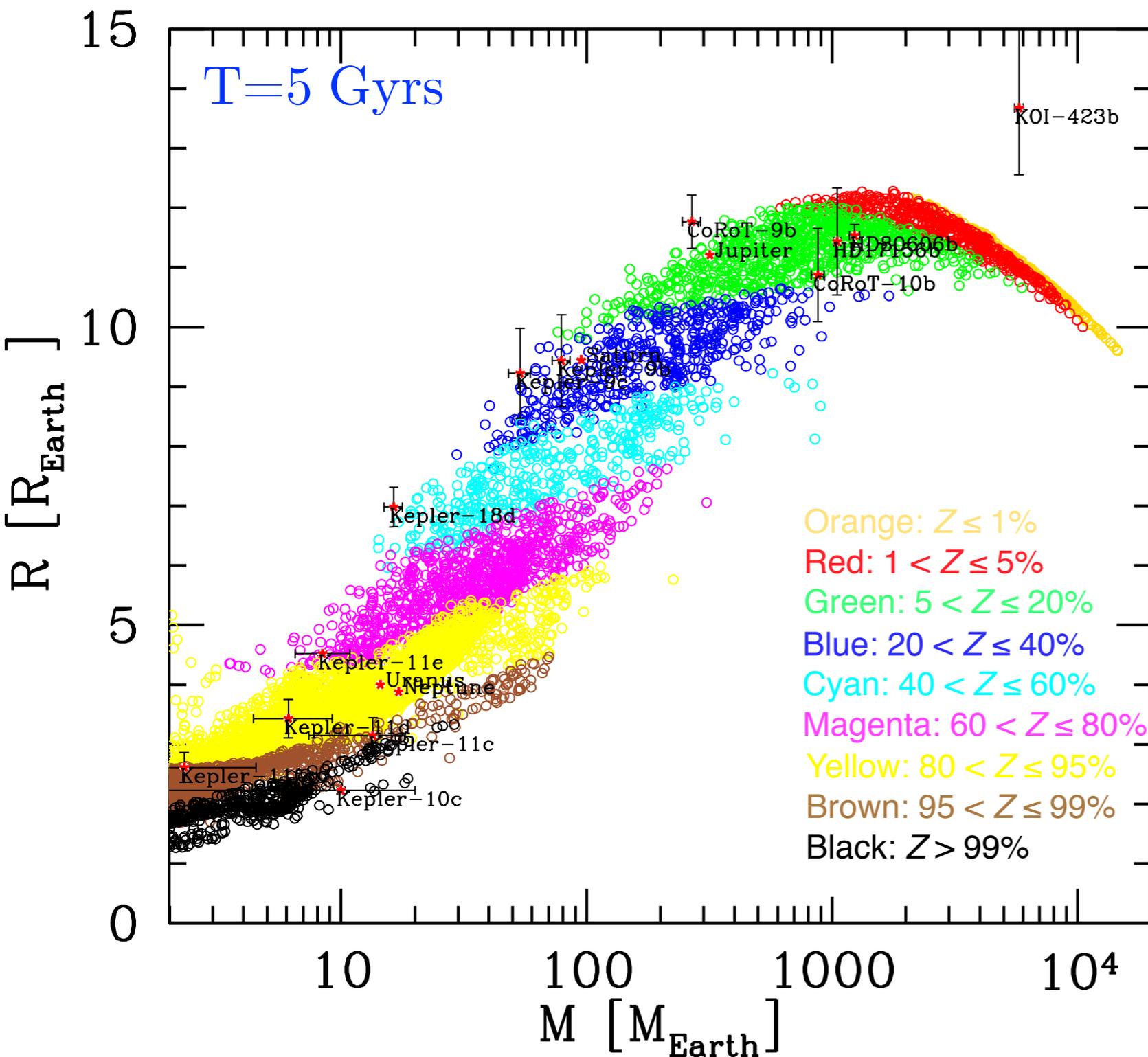
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# $M$ - $R$ diagram: comparison w. observations

Mordasini+2012b



All synthetic planets and all planets with known M-R outside 0.1 AU.

- S-shape with forbidden zones
  - low  $M \Rightarrow$  high  $Z \Rightarrow$  small  $R$
  - high  $M \Rightarrow$  low  $Z \Rightarrow$  large  $R$
- Imprint from core accretion & EOS
- Diversity in  $R$  at one  $M$
- Comparison with observations fine except for KOI-423b. To be tested with future observation.
- Expect divergence in future at small masses (only H/He!)

