



Magnetic flux emergence in fast rotating stars

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Abstract. Fast rotating cool stars are characterised by high magnetic activity levels and frequently show dark spots up to polar latitudes. Their distinctive surface distributions of magnetic flux are investigated in the context of the solar-stellar connection by applying the solar flux eruption and surface flux transport models to stars with different rotation rates, mass, and evolutionary stage. The rise of magnetic flux tubes through the convection zone is primarily buoyancy-driven, though their evolution can be strongly affected by the Coriolis force. The poleward deflection of the tube's trajectory increases with the stellar rotation rate, which provides an explanation for magnetic flux eruption at high latitudes. The formation of proper polar spots likely requires the assistance of meridional flows both before and after the eruption of magnetic flux on the stellar surface. Since small radiative cores support the eruption of flux tubes at high latitudes, low-mass pre-main sequence stars are predicted to show high mean latitudes of flux emergence. In addition to flux eruption at high latitudes, main sequence components of close binary systems show spot distributions which are non-uniform in longitude. Yet these 'preferred longitudes' of flux eruption are expected to vanish beyond a certain post-main sequence evolutionary stage.

Key words. stars: magnetic activity – stars: rotation – stars: pre-main sequence – binaries: close – magnetohydrodynamics (MHD)

1. Introduction

The magnetic activity of cool stars has a crucial impact on their rotational evolution and on their appearance in different wavelength ranges. Detailed observations of solar activity phenomena lead to models for the cyclic re-generation of magnetic fields through self-sustained dynamo processes inside the convection zone, which are based on the interaction of convective motions and (differential) rotation (Ossendrijver 2003; Schüssler 2005, and references therein). Yet a comprehensive understanding of the sub-surface origin of magnetic

flux is only possible when the heterogeneity of other cool stars is taken into account as well, since the large ranges of stellar rotation rates, stellar masses, and evolutionary stages provide the testbed required to verify (or falsify) current theories.

Photometric and spectro-polarimetric observations allow for the reconstruction of stellar surface brightness and surface magnetic field distributions (e.g. Collier Cameron 2001; Hussain 2004, and references therein). The surface maps frequently show distinctive differences compared to the solar case like huge spots at high latitudes. In the context of the solar-stellar connection, these characteristic phenomena are investigated in the framework

of solar flux transport models. The present review focuses on the storage, transport, and eruption of magnetic flux in the convective envelope of fast rotating cool stars.

2. Magnetic flux of fast rotating stars

Zeeman-broadening of magnetic sensitive lines provides a measure for the surface-averaged magnetic flux (flux density times filling factor) of cool stars, which follows roughly a power law, $\Phi \propto \Omega^{n_\Phi}$, with a rate of increase $n_\Phi \simeq 1.2$ (Saar 2001); Ω is the stellar rotation rate. Excluding targets which might be in the regime of saturated dynamo operation, Schrijver et al. (2003) suggest a higher value, $n_\Phi \simeq 2.8$. In contrast, observations of young open stellar clusters of different age indicate a spin-down of stellar rotation due to magnetic braking, which is consistent with a linear increase of the *open* magnetic flux (e.g. Holzwarth & Jardine 2005, and references therein). The combination of an empirical relationship between unsigned magnetic flux and coronal X-ray emission (Pevtsov et al. 2003) with empirical activity-rotation-relations (e.g. Pizzolato et al. 2003) implies an intermediate value $n_\Phi \sim 2$.

In the course of its 11-year activity cycle, the total spot coverage of the Sun may reach 0.5% of the visible hemisphere. In contrast, the spot coverage of rapidly rotating stars can be over two orders of magnitude larger (O’Neal et al. 2004). Whereas sunspots appear within an equatorial belt between about $\pm 35^\circ$, fast rotating stars frequently show huge spots at high and polar latitudes as well (e.g. Barnes et al. 2001, 2004; Oláh et al. 2002; Kovári et al. 2004; Jeffers et al. 2007; see also Strassmeier 2002, and references therein). Furthermore, spectropolarimetric Zeeman-Doppler imaging observations indicate that the magnetic field pattern of rapidly rotating stars is characterised by a significant mixture of polarities, which is in contrast to the unipolar field around the solar poles (Donati et al. 2003).

