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Radiation-driven winds of hot luminous stars

XI. Frictional heating in a multicomponent stellar wind plasma and decoupling of radiatively accelerated ions

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Abstract. It is shown that the usual assumption of regarding radiatively driven winds of hot stars as a one-component fluid is wrong under certain circumstances. A detailed investigation of the mechanism of momentum transfer from radiatively accelerated ions to the bulk matter of a stellar wind plasma via Coulomb collisions shows that, at least for thin winds (low mass loss rates $\dot{M}$ and high terminal velocities v_∞), the one-fluid description is not justified. Instead, for objects with thin winds (candidates are late OV and early BV stars, Central Stars of Planetary Nebulae and Subdwarf O-stars) a multicomponent model is required because ionic decoupling occurs, which leads to a "runaway mechanism" for the accelerated ions and hence terminates the momentum transfer from ions to the bulk matter of the wind (e.g. H and He). As a consequence the predicted one-fluid terminal wind velocities are significantly reduced. This is shown for the late main sequence O-star τ Scorpii (O9.5V). Furthermore, the collisionally induced momentum transfer is inevitably accompanied by the production of entropy in the form of frictional heating, which dominates the energy balance in the case of thin winds and thus enhances the runaway mechanism.

Key words: early-type stars – mass loss – winds

1. Introduction

Since the fundamental work of Lucy & Solomon (1970), Castor et al. (1975) and Abbott (1980) the modelling of *stationary* radiatively driven stellar winds has been continuously improved (a comprehensive compilation of references was recently given by Kudritzki & Hummer (1990) and Owocki (1990). Concerning our approach to the theory, we think that with the solution of the line and continuum transfer and hydrodynamics problems, including several thousand NLTE rate equations, a quantitative description is feasible. With this first quantitative approach as developed in the earlier papers of the present series, the theory of radiatively driven stellar winds is on the way to becoming an important diagnostic tool for understanding hot luminous stars and seems capable of accounting not only for characteristic wind features such as mass loss rates and terminal velocities, but also for stellar parameters such as masses, radii and distances (Pauldrach & Puls 1990, hereafter Paper VIII; Kudritzki et al. 1991, hereafter Paper X) in a direct and independent way.

A major drawback up to now has been the poor description of the energy balance: computed models had to be based on either an assumed (mainly constant) temperature stratification (Pauldrach 1987, hereafter Paper III; Pauldrach et al. 1990a, hereafter Paper VIII) or on radiative equilibrium calculations of Gabler et al. (1989, 1990) and Drew (1985); see also Papers III, VIII and Pauldrach et al. (1990b; hereafter Paper IX). However, there are indications that energy is transported not only by radiation but also by dissipative mechanisms, which might be connected to either instabilities in the wind (Owocki 1990) or the momentum transfer from accelerated ions to non-accelerated particles (see Paper IX). The objective of the present paper is to investigate in detail whether the latter mechanism can affect or even invalidate the assumption of radiative equilibrium. If the collisional decoupling is strong enough to affect the radiative equilibrium, then a multicomponent flow model is necessary to describe stellar winds precisely. Thus, we must first examine anew this one-component assumption which is fundamental in the standard model.

For this purpose we develop a multicomponent hydrodynamical description in order to investigate the scope and limitations of the standard view of the wind as a one-component single-fluid medium. This description has become the standard one since the work of Lucy & Solomon (1970). Although Johnson (1925, 1926) and Milne (1926) proposed the possibility that the ions alone (plus an appropriate number of electrons) could be expelled from the atmosphere, McCrea (1935) found from another schematic calculation that this would not occur. This conclusion was also reached by Castor et al. (1976), who found from simple estimates that the different particle species move essentially with the same velocity in the wind, because of Coulomb collisional coupling plus an electrostatic polarisation field. This results directly from the diffusion theory of Aller & Chapman (1960) which is valid when the drift speed (velocity difference between ions and protons) is small compared to the thermal velocity of the protons. Earlier still and in the same spirit, Lamers & Morton (1976) addressed the problem of momentum transfer and concluded likewise that radiation pressure acting on

Send offprint requests to: U.W.E. Springmann selective ions is capable of accelerating the whole plasma to large distances from the star.

