

Combined models of planet formation and evolution:

The planetary mass-radius relationship

Christoph Mordasini

Max-Planck Institute for Astronomy
Heidelberg

München, 5. 9. 2012

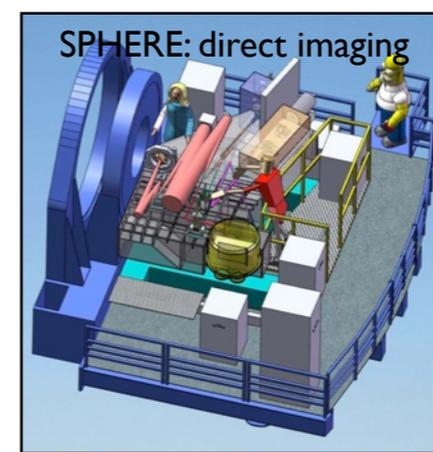
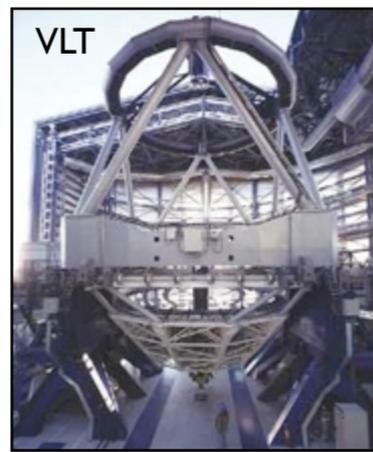
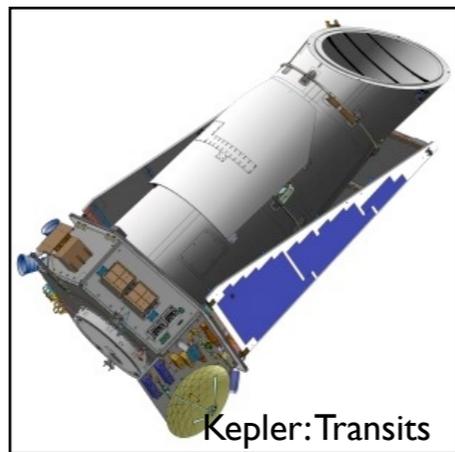


K. M. Dittkrist, P. Molliere, S. Jin, H. Klahr, T. Henning

Y. Alibert, A. Fortier, W. Benz

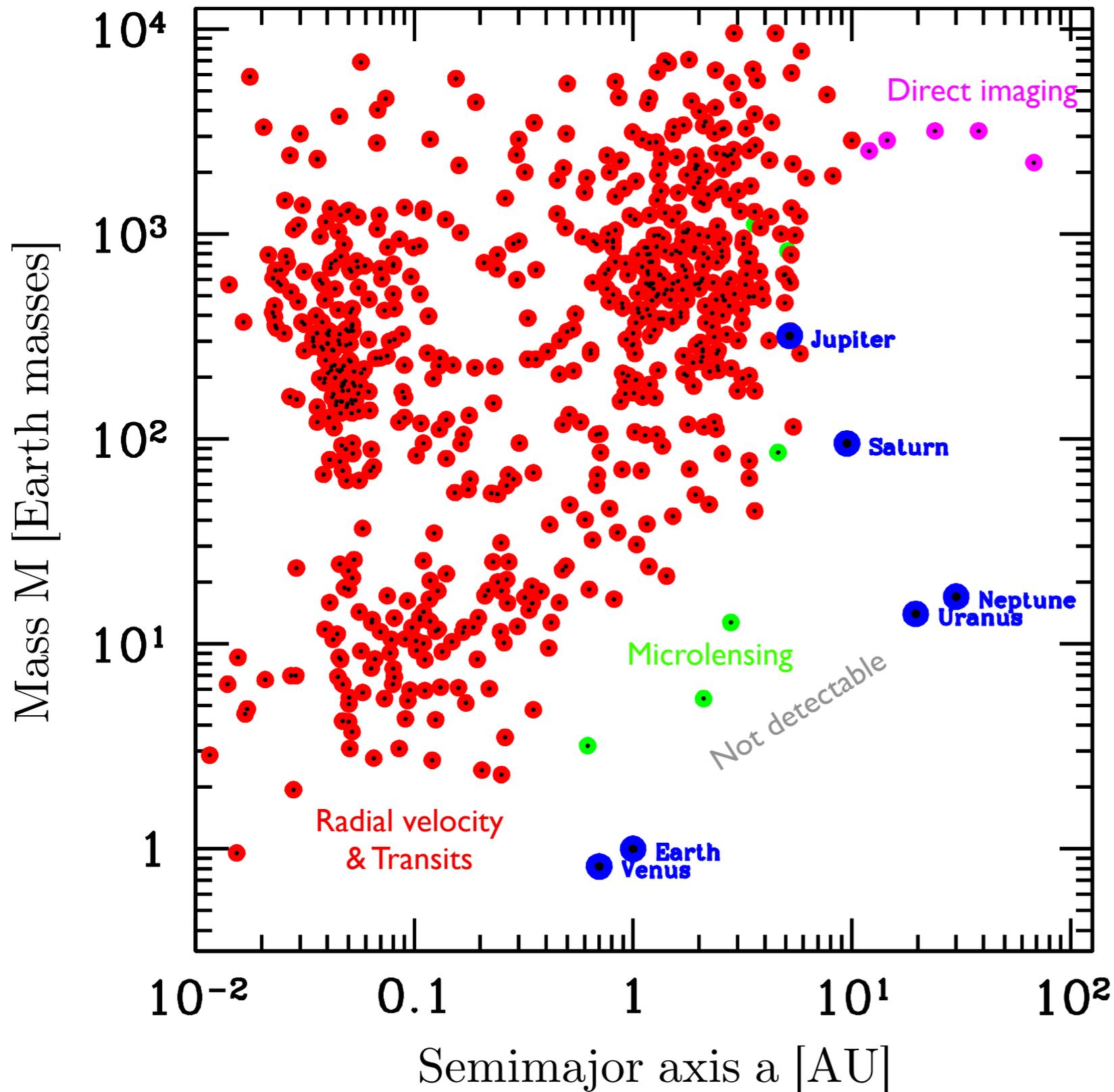
Linking observations and planet formation

- **Large** number of observations from space mission (transits, spectra) and ground (radial velocity, transits, spectra, direct imaging). More to come (SPHERE, Gaia, ESPRESSO, CHEOPS, EChO..)



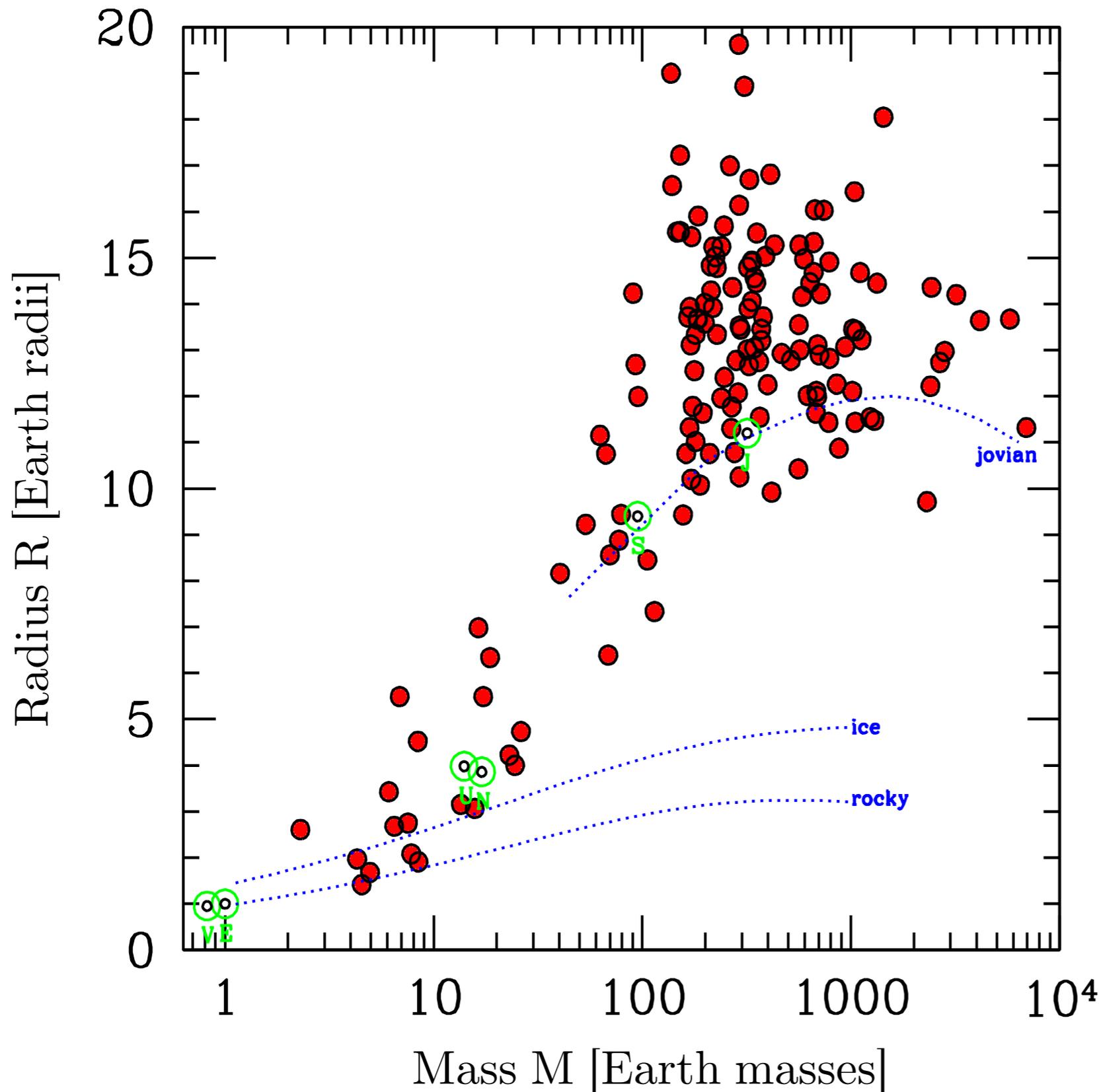
- Improve understanding of **planet formation by comparing theory and observation.**
- For a sufficiently large number of exoplanets: treat as a **statistical** ensemble.
- Planetary population synthesis: **statistical** comparison
- Difficulties: 1) different techniques constrain **different** aspects of the theory.
2) between formation and observation: **Myrs-Gyrs of evolution.**

From the a - M



- Many fundamental constraints from the a - M diagram.