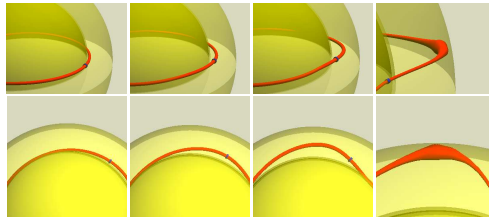


Fig. 1. Rise of a magnetic flux loop inside the convection zone, from the onset of the instability (*left*) to its eruption at the stellar surface (*right*). The flux tube radius is shown 5 \times magnified.

3. The Solar Paradigm

The magnetic field permeating the solar atmosphere is expected to originate from the bottom of the convection zone. The field is amplified in the tachocline and stored in the stably stratified overshoot region at the interface to the radiative core (van Ballegoijen 1982; Moreno-Insertis 1986). When the field strength is larger than a critical value, perturbations lead to the formation of rising flux loops (Fig. 1), which eventually emerge at the surface (Choudhuri & Gilman 1987; Fan et al. 1994; Schüssler et al. 1994).

After the dynamical disconnection from its sub-surface roots (Schrijver & Title 1999; Schüssler & Rempel 2005), the surface magnetic flux feature follows the differential rotation and the meridional flow in the photosphere. During its transport toward the pole, surface magnetic flux merges and annihilates with ambient flux features and eventually dissolves through diffusion and through the convective turnover of supergranular motions (DeVore et al. 1984; van Ballegoijen et al. 1998; Baumann et al. 2004).

The decapitated flux tube below the surface disintegrates in magneto-convective motions and may be transported by meridional circulations and through convective pumping back to the bottom of the convection zone for recurrent amplification (Tobias et al. 2001).

Calculations based on the solar paradigm and carried out in the framework of the thin flux tube approximation (Spruit 1981) predict critical field strengths, eruption latitudes, tilt

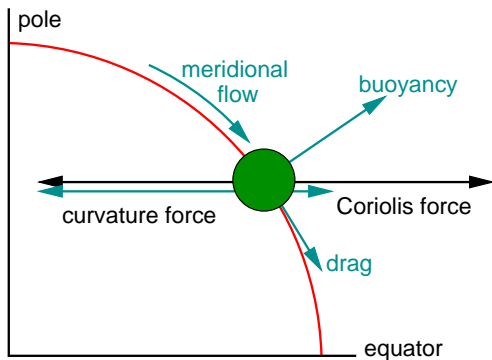


Fig. 2. Force balance of a magnetic flux ring in mechanical equilibrium in the absence (*black arrows*) and in the presence (*gray arrows*) of an equatorward meridional flow.

angles, and proper motions of spots which are in agreement with observed properties of emerging bipolar spot groups on the Sun (D’Silva & Choudhuri 1993; Fan et al. 1994; Caligari et al. 1995; Schüssler et al. 1996).

4. Flux tubes in fast rotating stars

Theoretical investigations of magnetic flux eruption in cool stars are based on analyses of the equilibrium, stability, and rise of flux tubes for different stellar rotation rates, masses, and evolutionary stages.

Equilibrium properties The magnetic flux tubes are assumed to be initially situated inside the overshoot region, stored in mechanical equilibrium parallel to the equatorial plane (Spruit & van Ballegoijen 1982; Moreno-Insertis et al. 1992). The flux ring is in pressure equilibrium with its environment and, in the absence of meridional circulations, non-buoyant. The magnetic tension force pointing toward the axis of rotation is balanced by the Coriolis force (Fig. 2), which is caused by an internal prograde flow along the flux ring.