# *M-R diagram: effect of grain opacity*

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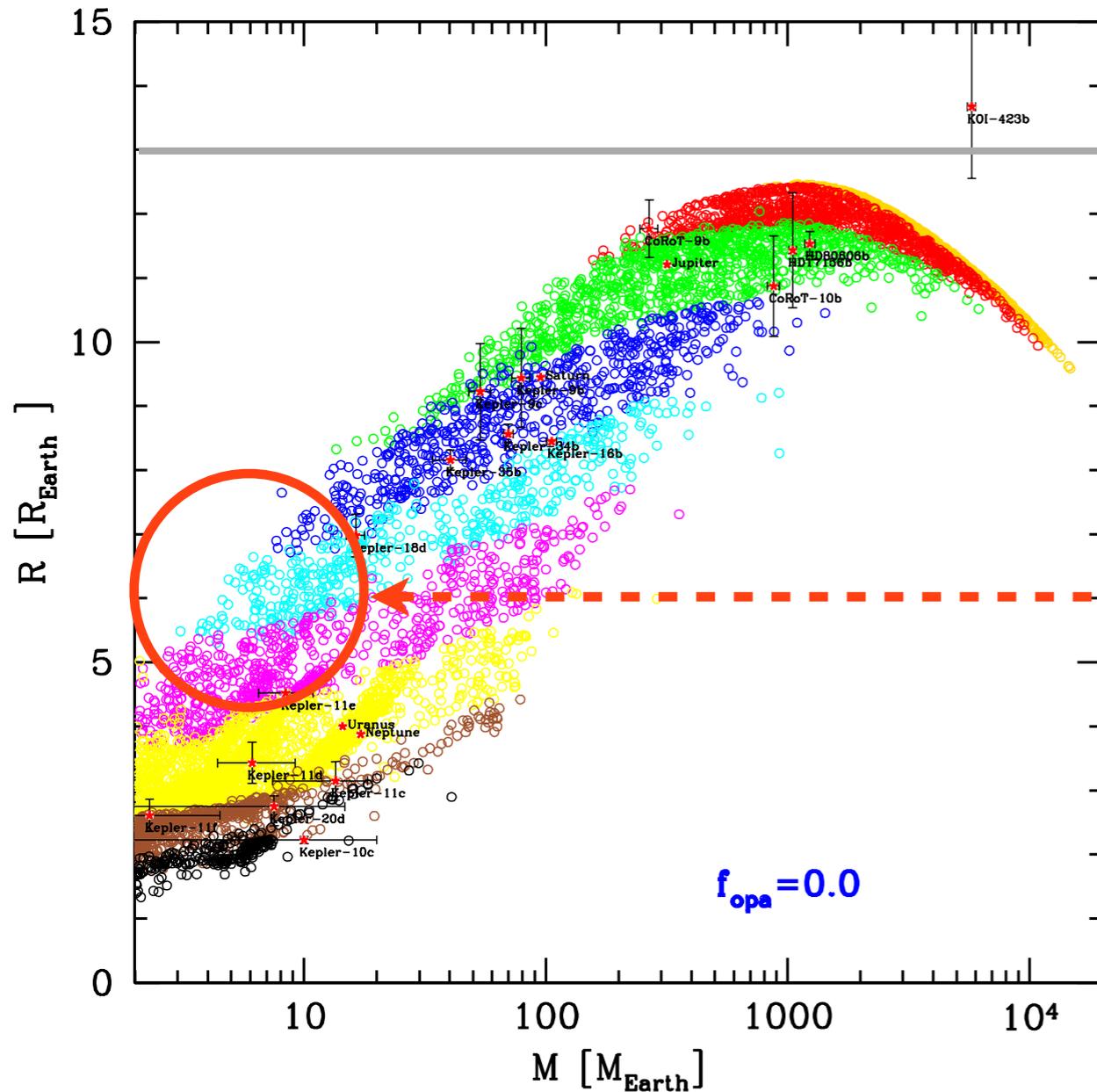
Efficiency of accretion of H/He by cores:  
Controlled by opacity due to grains in the  
envelope during formation. Grains evolve.  
Low opacity  $\Rightarrow$  high  $M_{\text{envelope}} \Rightarrow$  large  $R$ .  
High opacity  $\Rightarrow$  low  $M_{\text{envelope}} \Rightarrow$  small  $R$ .