From the a - M to the M - R diagram



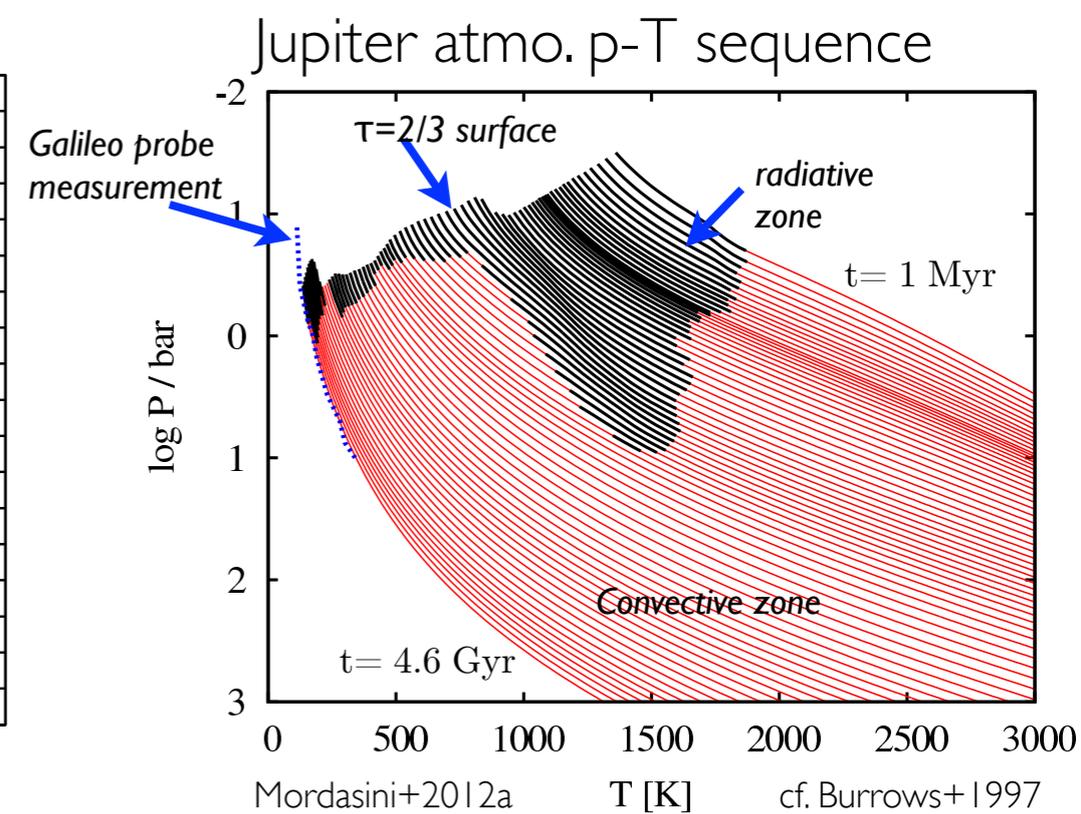
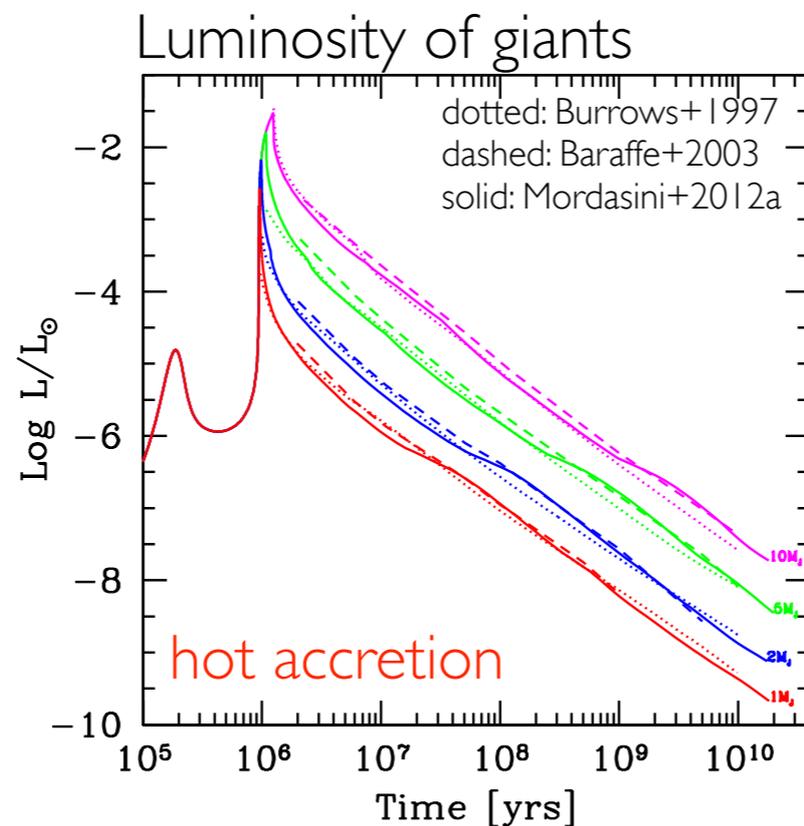
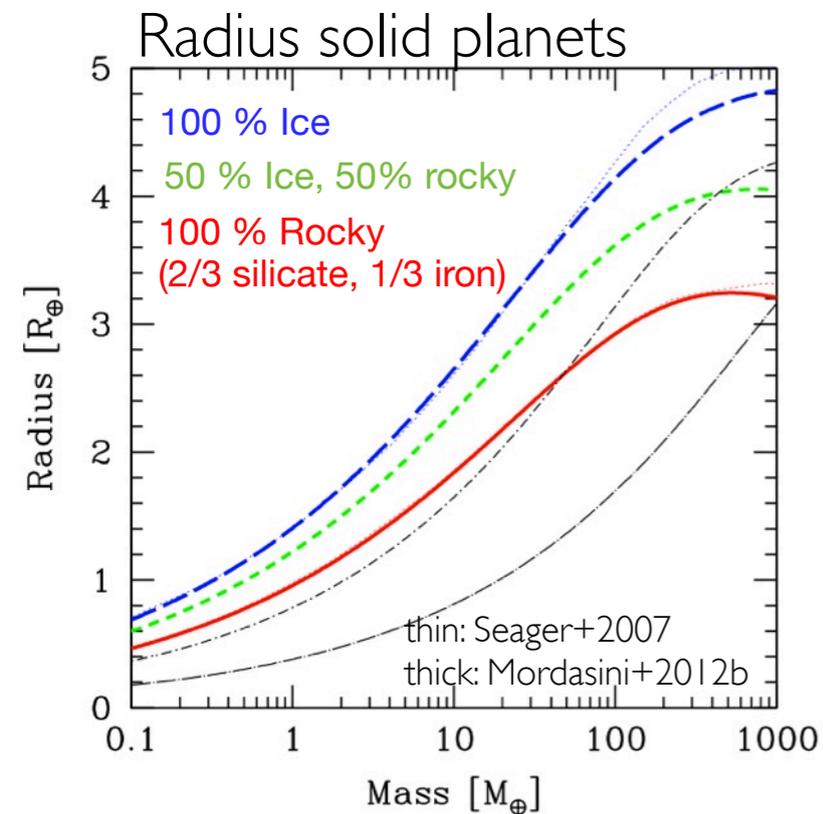
- M-R diagram: **diversity**, too!
- Bulk **composition**
- **Constraints** for formation beyond the a - M :
 - migration (icy planets close-in?)
 - efficiency of H/He accretion & loss
 - runaway when? opacity?
- **Understandable** with theoretical models?

Adding planet evolution

Formation: Based on [core accretion](#) paradigm, growth of seed embryo accreting gas and planetesimals in an evolving protoplanetary disk, undergoing orbital migration.

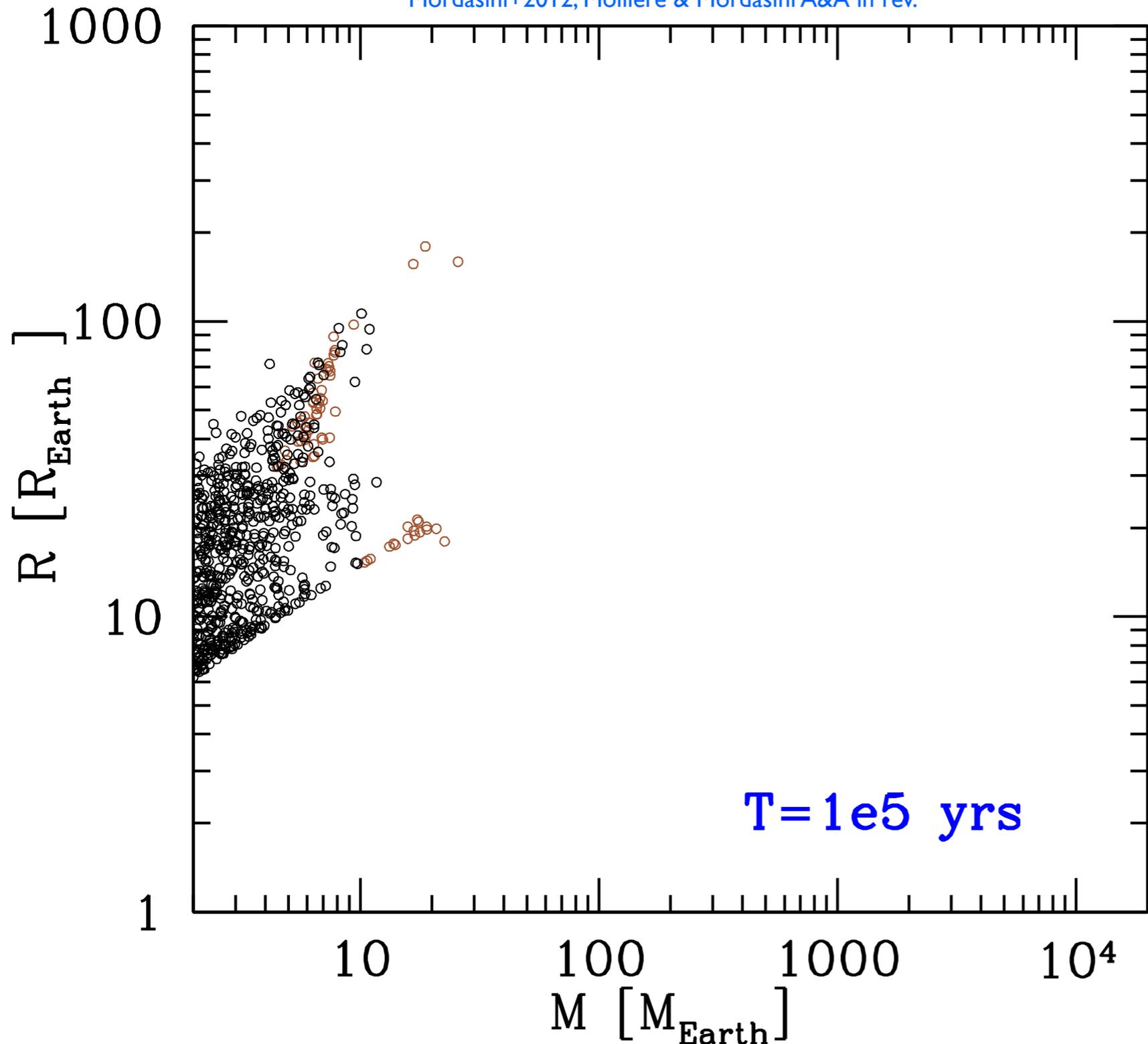
Evolution (after disk is gone): couple self consistently