A relative motion between a flux tube and its environment perpendicular to the tube’s axis gives rise to a hydrodynamic drag. In the presence of an equatorward meridional flow, the resulting drag is balanced by assuming a buoyant flux tube with a somewhat lower internal flow

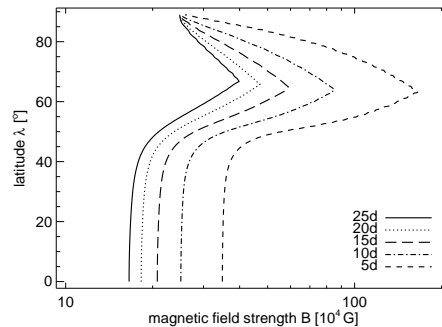


Fig. 3. Buoyancy-driven instability with a growth time of 100 d for pre-main sequence stars ($M = 1 M_{\odot}$, $R = 1.4 R_{\odot}$, $t = 4.7$ Myr) with different rotation periods.

velocity (van Ballegoijen & Choudhuri 1988; Holzwarth et al. 2006)

Stability properties In the case of the Sun, magnetic flux tubes in the overshoot region are (linearly) stable against perturbations for field strengths $\lesssim 10^5$ G (Spruit & van Ballegoijen 1982; Ferriz-Mas & Schüssler 1995). Beyond that critical value, buoyancy-driven instabilities (Parker 1966) lead to the onset of rising flux loops with characteristic growth times of less than a few hundred days. Fast stellar rotation increases the stability of a flux ring, since its enhanced angular momentum hampers perturbations perpendicular to the rotation axis. For otherwise unchanged equilibrium conditions, higher field strengths are required to obtain buoyancy-driven instabilities with comparable growth times (Fig. 3).

Eruption properties The eruption latitude of magnetic flux loops is mainly determined by the ratio between magnetic buoyancy and Coriolis force. Magnetic buoyancy is sustained through a net downflow of plasma inside the flux tube from its crest into the lower segments remaining in the overshoot region. The downflow and the density contrast increase with the magnetic field strength. If the rise is dominated by magnetic buoyancy, the trajectory will be radial and the eruption latitude similar to the

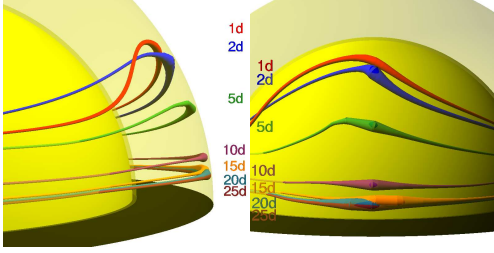


Fig. 4. Poleward deflection (*left*) and tilt angle (*right*) of erupting flux tubes. All flux tubes start with the same initial conditions ($B_0 = 2 \cdot 10^5$ G, $\lambda_0 = 5^\circ$, $r_0 = 5.07 \cdot 10^{10}$ cm).

initial latitude of the flux ring in the overshoot region. The azimuthal flow velocity of plasma within a rising flux loop decreases, owing to the (quasi-)conservation of angular momentum. The associated decrease of the Coriolis force reduces the outward directed net force perpendicular to the rotation axis and entails a deflection of the loop's trajectory to higher latitudes. The dependence of the Coriolis force on the stellar rotation rate causes the poleward deflection (of flux tubes with comparable field strength and equilibrium position) to be larger in more rapidly rotating stars (Fig. 4, left).

The deflection mechanism applies to each tube segment as well. The downflow velocity in the leading leg (relative to the direction of rotation) is larger than the upflow velocity in the following leg, so that the former is less deflected toward the pole than the latter. The asymmetric deflection of the two legs causes a twist of the rising loop and a tilt of the emerging bipolar spot group at the surface with respect to the East-West direction (Fig. 4, right).