However, the single-velocity situation does not necessarily apply, as we show in Sect. 3. Especially for thin winds the different acceleration mechanisms cause an appreciable drift between the radiatively driven ions and the bulk matter of the wind. The special conditions of these winds invalidate the reasoning of Castor et al. (1976) and give rise to effects like ion runaway and frictional heating. In Sect. 2 we present a multicomponent hydrodynamical description for steady-state, spherically symmetric winds and derive a nonlinear diffusion equation for the drift velocity between ions and protons. The runaway effect and its consequences are discussed in Sect. 3, whereas the influence of the frictional heating on the energy balance is considered in Sect. 4. The main conclusions are summarized in Sect. 5.

2. Multicomponent hydrodynamical model equations

To investigate the dynamical behaviour of the different components of the wind, we extend the usual one-component hydrodynamics to a three-component model consisting of metal ions (a minor species), electrons and bulk matter (protons and α-particles). The acceleration of the electrons and the bulk matter (hereafter called the "passive plasma", mainly consisting of protons) by external forces, i.e. Thomson scattering acting on electrons, can only reduce the influence of gravity somewhat, but does not suffice to drive the wind. Hence the bulk matter is mainly accelerated by internal bulk–ion collisional coupling which can macroscopically be viewed as a frictional force that depends on the velocity difference ("drift speed") between the ions and the passive plasma. If the drift speed approaches the average thermal velocity of the wind the well-known effect of ion runaway (Dreicer 1959, 1960), that is, the collisional decoupling of the radiatively accelerated ions from the plasma becomes important. To investigate this question we derive an equation for the drift speed between the ions and the passive plasma.

In the following we assume steady-state, spherically symmetric stellar winds without magnetic fields and rotation. The validity of these assumptions is discussed in Pauldrach et al. (1986; hereafter Paper I).

2.1. Basic equations

The time-independent, one-dimensional equations are as follows (the index j labels the particle species, $j=\mathrm{i}$, e, p for ions, electrons and passive plasma, which is represented by a mass-weighted mean of protons and α-particles):

(1) Equation of continuity:

$$\mathrm{div}_r(\rho_j v_j)=0. \tag{1}$$

(2) Equations of motion:

$$v_\mathrm{i}\partial_r v_\mathrm{i}=Z_\mathrm{i}eE/m_\mathrm{i}-g+g_\mathrm{i}^\mathrm{rad}-(1/n_\mathrm{i}m_\mathrm{i})R_\mathrm{pi}, \tag{2a}$$

$$v_\mathrm{p}\partial_r v_\mathrm{p}=eE/m_\mathrm{p}-g+(1/n_\mathrm{p}m_\mathrm{p})R_\mathrm{pi}, \tag{2b}$$

$$v_\mathrm{e}\partial_r v_\mathrm{e}=-eE/m_\mathrm{e}-g+g_\mathrm{e}^\mathrm{Thom}. \tag{2c}$$

Here ρ_j is the mass density, n_j the particle density, m_j the mass, e the electronic charge, $Z_\mathrm{i}e$ the ionic charge and v_j the velocity of each species. E is an electric polarisation field, g the gravitational acceleration, $g_\mathrm{i}^\mathrm{rad}$ the radiative acceleration by absorption of radiation by atomic transitions of the ions, and $g_\mathrm{e}^\mathrm{Thom}$ represents the Thomson scattering contribution, which is important only for electrons. R_{jk} is the frictional force(-density) exerted by species k on species j, which arises from a multitude of small-angle Coulomb scattering events. For reasons discussed below the collisions with electrons can be neglected.

Since the drift velocities are always small compared with the common streaming velocity of the wind, at least as long as an effective coupling holds the different species together, we take the left-hand sides of Eqs. (2) to be essentially equal.

Pressure gradients, heat fluxes and effects of viscosity can be neglected in the supersonic part of the wind, as discussed by Castor et al. (1975, 1976).