- ◆ Solve 1D (radial) structure equations for the thermal evolution of the H/He envelope on Gyrs (cooling & contraction), including effects of stellar irradiation and radiogenic heating. Gray atmosphere.
- ◆ Solve 1D internal structure equations for the solid core, assuming a differentiated interior.



Population synthesis: Formation of M-R

Mordasini+2012, Molliere & Mordasini A&A in rev.



Fraction Z of solids (rest H/He)

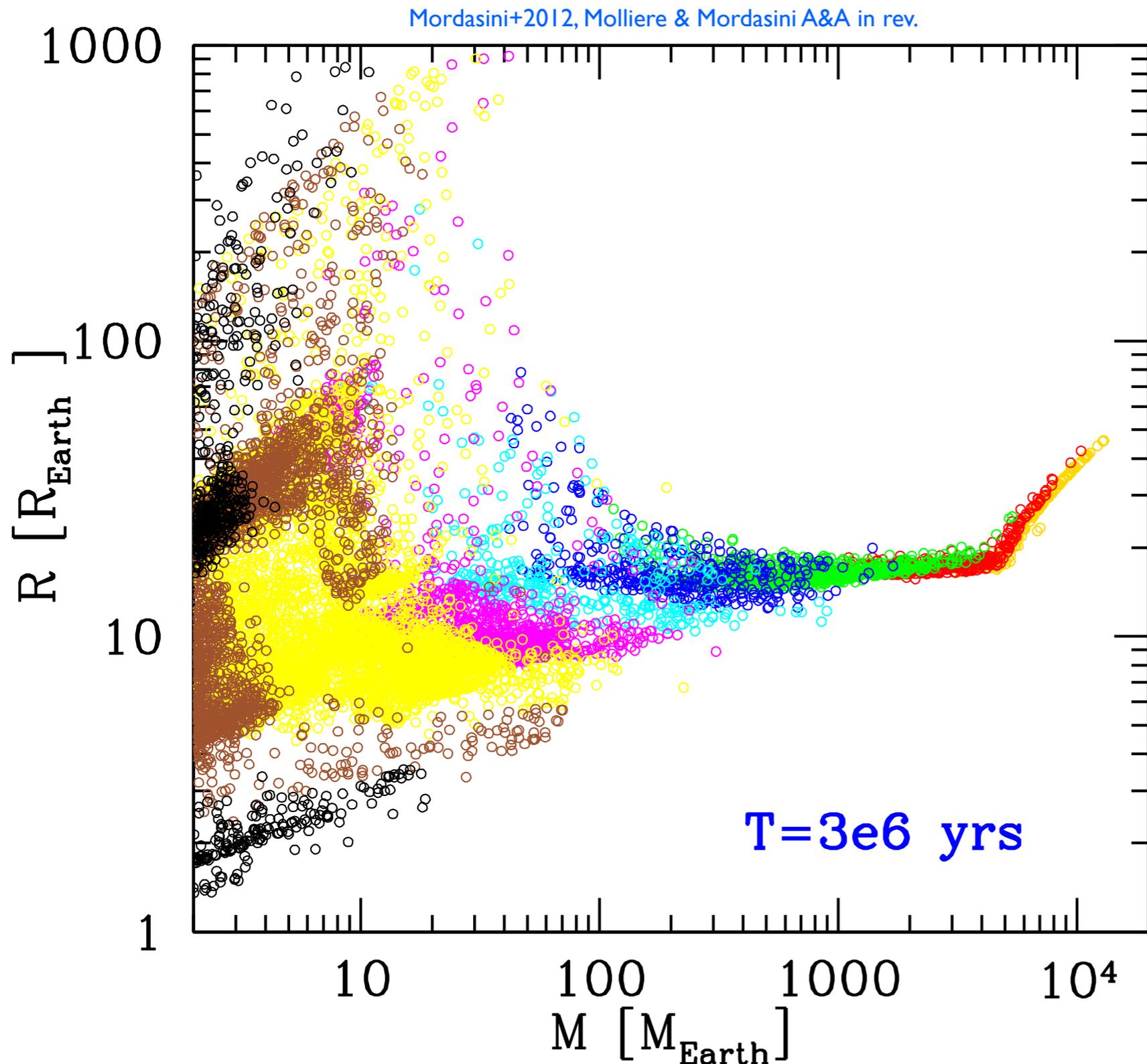
- Orange: $Z \leq 1\%$
- Red: $1 < Z \leq 5\%$
- Green: $5 < Z \leq 20\%$
- Yellow: $80 < Z \leq 95\%$
- Brown: $95 < Z \leq 99\%$
- Black: $Z > 99\%$
- Blue: $20 < Z \leq 40\%$
- Cyan: $40 < Z \leq 60\%$
- Magenta: $60 < Z \leq 80\%$

- Rapid collapse at $\sim 0.2 M_J$ when $Z \approx 0.5$ (runaway gas accretion)
- After disk dispersal ($T > 10$ Myrs), slow contraction.

Nominal Model. $M_{\text{star}} = 1 M_{\text{sun}}$. $a > 0.1 \text{ AU}$.

Non-isothermal Type I. Cold accretion. 1 embryo/disk

Population synthesis: Formation of M-R



Fraction Z of solids (rest H/He)

Orange: $Z \leq 1\%$ Blue: $20 < Z \leq 40\%$
Red: $1 < Z \leq 5\%$ Cyan: $40 < Z \leq 60\%$
Green: $5 < Z \leq 20\%$ Magenta: $60 < Z \leq 80\%$
Yellow: $80 < Z \leq 95\%$
Brown: $95 < Z \leq 99\%$
Black: $Z > 99\%$

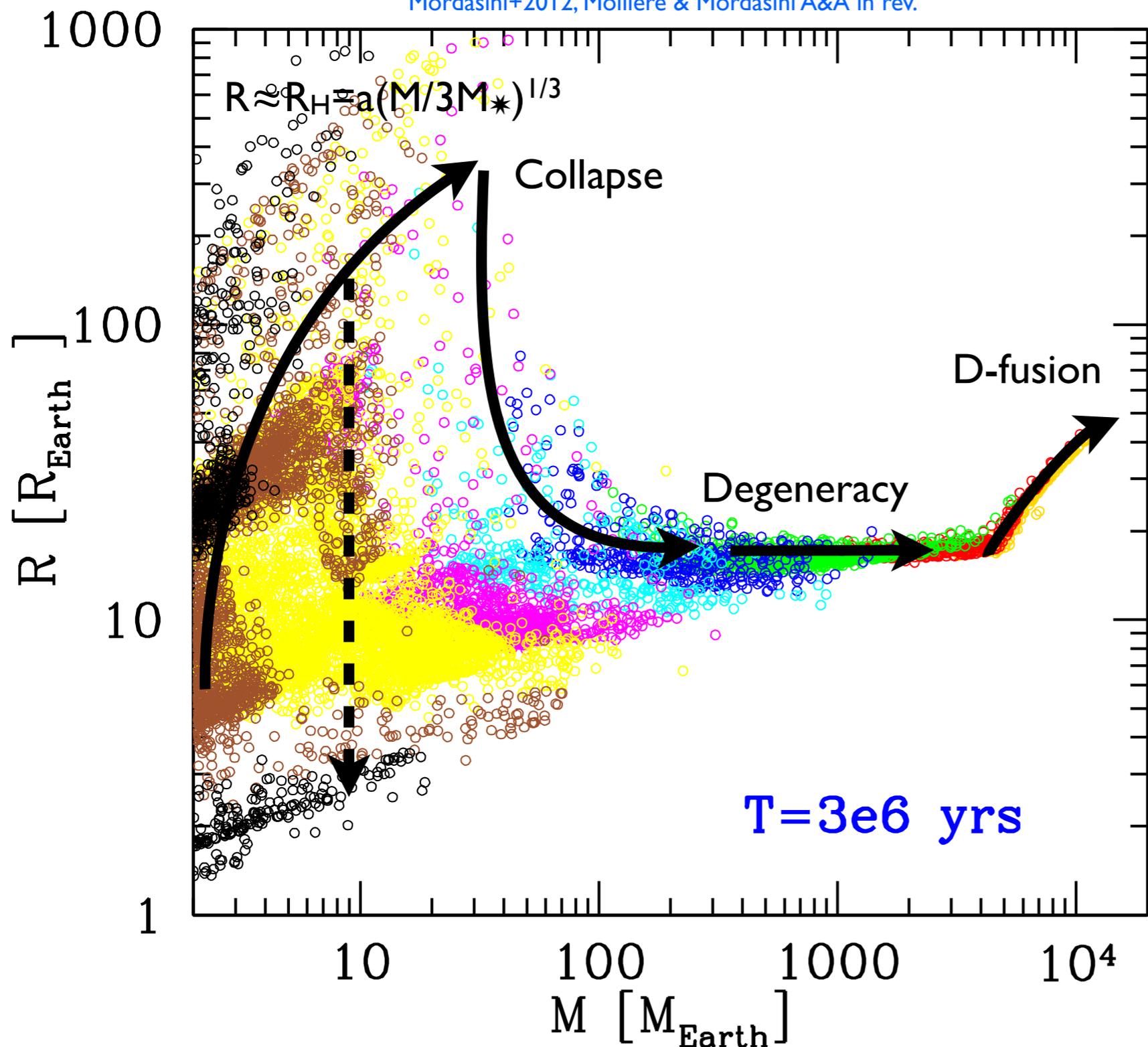
- Rapid collapse at $\sim 0.2 M_J$ when $Z \approx 0.5$ (runaway gas accretion)
- After disk dispersal ($T > 10$ Myrs), slow contraction.