Both the poleward deflection and the tilt angle of emerging bipoles depend on the ratio between buoyancy and Coriolis force, i.e. eruption timescale and rotation period, respectively. The eruption times of magnetic flux tubes with the same initial field strength and equilibrium position increase with the rotation rate (Fig. 5), which confirms the influence of the enhanced angular momentum on the sub-surface evolution of magnetic flux indicated by the linear stability analysis.

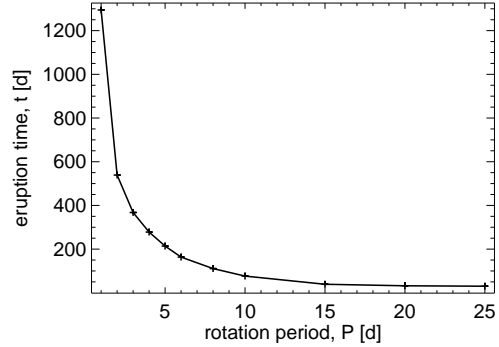


Fig. 5. Eruption times of magnetic flux tubes. The solar-like stellar structure and initial equilibrium conditions ($B_0 = 2 \cdot 10^5$ G, $\lambda_0 = 5^\circ$, $r = 5.07 \cdot 10^{10}$ cm) are the same for all flux tubes.

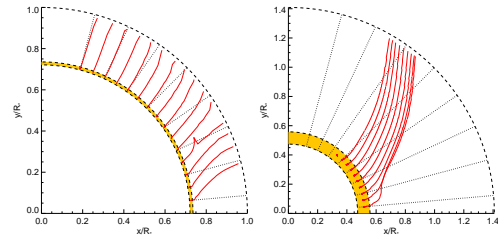


Fig. 6. Flux loop trajectories in a $1 M_\odot$ -main sequence star (*left*) and a pre-main sequence star (*right*) of age 4.7 Myr. The initial field strengths are $B_0 = 22 \cdot 10^4$ G in the former case and $B_0 = 30 \cdot 10^4$ G in the latter; the stellar rotation period is $P = 6$ d.

5. Formation of polar spots

Poleward deflection In fast rotating stars, the magnetic field strengths required for the onset of buoyancy-driven instabilities are significantly higher than in the solar case, which would imply a dominance of magnetic buoyancy and radial trajectories. Yet higher initial field strengths entail higher internal flow velocities to achieve mechanical equilibrium, which in conjunction with the large rotation rate increase the Coriolis force considerably. The resulting strong poleward deflection of rising flux loops (Fig. 6, left) provides an explanation for the occurrence of flux eruption at high latitudes (Schüssler & Solanki 1992; Buzasi 1997).

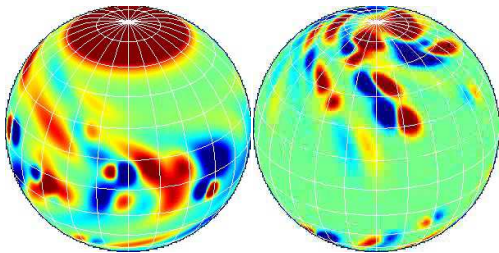


Fig. 7. Surface distributions of the radial magnetic field, assuming solar-like surface transport properties. The assumption of a 30 times solar flux eruption rate (*left*) yields a unipolar field at high latitudes, whereas the additional assumption of a larger latitudinal range of flux eruption and a fast poleward meridional flow yields a mixture of polarities (*right*). From Mackay et al. (2004).

Meridional circulation Since the poleward deflection decreases for larger initial latitudes, the formation of polar spots through bona fide flux eruption would require the presence (and possibly generation) of large amounts of magnetic flux at very high latitudes in the lower part of the convection zone. Although this possibility can a priori not be ruled out, it is more likely that the formation of polar spots is supported by an additional poleward transport of magnetic flux through meridional flows.