Podolak+2003, Movshovitz+2010, Hori & Ikoma 2010

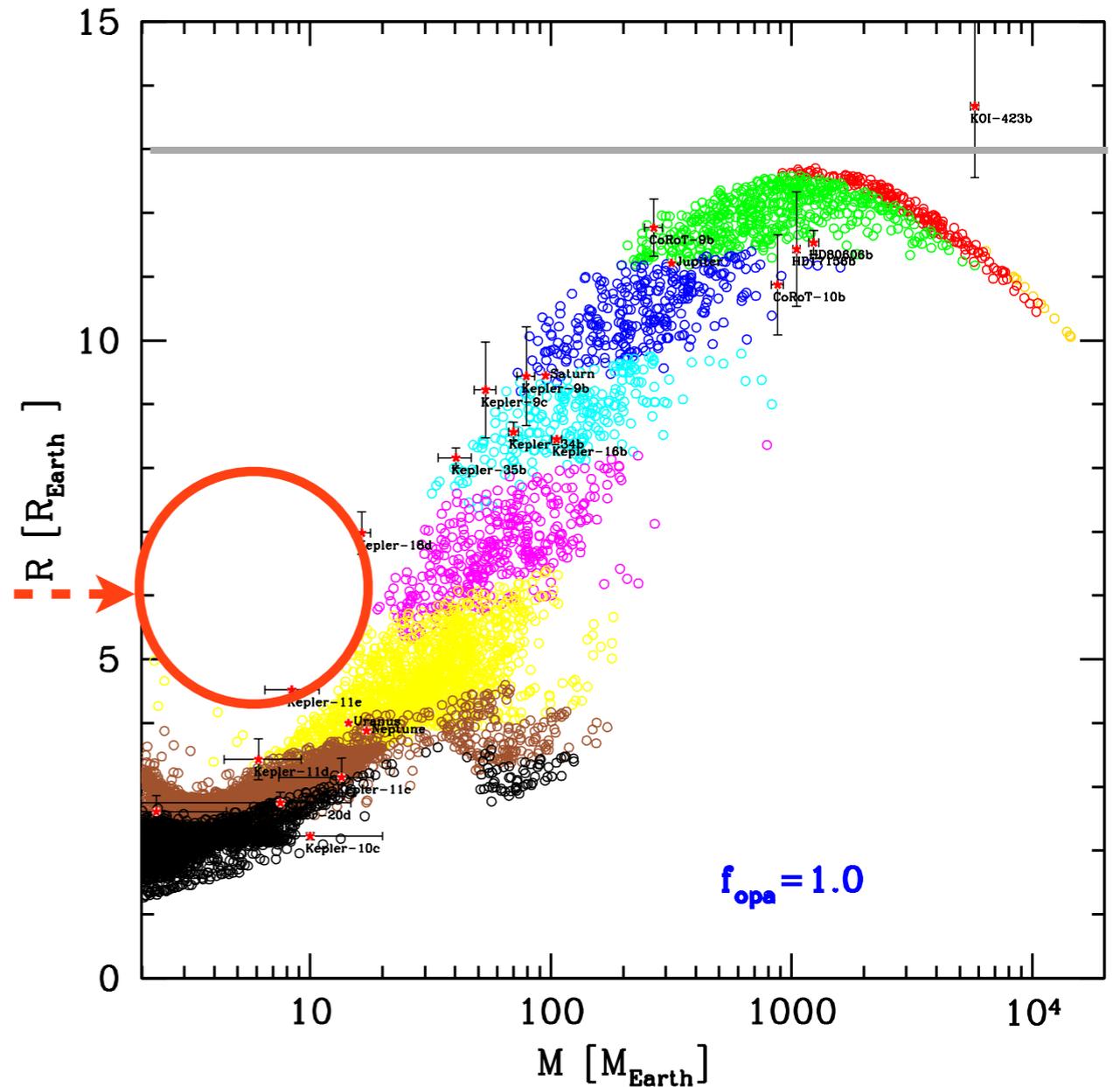
# M-R diagram: effect of grain opacity

Mordasini et al in rev.