Nominal Model. $M_{\text{star}}=1 M_{\text{sun}}$. $a > 0.1 \text{ AU}$.

Non-isothermal Type I. Cold accretion. 1 embryo/disk

Population synthesis: Formation of M-R

Mordasini+2012, Molliere & Mordasini A&A in rev.



Fraction Z of solids (rest H/He)

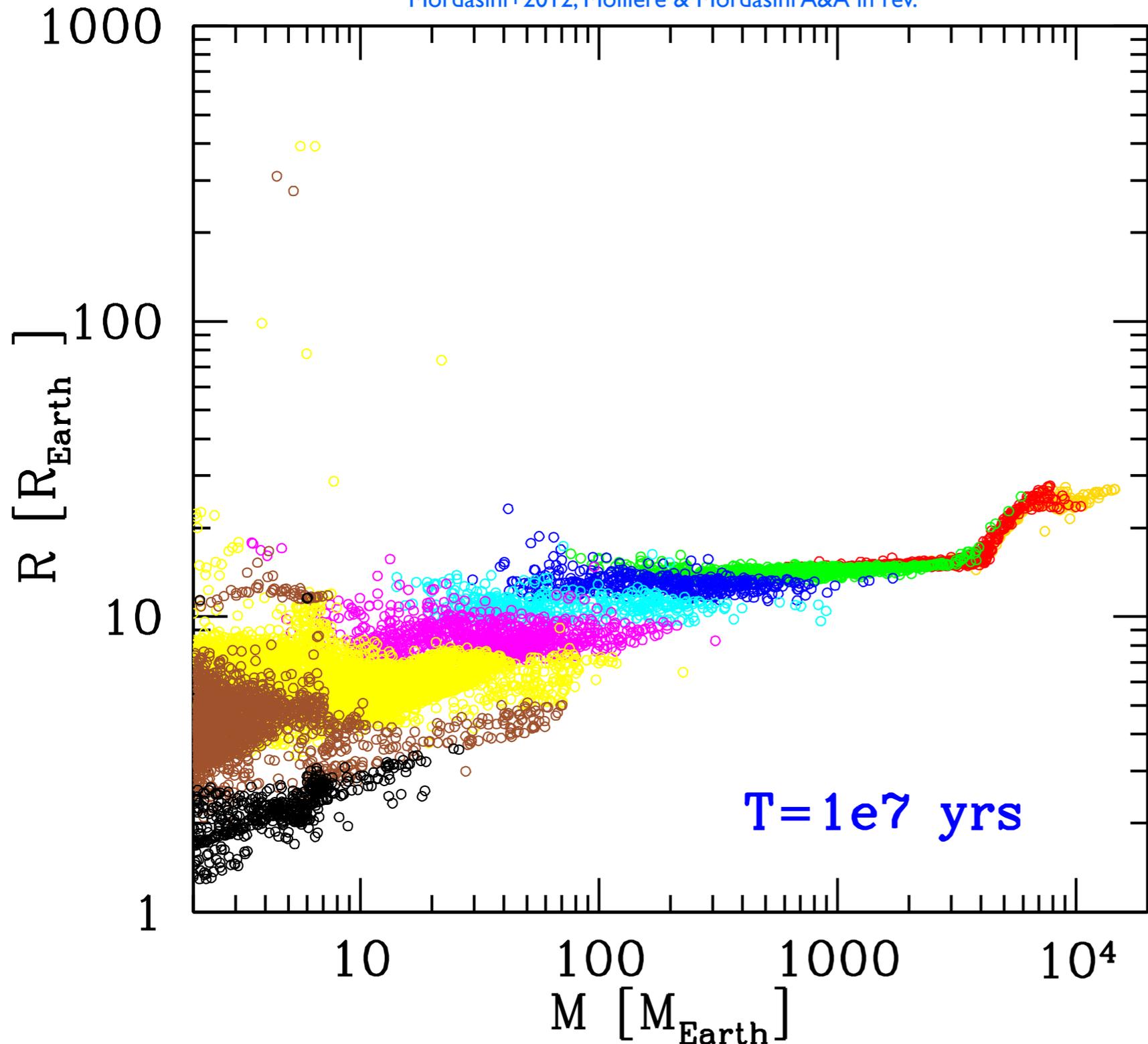
- Orange: $Z \leq 1\%$
- Red: $1 < Z \leq 5\%$
- Green: $5 < Z \leq 20\%$
- Yellow: $80 < Z \leq 95\%$
- Brown: $95 < Z \leq 99\%$
- Black: $Z > 99\%$
- Blue: $20 < Z \leq 40\%$
- Cyan: $40 < Z \leq 60\%$
- Magenta: $60 < Z \leq 80\%$

- Rapid collapse at $\sim 0.2 M_J$ when $Z \approx 0.5$ (runaway gas accretion)
- After disk dispersal ($T > 10$ Myrs), slow contraction.

Nominal Model. $M_{\text{star}} = 1 M_{\text{sun}}$. $a > 0.1 \text{ AU}$.
 Non-isothermal Type I. Cold accretion. 1 embryo/disk

Population synthesis: Formation of M-R

Mordasini+2012, Molliere & Mordasini A&A in rev.



Fraction Z of solids (rest H/He)

- Orange: $Z \leq 1\%$
- Red: $1 < Z \leq 5\%$
- Green: $5 < Z \leq 20\%$
- Yellow: $80 < Z \leq 95\%$
- Brown: $95 < Z \leq 99\%$
- Black: $Z > 99\%$
- Blue: $20 < Z \leq 40\%$
- Cyan: $40 < Z \leq 60\%$
- Magenta: $60 < Z \leq 80\%$

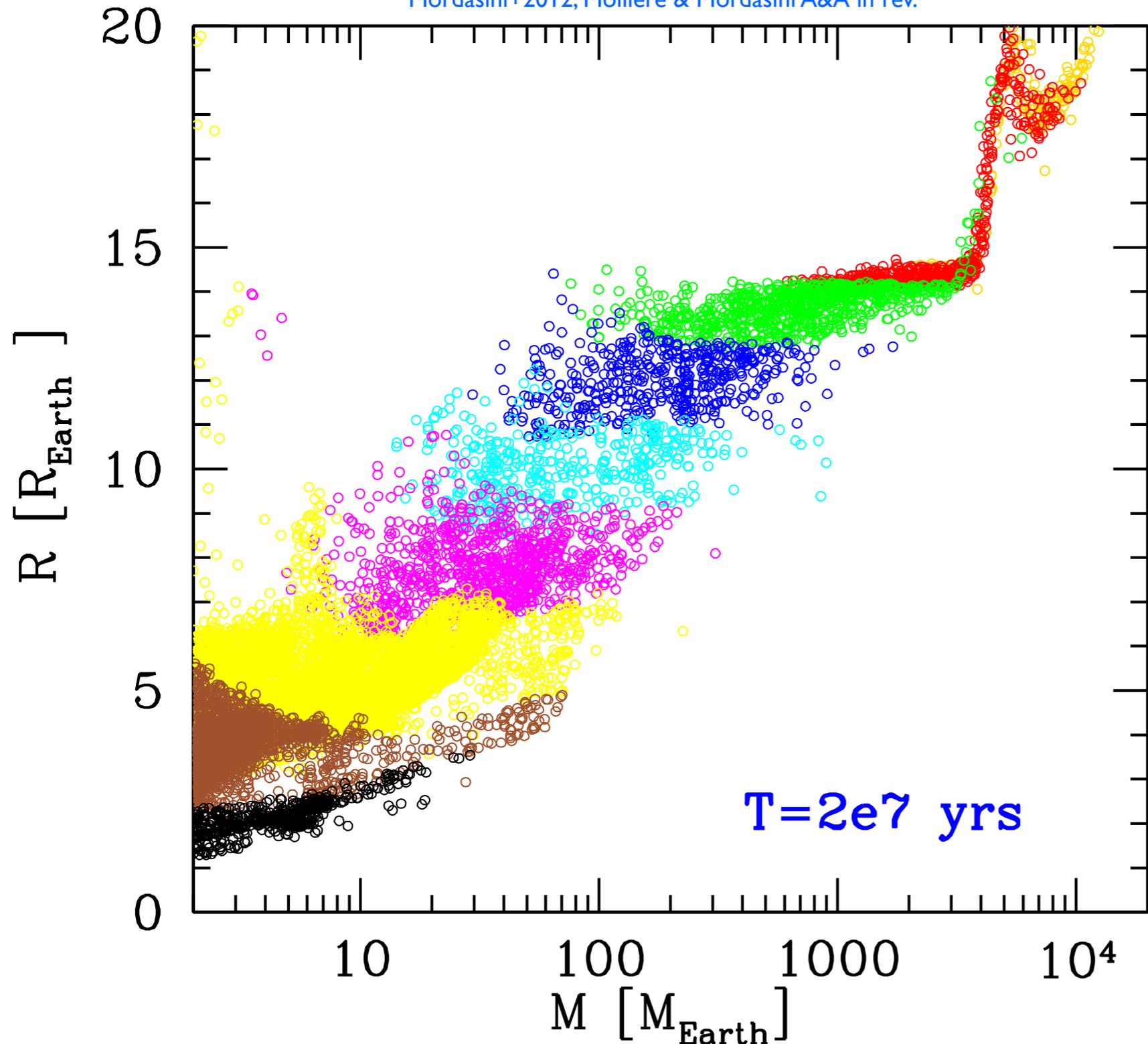
- Rapid collapse at $\sim 0.2 M_J$ when $Z \approx 0.5$ (runaway gas accretion)
- After disk dispersal ($T > 10$ Myrs), slow contraction.