The attempt to simulate the formation of polar spots on the basis of a solar surface flux transport model with a $30\times$ larger flux emergence rate generates unipolar magnetic flux at the poles (Fig. 7, left), which disagrees with the mixture of polarities observed on rapidly rotating stars (Schrijver & Title 2001; Donati et al. 2003). The additional assumption of a larger latitudinal range of flux emergence and a fast poleward meridional flow yields an intermingling of polarities (Fig. 7, right), which is in qualitative agreement with observations (Mackay et al. 2004). The assumed meridional flow velocities ($\gtrsim 100$ m/s) are significantly higher than in the solar case (11 m/s).

Strong meridional circulations increase the pre-eruptive poleward deflection of rising flux tubes (Fig. 8). The deflection is strongest for flux tubes originating from mid latitudes with low field strengths and small cross sections, which renders the wings of predicted stellar

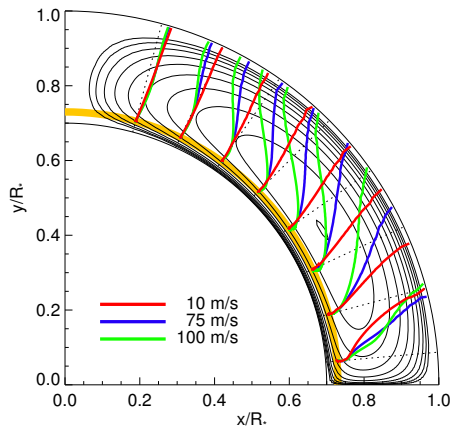


Fig. 8. Trajectories of rising flux loops with initial field strength $15 \cdot 10^4$ G and initial tube radius 100 km. The stellar rotation period is 6 d. The meridional flow is poleward at the surface and equatorward at the bottom of the convection zone. From Holzwarth et al. (2006).

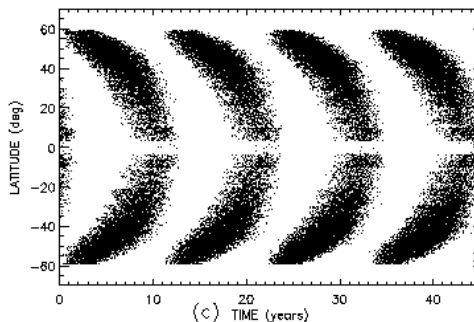


Fig. 9. Stellar butterfly diagram in the presence of a meridional circulation with 100 m/s flow velocity. From Holzwarth et al. (2006).

butterfly diagrams distinctively convex (Fig. 9). The enhanced pre-eruptive poleward deflection explains the required larger range of flux emergence latitudes (Holzwarth et al. 2006).

Influence of stellar structure The marginal case of poleward deflection corresponds to the rise of a flux ring parallel to the stellar rotation axis (e.g. Buzasi 1997), which enables flux emergence at high latitudes, depending on

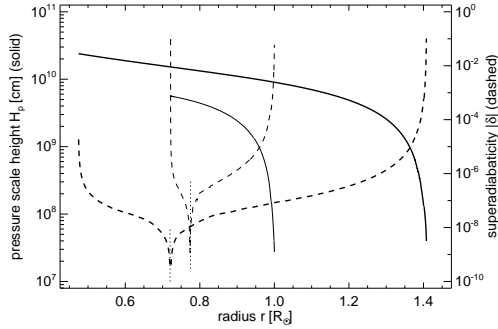


Fig. 10. Pressure scale height (*solid*) and superadiabaticity, $\delta = \nabla - \nabla_{\text{ad}}$, in the convection zone of a 4.7 Myr old pre-main sequence star (*thick lines*) and a solar-like main sequence star (*thin lines*); to the left of the vertical *dotted lines* the stratification is subadiabatic (i.e. $\delta < 0$).

the relative size of the radiative core. This dependence on the stellar structure makes rapidly rotating pre-main sequence stars optimal candidates for polar spots (Granzer et al. 2000; Granzer 2002).