Zero grain opacity



Full interstellar grain opacity



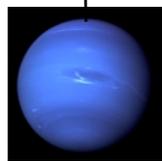
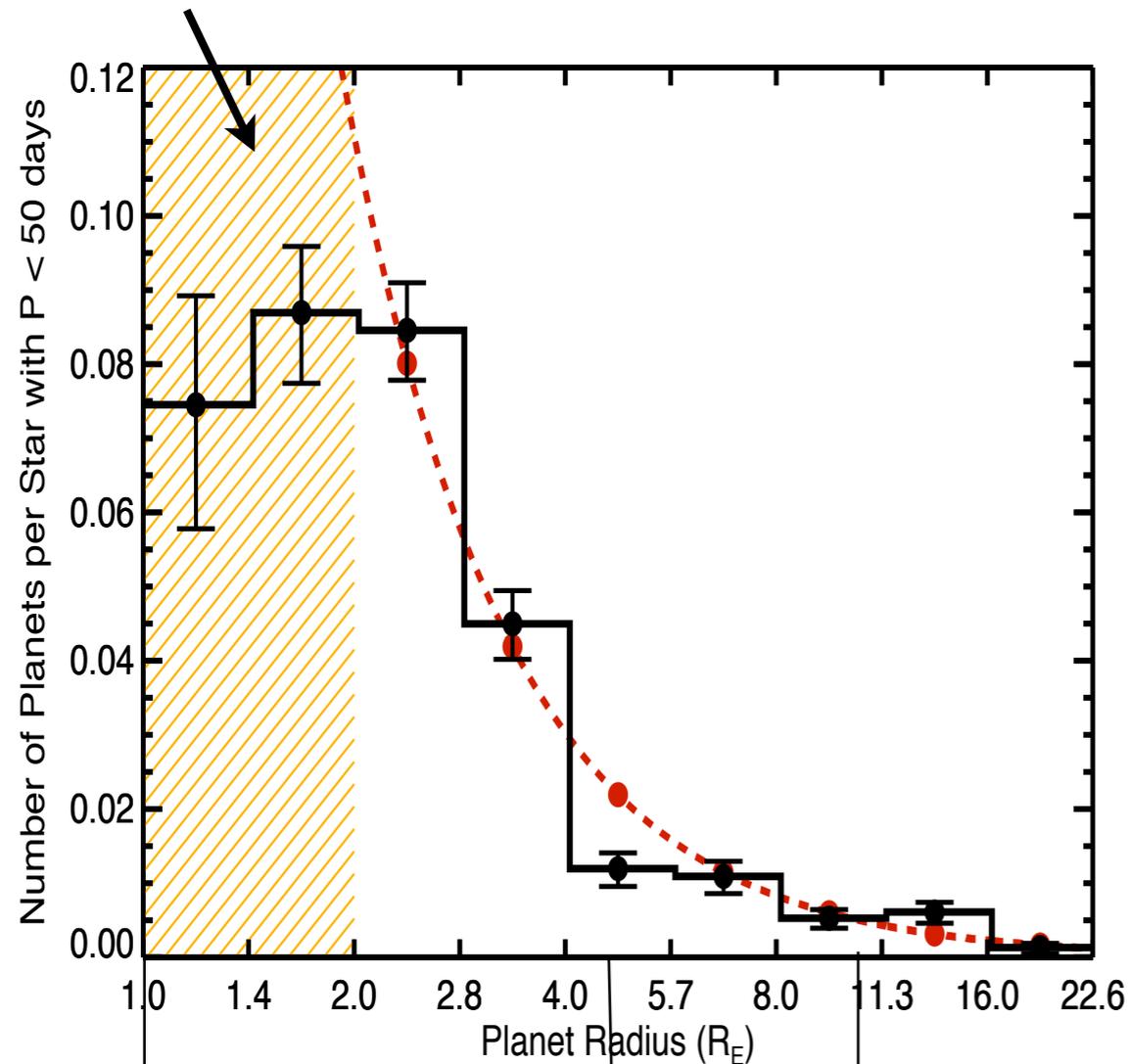
Link between ill known quantity important for formation and observations. Kepler-18d and Kepler-11e point towards small opacities.

Imprint of grain opacity on planetary mass-radius relationship.