Nominal Model. $M_{\text{star}} = 1 M_{\text{sun}}$. $a > 0.1 \text{ AU}$.

Non-isothermal Type I. Cold accretion. 1 embryo/disk

Population synthesis: Formation of M-R

Mordasini+2012, Molliere & Mordasini A&A in rev.



Fraction Z of solids (rest H/He)

- Orange: $Z \leq 1\%$
- Red: $1 < Z \leq 5\%$
- Green: $5 < Z \leq 20\%$
- Yellow: $80 < Z \leq 95\%$
- Brown: $95 < Z \leq 99\%$
- Black: $Z > 99\%$
- Blue: $20 < Z \leq 40\%$
- Cyan: $40 < Z \leq 60\%$
- Magenta: $60 < Z \leq 80\%$

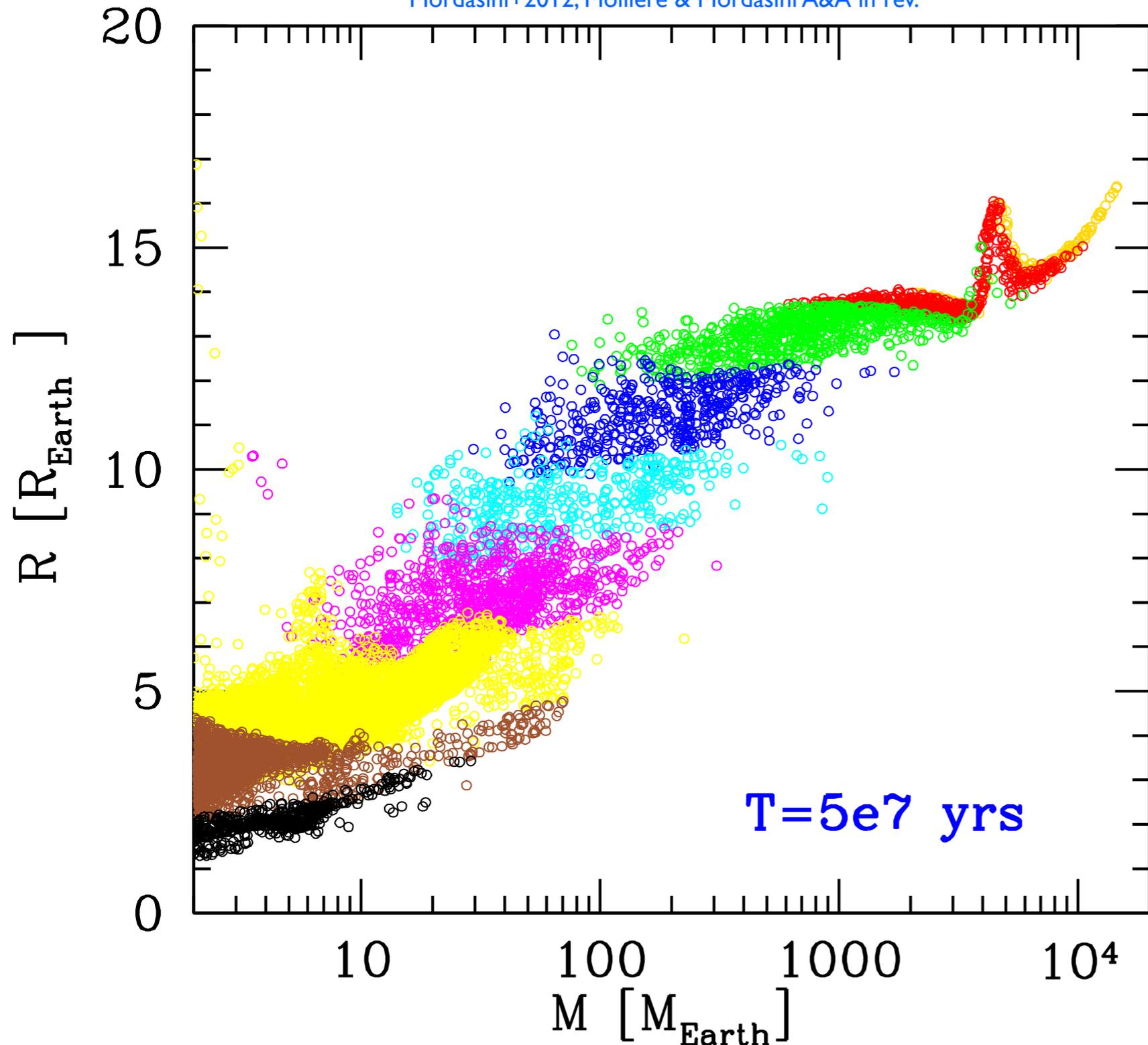
- Rapid collapse at $\sim 0.2 M_J$ when $Z \approx 0.5$ (runaway gas accretion)
- After disk dispersal ($T > 10$ Myrs), slow contraction.

Nominal Model. $M_{\text{star}} = 1 M_{\text{sun}}$. $a > 0.1 \text{ AU}$.

Non-isothermal Type I. Cold accretion. 1 embryo/disk

Population synthesis: Formation of M-R

Mordasini+2012, Molliere & Mordasini A&A in rev.



Fraction Z of solids (rest H/He)

- Orange: $Z \leq 1\%$
- Red: $1 < Z \leq 5\%$
- Green: $5 < Z \leq 20\%$
- Yellow: $80 < Z \leq 95\%$
- Brown: $95 < Z \leq 99\%$
- Black: $Z > 99\%$
- Blue: $20 < Z \leq 40\%$
- Cyan: $40 < Z \leq 60\%$
- Magenta: $60 < Z \leq 80\%$

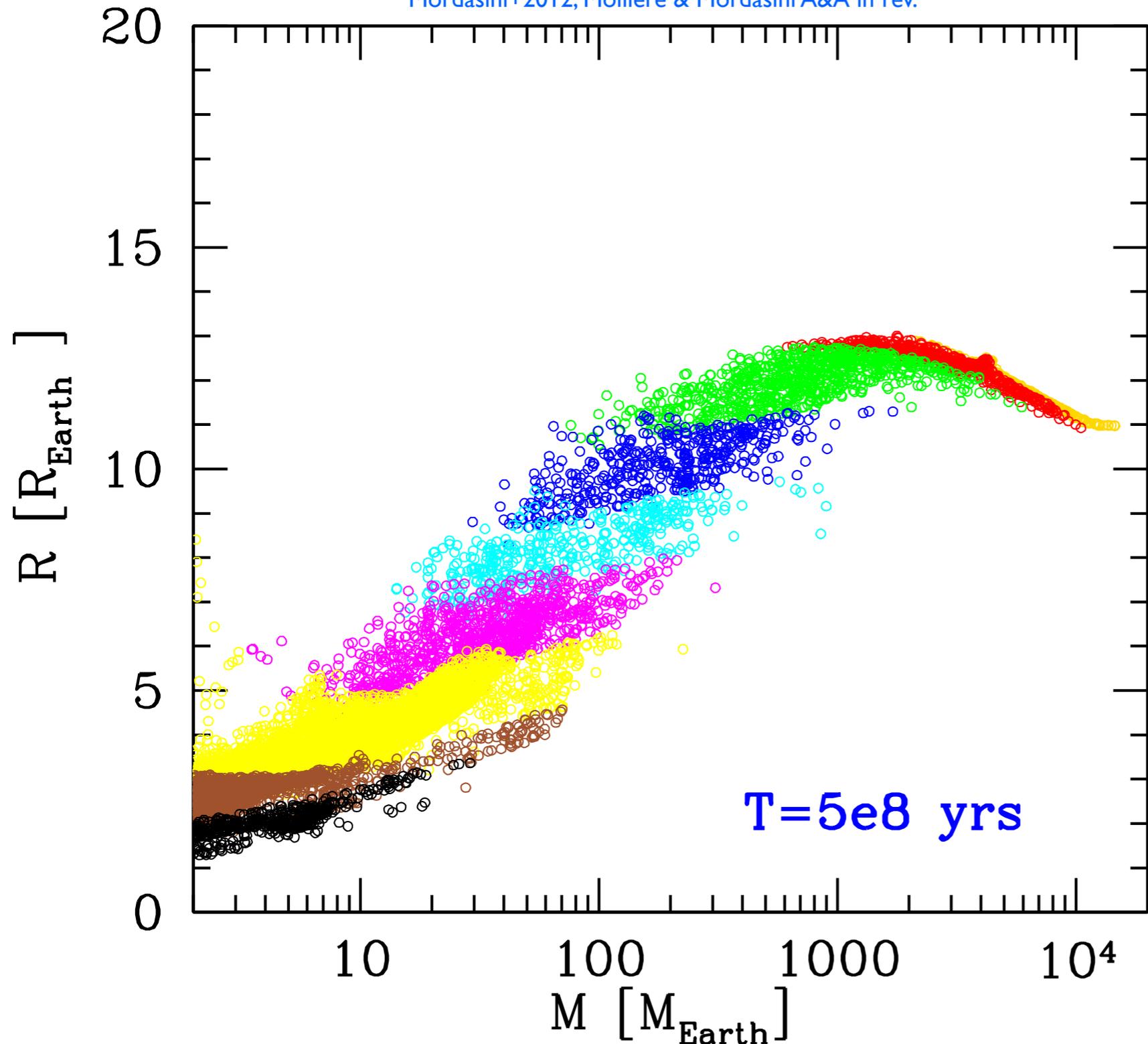
- Rapid collapse at $\sim 0.2 M_{\text{J}}$ when $Z \approx 0.5$ (runaway gas accretion)
- After disk dispersal ($T > 10$ Myrs), slow contraction.

Nominal Model. $M_{\text{star}}=1 M_{\text{sun}}$. $a > 0.1 \text{ AU}$.

Non-isothermal Type I. Cold accretion. 1 embryo/disk

Population synthesis: Formation of M-R

Mordasini+2012, Molliere & Mordasini A&A in rev.