The poleward deflection is supported by the stratification of the convection zone of young stars, which is characterised by larger pressure scales heights and lower superadiabaticities than in a main sequence star (Fig. 10). The smaller magnetic buoyancy enhances the influence of the Coriolis force, so that the poleward deflection is increased (Fig. 6, right). The larger stellar radii and deeper convection zones of pre-main sequence stars also imply longer rise times, during which the poleward deflection cumulates to higher eruption latitudes.

6. Distributions of flux eruption

Young stars Given the influence of the stellar structure and stratification on the formation of polar spots, the mean latitude of magnetic flux eruption is predicted to increase for stars of lower mass and of earlier evolutionary stage (Granzer et al. 2000; Granzer 2002). Comparing pre-main sequence stars of similar evolutionary stage, latitudinal probability distributions based on simulations of erupting

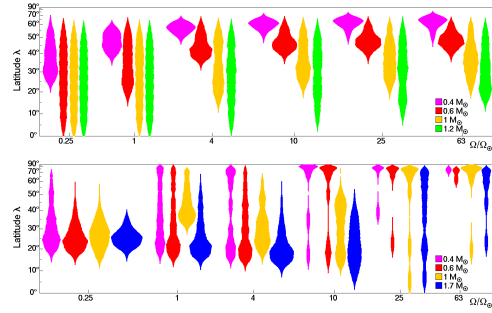


Fig. 11. Latitudinal probability distributions of flux eruption in stars of different stellar mass on the zero-age main sequence (*top*) and shortly after the Hayashi phase (*bottom*). From Granzer (2000).

flux tubes show that for lower mass stars the mean flux eruption latitude is very high already for rotation periods of a week, whereas for higher mass stars flux emergence can be expected to proceed at low and intermediate latitudes for rotation rates in the saturated regime (Fig. 11, top). For even earlier evolutionary stages, magnetic flux emergence can still occur down to low latitudes through a different eruption mechanism (Fig. 11, bottom). When the relative size of the radiative core is very small, unstable magnetic flux rings slip at the bottom of the convection zone to the pole and detach from the overshoot region. If the instability is axial symmetric, the flux ring can rise along the rotation axis and erupt at the pole; if the instability is non-axisymmetric, the flux tube drifts toward the equator and eventually emerges at lower latitudes (Granzer et al. 2000). The predicted eruption of magnetic flux down to low latitudes even in the case of rapid rotation is in agreement with observations.

Binary stars Close binaries with cool stellar components like RS CVn- or BY Dra-systems show strong magnetic activity, since tidal interactions maintain high rotation rates against magnetic braking. The presence of the companion star gives rise to tidal forces and a deformation of the stellar structure, which in the lowest order of approximation is π -periodic in longitude. The tidal effects modify the equilibrium, stability, and eruption properties of

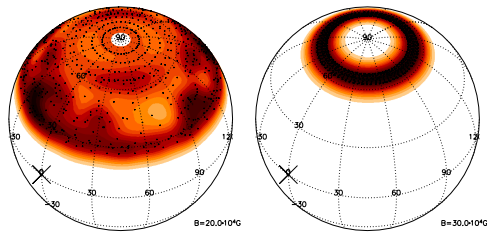


Fig. 12. Surface distributions of erupting flux tubes in a main sequence component (*left*) and a post-main sequence component (*right*) of a binary system with two $1 M_{\odot}$ -stars and a 2 d rotation period; the cross marks the direction to the companion star. From Holzwarth (2004).

magnetic flux tubes (Holzwarth & Schüssler 2003a,b). Albeit the tidal perturbations are rather small, their resonant interaction with double-looped (i.e. roughly π -periodic) flux tubes result in considerable non-uniformities in the surface probability distributions of magnetic flux eruption (Fig. 12, left). The orientation of the resulting π -periodic ‘preferred longitudes’ of flux eruption depends on the initial field strength and latitude of the original flux ring in the overshoot region.