Provided that the gradients of the velocity fields are essentially equal (which is our assumption at the moment and is justified below) one gets the usual one-component equation by multiplying each of the Eqs. (2) by their respective densities $n_j m_j$ and summing (2a)–(2c):

$$v\partial_r v=-g+\frac{n_\mathrm{e}m_\mathrm{e}}{nm}g_\mathrm{e}^\mathrm{Thom}+\frac{n_\mathrm{i}m_\mathrm{i}}{nm}g_\mathrm{i}^\mathrm{rad} \tag{2d}$$

or

$$v\partial_r v=-g(1-\Gamma_\mathrm{Thom}-\Gamma_\mathrm{L}), \tag{2e}$$

with the definitions $v:=\sum(\rho_j/\rho)v_j$ (the barycentric or bulk or streaming velocity), $\rho=\Sigma n_j m_j=nm$ (the total density) and $\Gamma_\mathrm{L}:=(\rho_\mathrm{i}/\rho)(g_\mathrm{i}^\mathrm{rad}/g)=g_\mathrm{L}^\mathrm{rad}/g$ (the radiative acceleration per unit density of *bulk matter*, divided by the gravitational acceleration). Note further that the internal contributions to the individual equations of motion cancel: For the frictional force this follows from Newton's third law ($R_\mathrm{ip}=-R_\mathrm{pi}$) and for the electrical force it depends on additional properties of the wind plasma. These are the *quasi-neutrality* and the *zero-current* condition. On a macroscopic scale, due to the mutual screening of positive and negative charges (Debye-screening) the plasma appears neutral and (1) the condition of *quasi-neutrality* can be applied (q_j is the charge of species j):

$$\sum q_j n_j=0, \tag{3}$$

(2) global conservation of charge leads to the *zero-current* condition:

$$\sum q_j n_j v_j=0. \tag{4}$$

(3) Energy balance:

For the energy balance we assume a common temperature T for all components of the wind. The energy balance (first law of thermodynamics) of a co-moving parcel of homogeneous matter including internal frictional heating is then

$$\mathrm{div}_r(uv)+\mathrm{p}\,\mathrm{div}_r v=Q^\mathrm{rad}-R_\mathrm{ip}(v_\mathrm{i}-v_\mathrm{p}). \tag{5}$$

The internal energy u is taken to be $3/2\ nkT$. Thus, the first term gives the advective contribution to the energy balance ($=3/2\ nvk\partial_r T$ as no sources or sinks for particles are present) and the second term describes adiabatic cooling by spherical expansion. $Q^\mathrm{rad}=\int\mathrm{d}\nu\int\mathrm{d}\Omega(\chi_\nu I_\nu-\eta_\nu)$ is the radiative heating term (sources minus sinks), where χ_ν and η_ν are the absorption and emission coefficient, respectively, while I_ν is the intensity of the radiation field.

The last term gives the frictional heating arising from the non-vanishing drift speed $v_i - v_p$ between the ions and the passive plasma. These two quantities, which are the subject of this paper, are present only in a multi-component picture.

Discussion of the frictional force. The frictional force R_{jk} of species k on species j can be written as (Braginskii 1965; Sivukhin 1966):

$$R_{jk} = -n_j n_k k_{jk} G(x_{jk}), \tag{6}$$

where k_{jk} is the coefficient of friction, given by

$$k_{jk} = \frac{4\pi \ln\Lambda q_j^2 q_k^2}{kT} \frac{v_i - v_k}{|v_j - v_k|} \tag{7}$$

and G is the Chandrasekhar function, which is defined in terms of the error function $\Phi(x)$:

$$G(x) = [\Phi(x) - x\partial_x \Phi(x)]/(2x^2). \tag{8}$$

The term $\ln\Lambda$ is called the Coulomb logarithm, because Λ is the small-angle cutoff for Coulomb scattering which originates from screening effects in the plasma (the usual Rutherford cross section diverges for small-angle scattering). The argument of the Chandrasekhar function is $x_{jk} = \sqrt{A_{jk}}|v_j - v_k|/a_p$, where $A_{jk} = A_j A_k/(A_j + A_k)$ is the reduced atomic mass and $a_p = (2kT/m_p)^{1/2}$ is the thermal velocity of the protons. Note that x_{ij}^2 is the ratio of relative kinetic energy of the particles i, j to the thermal energy of the protons.