Fraction Z of solids (rest H/He)

- Orange: $Z \leq 1\%$
- Red: $1 < Z \leq 5\%$
- Green: $5 < Z \leq 20\%$
- Yellow: $80 < Z \leq 95\%$
- Brown: $95 < Z \leq 99\%$
- Black: $Z > 99\%$
- Blue: $20 < Z \leq 40\%$
- Cyan: $40 < Z \leq 60\%$
- Magenta: $60 < Z \leq 80\%$

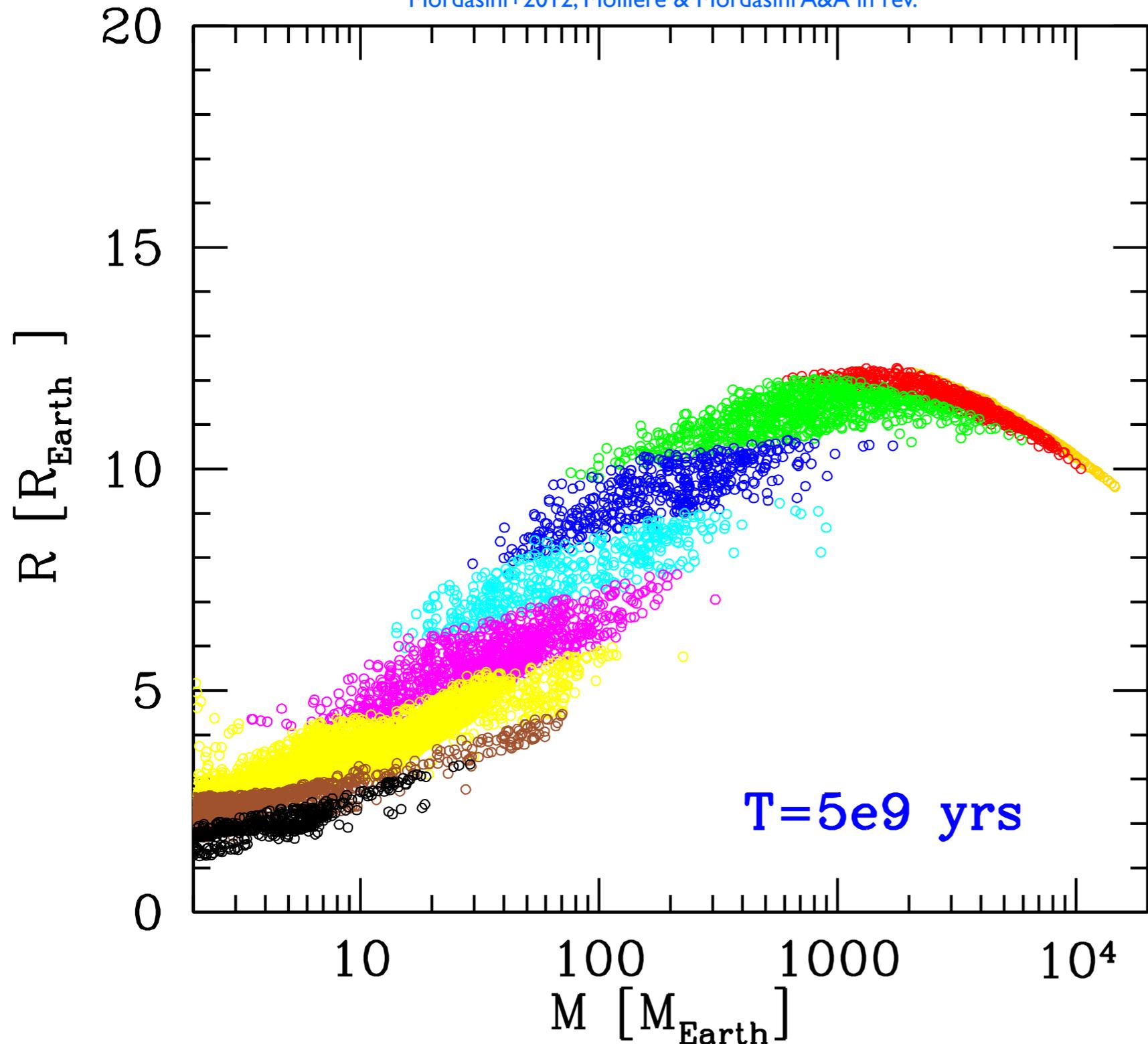
- Rapid collapse at $\sim 0.2 M_J$ when $Z \approx 0.5$ (runaway gas accretion)
- After disk dispersal ($T > 10$ Myrs), slow contraction.

Nominal Model. $M_{\text{star}} = 1 M_{\text{sun}}$. $a > 0.1 \text{ AU}$.

Non-isothermal Type I. Cold accretion. 1 embryo/disk

Population synthesis: Formation of M-R

Mordasini+2012, Molliere & Mordasini A&A in rev.



Fraction Z of solids (rest H/He)

- Orange: $Z \leq 1\%$
- Red: $1 < Z \leq 5\%$
- Green: $5 < Z \leq 20\%$
- Yellow: $20 < Z \leq 40\%$
- Brown: $40 < Z \leq 60\%$
- Black: $60 < Z \leq 80\%$
- Blue: $80 < Z \leq 95\%$
- Cyan: $95 < Z \leq 99\%$
- Magenta: $Z > 99\%$

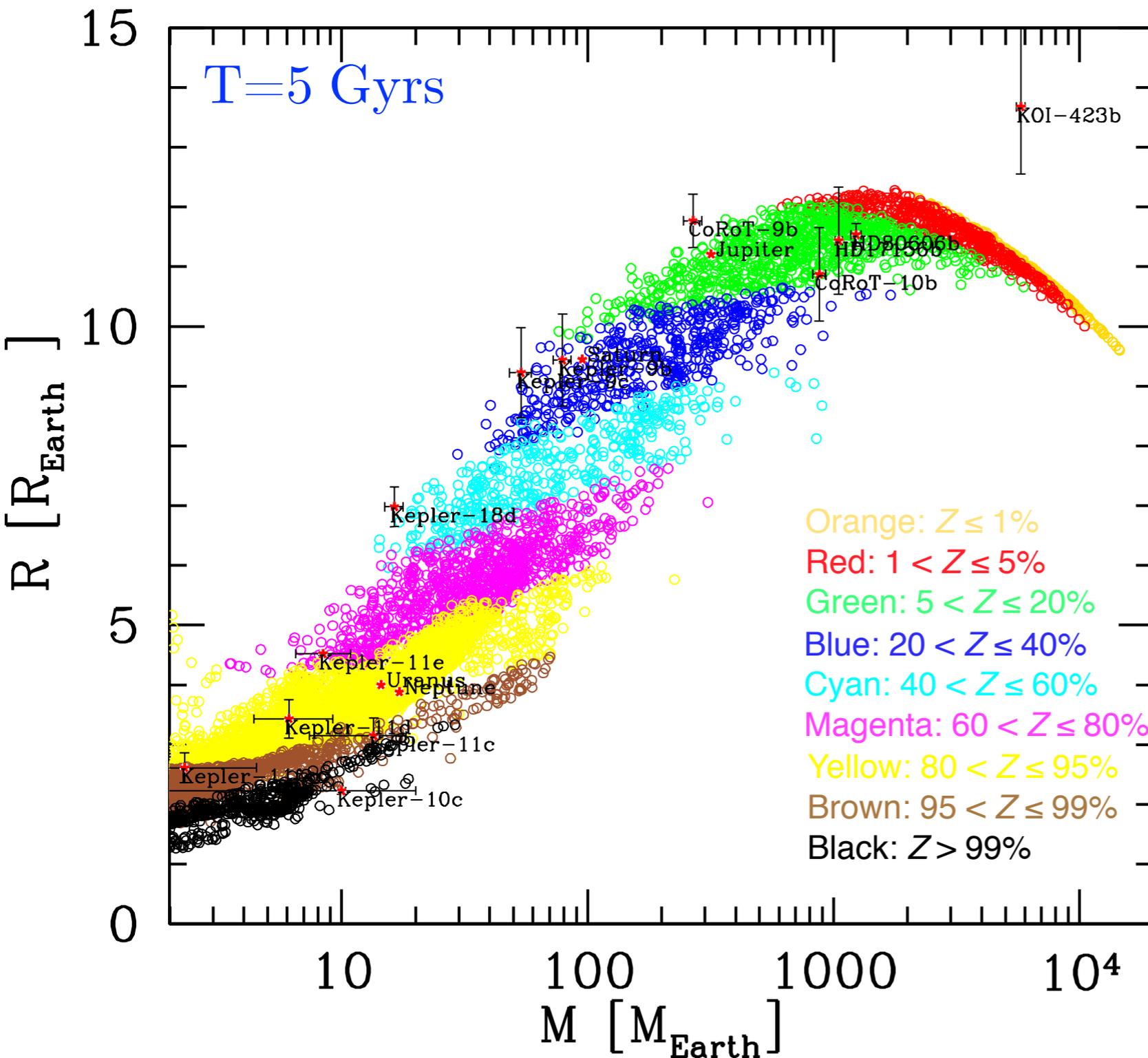
- Rapid collapse at $\sim 0.2 M_J$ when $Z \approx 0.5$ (runaway gas accretion)
- After disk dispersal ($T > 10 \text{ Myrs}$), slow contraction.

Nominal Model. $M_{\text{star}} = 1 M_{\text{sun}}$. $a > 0.1 \text{ AU}$.

Non-isothermal Type I. Cold accretion. 1 embryo/disk

M - R diagram: comparison w. observations

Mordasini+2012b



All synthetic planets and all planets with known M-R outside 0.1 AU.

- S-shape with forbidden zones
 - low $M \Rightarrow$ high $Z \Rightarrow$ small R
 - high $M \Rightarrow$ low $Z \Rightarrow$ large R
- Imprint from core accretion & EOS
- Diversity in R at one M
- Comparison with observations fine except for KOI-423b. To be tested with future observation.
- Expect divergence in future at small masses (only H/He!)