Beyond a certain post-main sequence evolutionary stage the different stellar structure and stratification change the stability properties of flux rings. Double-loop flux tubes are no longer the dominant mode of flux eruption but superseded by single-loop flux tubes (Holzwarth 2004). Since the latter are incongruent with the π -periodicity of the tidal interaction, they are less susceptible to the presence of the companion star. The resulting surface probability distribution is almost axial symmetric, showing hardly any signs of preferred longitudes (Fig. 12, right). Owing to the rapid rotation, flux eruption remains to occur at high latitudes.

7. Summary and Conclusion

High-latitude spots on fast rotating stars are ascribed to the combined pre-eruptive and post-eruptive poleward transport of magnetic flux originating from the bottom of the convection zone. The pre-eruptive poleward deflec-

tion and tilt of emerging spot groups is mainly determined through the ratio between magnetic buoyancy and Coriolis force, which depend on the stellar structure and rotation rate, respectively. Since the deflection typically increases with increasing stellar rotation rate and decreasing size of the radiative core, rapidly rotating pre-main sequence stars are expected to have high mean latitudes of flux eruption, though low latitude spots are still possible.

The identification of characteristic activity features on fast rotating stars may provide the key for our understanding of essential dynamo processes. Solar activity models describing the pre-eruptive and post-eruptive transport of magnetic flux have demonstrated their applicability in the investigations of stellar activity signatures and their dependence on stellar structure and rotation rate. Yet there are activity phenomena unaccounted for by the current flux eruption model, for example, regarding the high activity levels of fully convective low-mass stars (e.g. Donati et al. 2006) or the tentative signs for preferred longitudes on some single stars including the ‘flip-flop’ phenomenon (Jetsu et al. 1994; Korhonen et al. 2001; Berdyugina 2004).

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References

- Barnes, J. R., Collier Cameron, A., James, D. J., & Donati, J.-F. 2001, *Mon. Not. Royal Astron. Soc.*, 324, 231
- Barnes, J. R., Lister, T. A., Hilditch, R. W., & Collier Cameron, A. 2004, *Mon. Not. Royal Astron. Soc.*, 348, 1321
- Baumann, I., Schmitt, D., Schüssler, M., & Solanki, S. K. 2004, *A&A*, 426, 1075
- Berdyugina, S. V. 2004, *Sol. Phys.*, 224, 123
- Buzasi, D. L. 1997, *ApJ*, 484, 855
- Caligari, P., Moreno-Insertis, F., & Schüssler, M. 1995, *ApJ*, 441, 886
- Choudhuri, A. R. & Gilman, P. A. 1987, *ApJ*, 316, 788
- Collier Cameron, A. 2001, *LNP Vol. 573: Astrotomography, Indirect Imaging*