# Comparison: KEPLER radius distribution

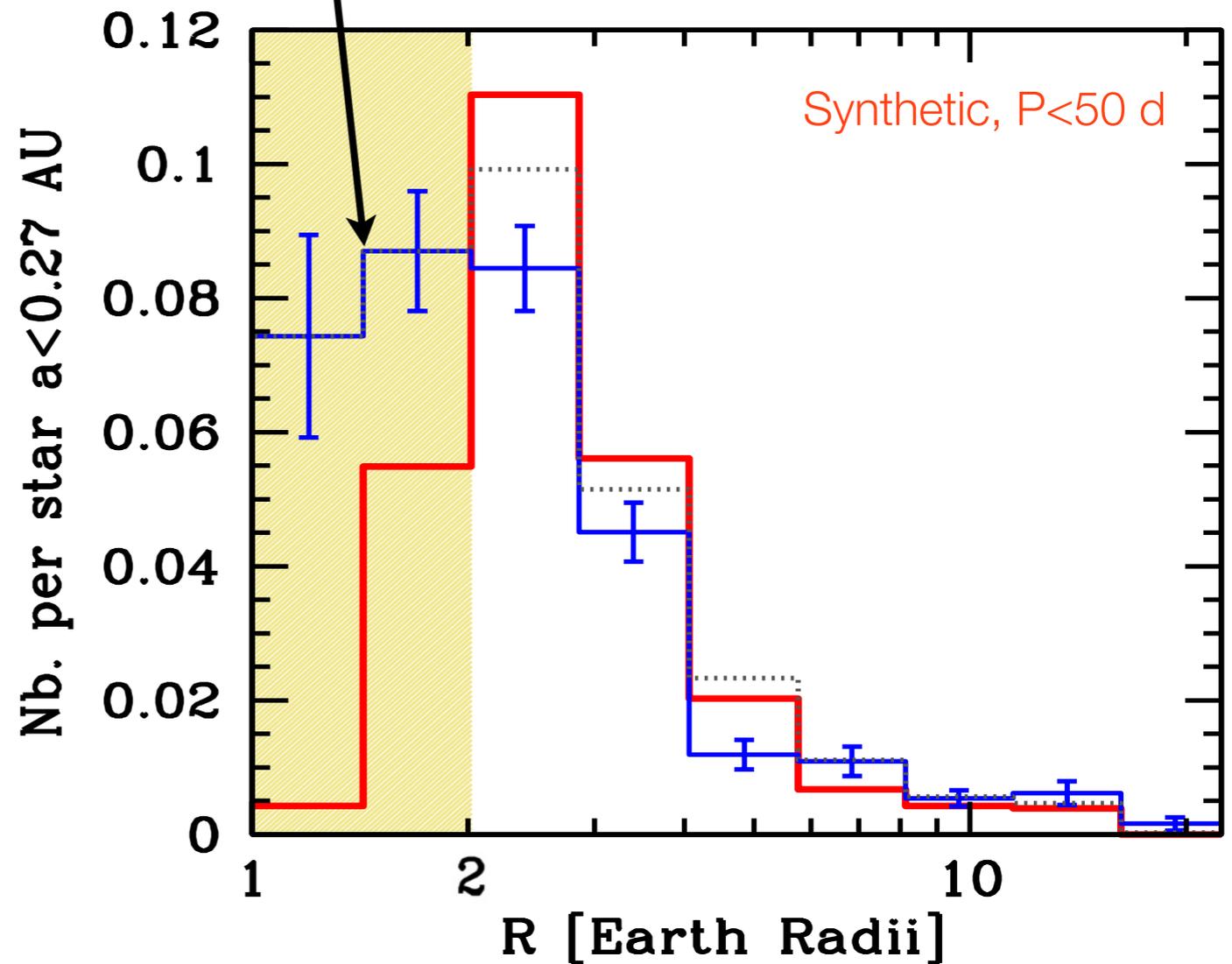
Incompleteness

Howard et al. 2011



Corrected for observational bias

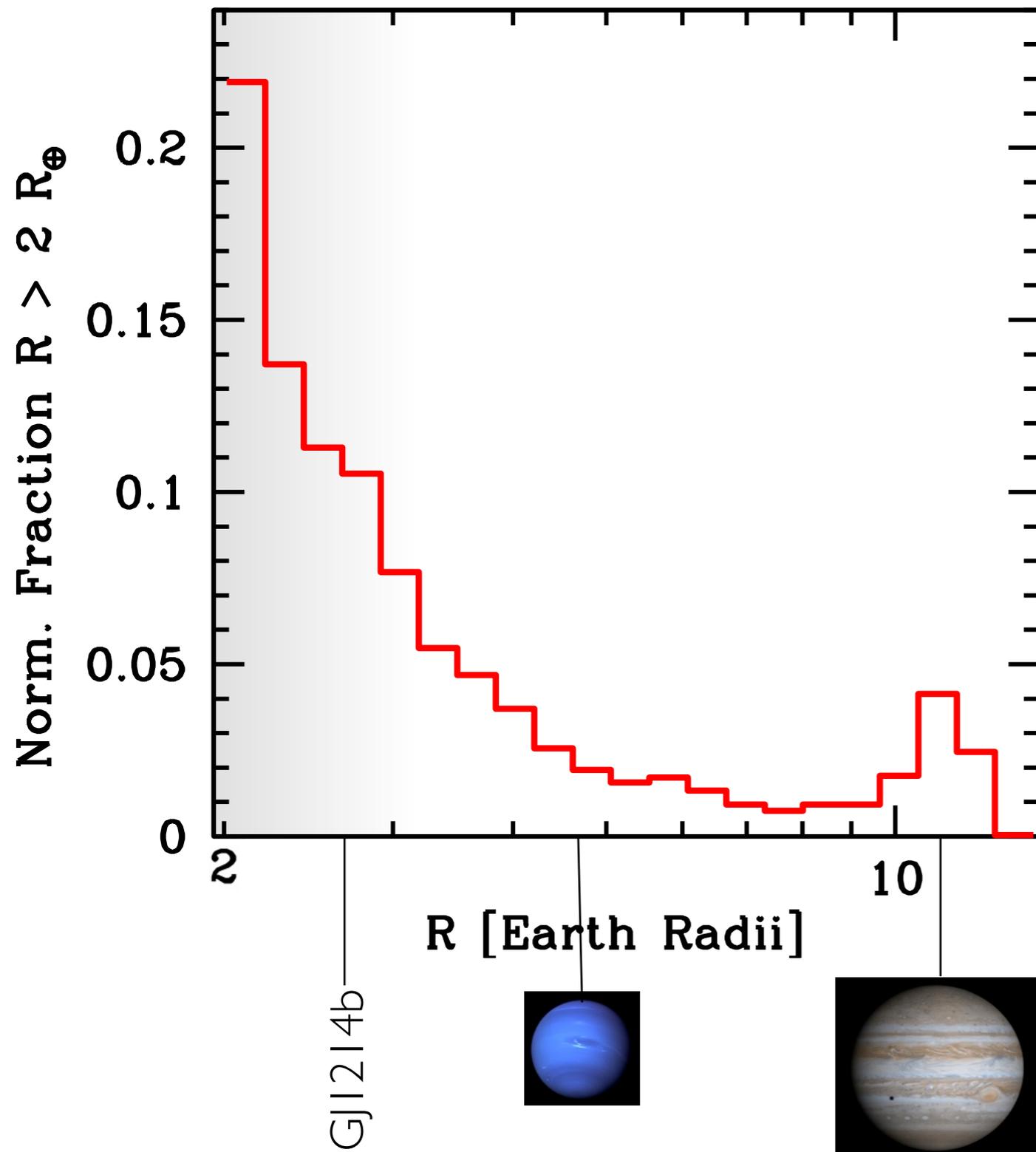
Not H/He atmospheres?



- Tentative agreement for  $R > 2 R_E$ 
  - Many low radii, Hot Jupiter 0.5-1%
  - Sensitive to type I migration model
- Divergence for  $R < 2 R_E$ :
  - Low mass H/He planets have large radii.
  - Dividing line mini-Neptunes vs. super-Earth?

# Bimodal planetary radius distribution

Mordasini+2012



*all a, finer bins*

- Radius distribution is *bimodal* (cf. Schlaufmann+2010, Wuchter2011)
- Peak at lowest radii. Most seeds don't grow much, and have large  $Z$ .
- Peak at  $\sim 1 R_J \Leftrightarrow$  Giant planets have all approx. *the same radius independent of mass* (degeneracy!)
- **Prediction:** Kepler should detect the *second, local maximum* at  $\sim 1 R_J$  (except ....)

# Summary

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- 1) Added self-consistently evolution to c.a. formation model, giving radius and luminosity besides  $a$ ,  $M$ ,  $e$ .
- 2) Calculated population wide M-R relationship.
- 3) Compared with observation, finding good agreement for the general shape. Many imprints of formation.
- 4) Calculated planetary radius distribution. Bimodal, w. strong increase to small  $R$ , and second maximum at  $\sim 1 R_J$ .
- 5) Compared with Kepler  $R$  distribution. Similar general shape. We predict the  $\sim 1 R_J$  maximum to be found in future.