M-R diagram: effect of grain opacity

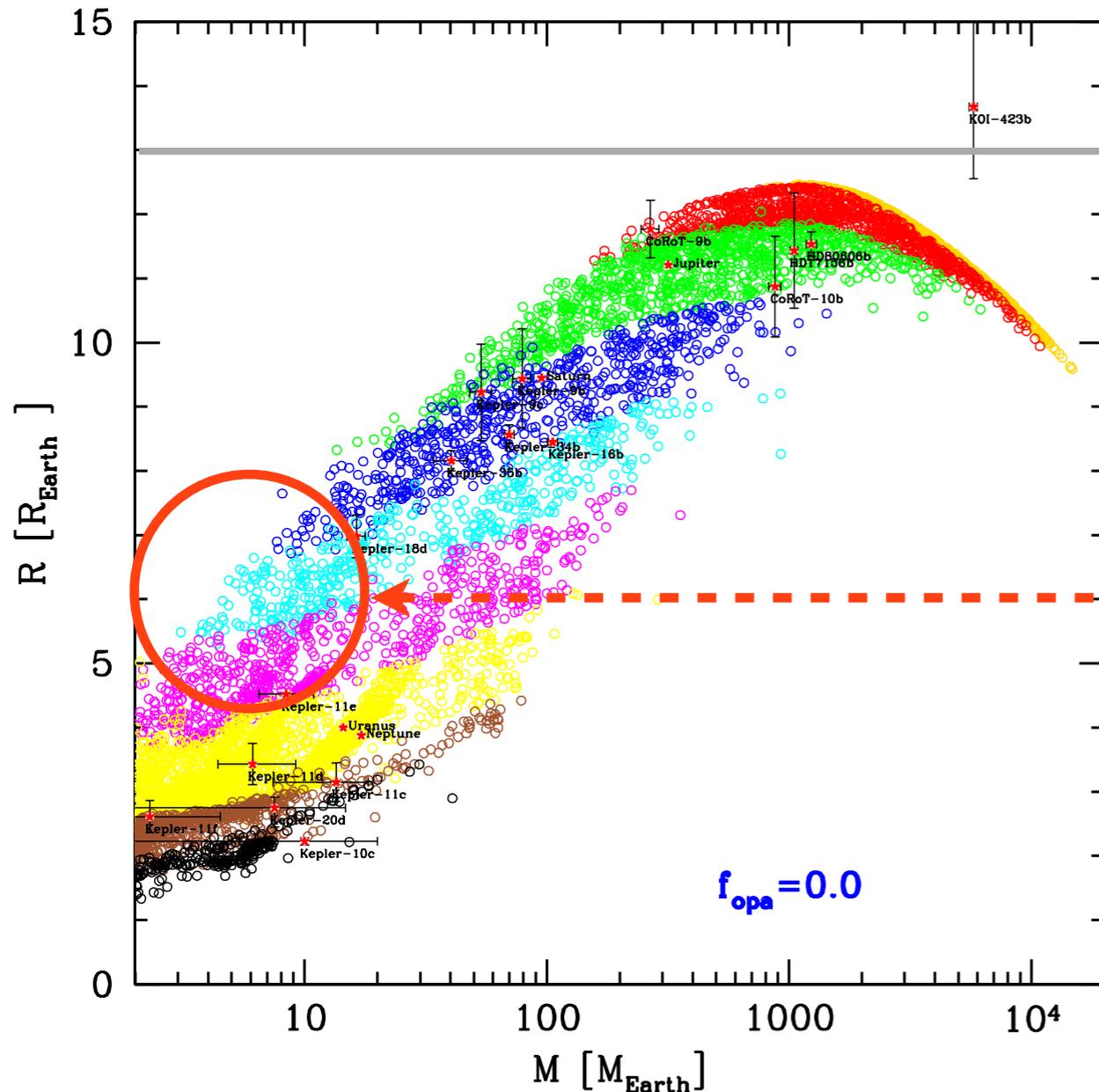
Efficiency of accretion of H/He by cores:
Controlled by opacity due to grains in the
envelope during formation. Grains evolve.
Low opacity \Rightarrow high $M_{\text{envelope}} \Rightarrow$ large R .
High opacity \Rightarrow low $M_{\text{envelope}} \Rightarrow$ small R .

Podolak+2003, Movshovitz+2010, Hori & Ikoma 2010

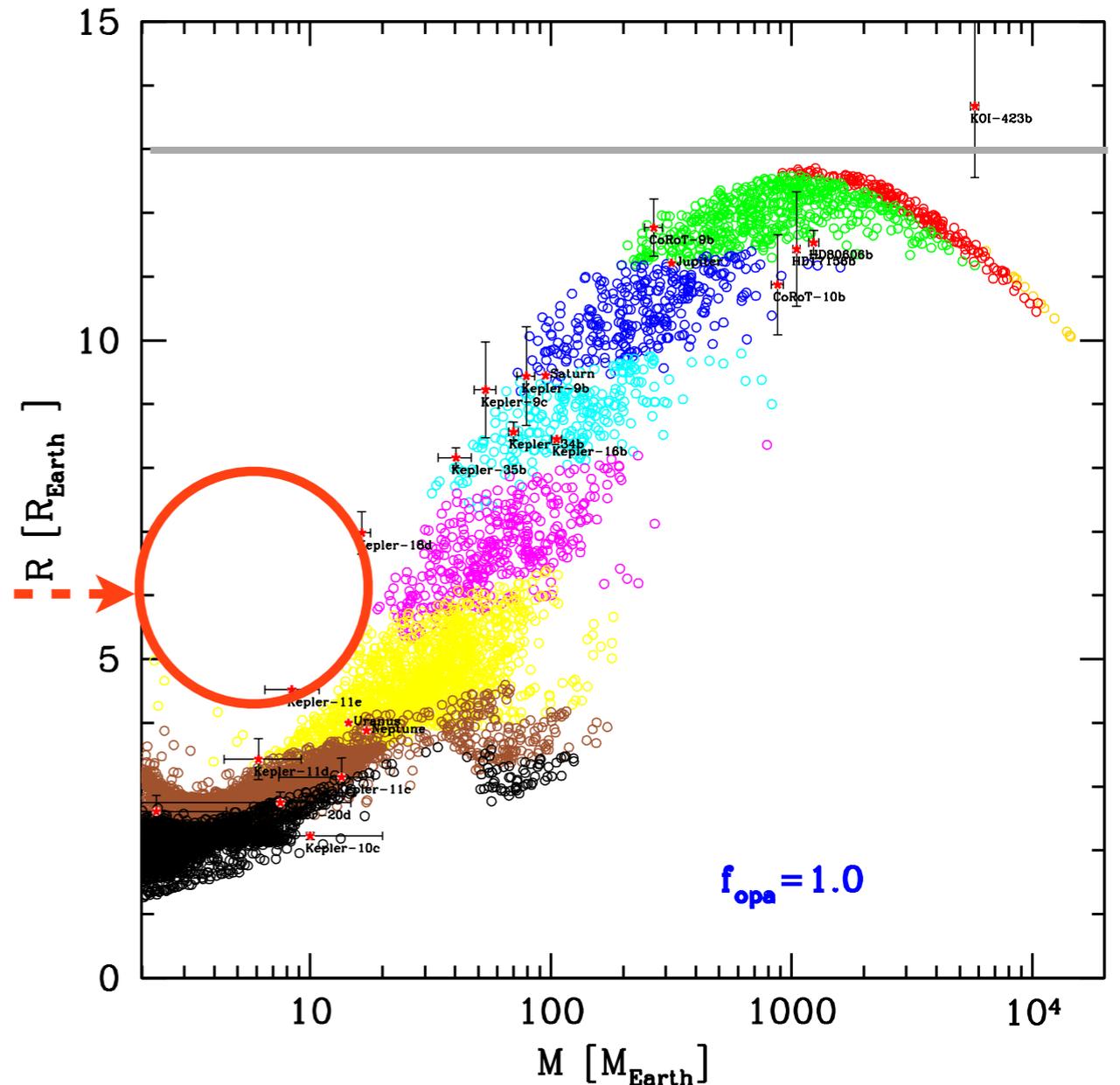
M-R diagram: effect of grain opacity

Mordasini et al in rev.

Zero grain opacity



Full interstellar grain opacity



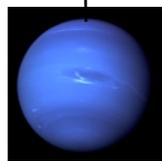
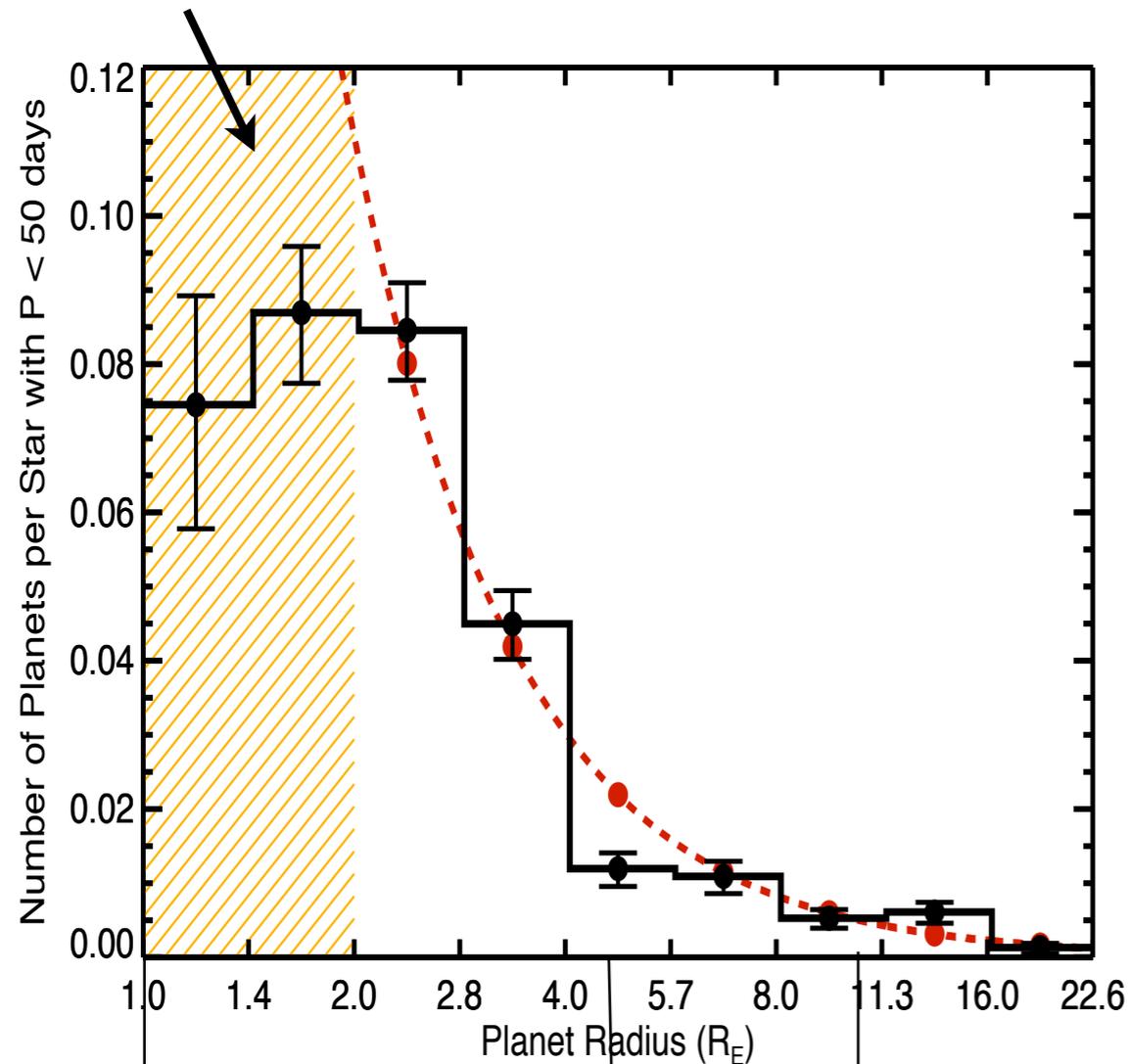
Link between ill known quantity important for formation and observations. Kepler-18d and Kepler-11e point towards small opacities.