- Methods in Observational Astronomy, 183
- DeVore, C., Sheeley, N., & Boris, J. 1984, *Sol. Phys.*, 21, 1
- Donati, J.-F., Cameron, A. C., Semel, M., et al. 2003, *Mon. Not. Royal Astron. Soc.*, 345, 1145
- Donati, J.-F., Forveille, T., Cameron, A. C., et al. 2006, *Science*, 311, 633
- D'Silva, S. & Choudhuri, A. R. 1993, *A&A*, 272, 621
- Fan, Y., Fisher, G. H., & McClymont, A. N. 1994, *ApJ*, 436, 907
- Ferriz-Mas, A. & Schüssler, M. 1995, *Geophys. Astrophys. Fluid Dyn.*, 81, 233
- Granzer, T. 2000, PhD thesis, Universität Wien
- Granzer, T. 2002, *Astronomische Nachrichten*, 323, 395
- Granzer, T., Schüssler, M., Caligari, P., & Strassmeier, K. G. 2000, *A&A*, 355, 1087
- Holzwarth, V. 2004, *Astronomische Nachrichten*, 325, 408
- Holzwarth, V. & Jardine, M. 2005, *A&A*, 444, 661
- Holzwarth, V., Mackay, D. H., & Jardine, M. 2006, *Mon. Not. Royal Astron. Soc.*, 369, 1703
- Holzwarth, V. & Schüssler, M. 2003a, *A&A*, 405, 291
- Holzwarth, V. & Schüssler, M. 2003b, *A&A*, 405, 303
- Hussain, G. A. J. 2004, *Astronomische Nachrichten*, 325, 216
- Jeffers, S. V., Donati, J.-F., & Collier Cameron, A. 2007, *Mon. Not. Royal Astron. Soc.*, 1492
- Jetsu, L., Tuominen, I., Grankin, K. N., Mel'Nikov, S. Y., & Schevchenko, V. S. 1994, *A&A*, 282, L9
- Korhonen, H., Berdyugina, S. V., Strassmeier, K. G., & Tuominen, I. 2001, *A&A*, 379, L30
- Kovári, Z., Strassmeier, K. G., Granzer, T., et al. 2004, *A&A*, 417, 1047
- Mackay, D. H., Jardine, M., Cameron, A. C., Donati, J.-F., & Hussain, G. A. J. 2004, *Mon. Not. Royal Astron. Soc.*, 354, 737
- Moreno-Insertis, F. 1986, *A&A*, 166, 291
- Moreno-Insertis, F., Schüssler, M., & Ferriz-Mas, A. 1992, *A&A*, 264, 686
- Oláh, K., Strassmeier, K. G., & Weber, M. 2002, *A&A*, 389, 202
- O'Neal, D., Neff, J. E., Saar, S. H., & Cuntz, M. 2004, *Astron. J.*, 128, 1802
- Ossendrijver, M. 2003, *A&AR*, 11, 287
- Parker, E. N. 1966, *ApJ*, 145, 811
- Pevtsov, A. A., Fisher, G. H., Acton, L. W., et al. 2003, *ApJ*, 598, 1387
- Pizzolato, N., Maggio, A., Micela, G., Sciortino, S., & Ventura, P. 2003, *A&A*, 397, 147
- Saar, S. H. 2001, in *ASP Conf. Ser. 223: 11th Cambridge Workshop on Cool Stars, Stellar Systems and the Sun*, ed. R. J. Garcia Lopez, R. Rebolo, & M. R. Zapaterio Osorio, 292–299
- Schrijver, C. J., DeRosa, M. L., & Title, A. M. 2003, *ApJ*, 590, 493
- Schrijver, C. J. & Title, A. M. 1999, *Sol. Phys.*, 188, 331
- Schrijver, C. J. & Title, A. M. 2001, *ApJ*, 551, 1099
- Schüssler, M. 2005, *Astronomische Nachrichten*, 326, 194
- Schüssler, M., Caligari, P., Ferriz-Mas, A., & Moreno-Insertis, F. 1994, *A&A*, 281, L69
- Schüssler, M., Caligari, P., Ferriz-Mas, A., Solanki, S. K., & Stix, M. 1996, *A&A*, 314, 503
- Schüssler, M. & Rempel, M. 2005, *A&A*, 441, 337
- Schüssler, M. & Solanki, S. K. 1992, *A&A*, 264, L13
- Spruit, H. C. 1981, *A&A*, 102, 129
- Spruit, H. C. & van Ballegoijen, A. A. 1982, *A&A*, 106, 58
- Strassmeier, K. G. 2002, *Astron. Nachr.*, 323, 309
- Tobias, S. M., Brummell, N. H., Clune, T. L., & Toomre, J. 2001, *ApJ*, 549, 1183
- van Ballegoijen, A. A. 1982, *A&A*, 113, 99
- van Ballegoijen, A. A., Cartledge, N. P., & Priest, E. R. 1998, *ApJ*, 501, 866
- van Ballegoijen, A. A. & Choudhuri, A. R. 1988, *ApJ*, 333, 965