For small x_{jk} (drift speed small compared to a_p) $G(x)$ is proportional to x and the frictional force is proportional to the velocity, the so-called Stokes law. $G(x)$ has a maximum for $x \approx 1$ and for $x \gg 1$ it decreases as $1/x^2$ (Fig. 1). In this last case, the "runaway-regime", the driving force can no longer be balanced by the frictional response of the system. Since an increase in ion velocity further decreases the frictional force the ions simply "run away". *In fact, friction rapidly decreases and the radiatively driven species can accelerate freely.*

Since in the linear regime (small drift speed) the drift velocity depends upon the wind parameters as $v/\dot{M}$, where v is the velocity and $\dot{M}$ the mass loss rate of the wind, the runaway effects are expected to occur for stars with small $\dot{M}$ and high v_∞.

In the linear case the frictional force depends on the square root of the reduced mass (see above), and collisions of the ions with electrons are thus less effective by a factor of 43 than collisions of the ions and the passive plasma. These interactions can therefore be neglected. Likewise the collisions of the electrons with the passive plasma can be neglected because the zero-current condition forces the relative drift speed between these two species to be very small. Thus only the frictional force R_{pi} of the ions on the passive plasma appears in Eqs. (2).

For a derivation of the hydrodynamical equations starting from the Boltzmann equation, see e.g. the review articles by Trubnikov (1965), Braginskii (1965) and Sivukhin (1966).

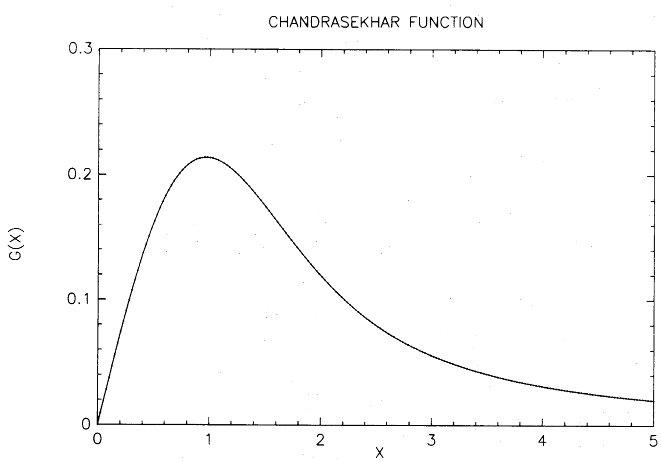

Fig. 1. The Chandrasekhar function $G(x)$ which gives the frictional force on test particles by field particles of unit density for an inverse square law of Coulomb interaction. The variable x is essentially the ratio of the velocity of the test particles in the rest frame of the field particles to the thermal velocity of the field particles (see text). The limiting cases are $G(x) \sim x$ for $x \ll 1$ and $G(x) \sim x^{-2}$ for $x \gg 1$

The role of the electrons. In the equation for the electrons (2.3), all terms can be neglected except the E-term and g_e^{Thom}, since these are about 2000 times larger than the others. (This follows from the fact that the left-hand sides of Eqs. (2) are about equal and of the order of several g. Note, that g_e^{Thom} gives the acceleration of Thomson scattering on electrons alone). Hence, we can estimate an upper limit for the electric polarisation field by simply setting $eE = m_e g_e^{\mathrm{Thom}}$ so that

$$E = 8.75\,10^{-7}(L_*/10^6 L_\odot)(R_*/R_\odot)^{-2}(r/R_*)^{-2}\ \mathrm{V\,cm^{-1}}, \tag{9}$$

where L, R and r denote stellar luminosity and radius, and radial coordinate, respectively. For typical winds, E is clearly too small to play any role other than that of preventing the electrons from being accelerated more than the ions and the passive plasma. In the absence of collisional coupling to the electrons, this polarization field is the origin of the zero-current condition [Eq. (3)] that forces the electron velocity to lie between the velocities of the ions and the passive plasma.