Imprint of grain opacity on planetary mass-radius relationship.

Comparison: KEPLER radius distribution

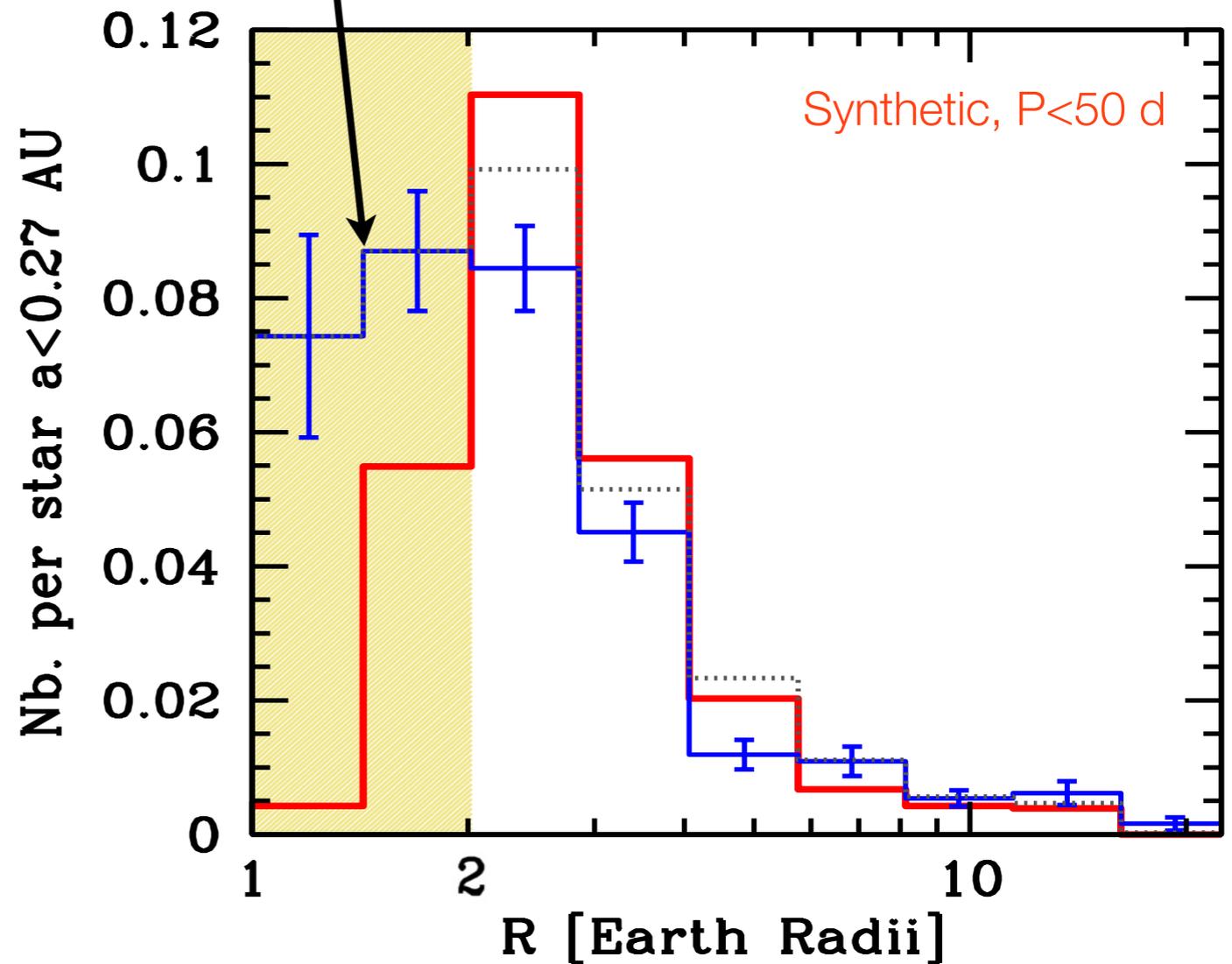
Incompleteness

Howard et al. 2011



Corrected for observational bias

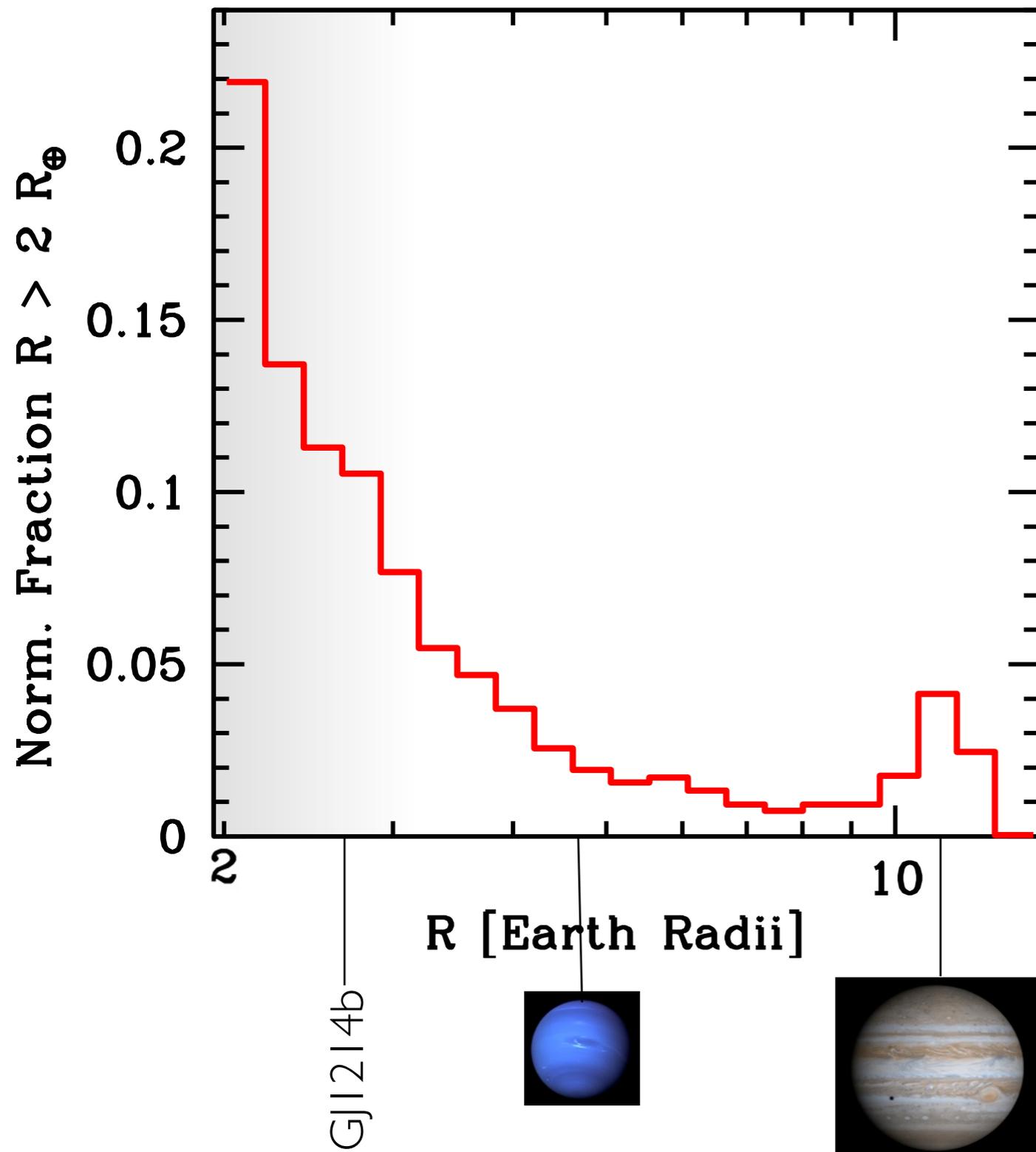
Not H/He atmospheres?



- Tentative agreement for $R > 2 R_E$
 - Many low radii, Hot Jupiter 0.5-1%
 - Sensitive to type I migration model
- Divergence for $R < 2 R_E$:
 - Low mass H/He planets have large radii.
 - Dividing line mini-Neptunes vs. super-Earth?

Bimodal planetary radius distribution

Mordasini+2012



all a, finer bins

- Radius distribution is *bimodal* (cf. Schlaufmann+2010, Wuchter2011)
- Peak at lowest radii. Most seeds don't grow much, and have large Z .
- Peak at $\sim 1 R_J \Leftrightarrow$ Giant planets have all approx. *the same radius independent of mass* (degeneracy!)
- **Prediction:** Kepler should detect the *second, local maximum* at $\sim 1 R_J$ (except)

Summary

- 1) Added self-consistently evolution to c.a. formation model, giving radius and luminosity besides a , M , e .
- 2) Calculated population wide M-R relationship.
- 3) Compared with observation, finding good agreement for the general shape. Many imprints of formation.
- 4) Calculated planetary radius distribution. Bimodal, w. strong increase to small R , and second maximum at $\sim 1 R_J$.
- 5) Compared with Kepler R distribution. Similar general shape. We predict the $\sim 1 R_J$ maximum to be found in future.