These considerations allow a reduced system to be defined, which consists of two components, in which only one drift speed has to be calculated. Until the runaway-regime is reached, the drift speed is smaller than the thermal speed of the passive plasma, and since decoupling can only occur in the hypersonic part of the wind, the drift speed is always small compared to the streaming velocity. Thus, we can assume that the velocity field of the wind is not affected by the fact that the true radiation force depends on the velocity of the ions, which is slightly higher than the barycentric velocity. Numerical computations as well as linear stability analysis show that this is a good assumption until the drift speed becomes comparable to the thermal speed of the wind. (Work is under way in our group to verify this quantitatively by solving the equations of motion with the correct velocity dependence of the radiation force.)

2.2. *The diffusion equation for the drift velocity*

On the basis of the assumptions given above we need not distinguish between the $v_j \partial_r v_j$-terms for the ions and the passive plasma, and can therefore use the models of Paper III to determine the local conditions in the wind, which we need for solving the resulting nonlinear diffusion equation [subtract Eqs. (2a) and (2b) for ions and passive plasma; the left-hand sides are practically equal]:

$$[1/(n_i m_i) + 1/(n_p m_p)]\, R_{pi} = g_i^{\mathrm{rad}}$$

or

$$g_i^{\rm rad} = k_{\rm ip}(n_{\rm H}/m_{\rm H})[(1+Y_{\rm He})/A_i + Y_i/A_p]\,G(x_{\rm ip}), \tag{10}$$

where H and He denote hydrogen and helium, respectively. This equation states that the driving force of line-absorbed radiation on the ions is balanced locally by the frictional force resulting from collisions with protons and α-particles. For small drift speeds (small $x_{\rm ip}$), the equation reduces to an ordinary diffusion equation, which can be solved for the drift speed $w := v_p - v_i$. In the following we concentrate on conditions in which this simplifying assumption may not apply.

3. The effect of ion decoupling ("runaway")

Since the drift results from the radiative acceleration on the ions, the condition for ion runaway is expressed in terms of Γ_L, the ratio of radiation line force (acting on the ions alone) to gravity (acting on all three components), i.e. $\Gamma_L = \rho_i g_i^{\rm rad}/(\rho g) = g_L^{\rm rad}/g$, where the radiative line force acting on the ions is scaled to the density of the whole wind. Note that Γ_L depends on the radial coordinate r.

In analogy to Γ_L, a parameter Γ_B can be defined which describes the situation in which the drift velocity reaches the thermal speed, i.e.

$$\Gamma_B = \frac{g_L^{\rm rad}|_{x_{\rm ip}=1}}{g} \tag{11}$$

(the subscript B stands for breakthrough into the runaway regime). According to Eq. (10), this gives

$$\Gamma_B = \frac{\rho_i}{\rho} k_{\rm ip}(n_{\rm H}/m_{\rm H})(Y_p/A_i + Y_i/A_p)\,G_{\max}/g(r) \tag{12}$$

or in scaled form,

$$\Gamma_B = 8\,10^6\, Y_i Z_i^2 \ln\Lambda (10^4\,K/T)(M_\odot/M_*) \times (10^3\ {\rm km\,s^{-1}}/v)(\dot{M}/10^{-6} M_\odot\ {\rm yr^{-1}}). \tag{13}$$

In the last equation we used $Y_{\rm He} = 0.1$, $A_i = 12$. Z_i is the charge of the ions in terms of the electron charge. In comparing Γ_B with Γ_L one can use for a first estimate the supersonic form of the equation of motion for a one component fluid: $v\partial_r v = g(r)\Gamma_L$. If we assume for the wind a β-velocity field of the form $v = v_\infty(1 - R_*/r)^\beta$ [which is not a bad approximation to the real velocity structure, see Pauldrach et al. (1986; hereafter Paper I)] we get

$$\Gamma_L = 5.24\beta(v_\infty/10^3\ {\rm km\,s^{-1}})^2 (R_*/R_\odot)(M_\odot/M_*) \times (1 - R_*/r)^{2\beta-1}. \tag{14}$$

The condition for the runaway regime is

$$\Gamma_B < \Gamma_L. \tag{15}$$

In a more general way we can also give a condition for breakthrough in terms of the density: Eq. (12) may then be used to give the following condition:

$$n_{\rm H} \le 10^7 \frac{\Gamma_B/20}{(Y_i/10^{-3})(\ln\Lambda Z_i^2)/300}\, g_4(r)\, T_4\ {\rm cm^{-3}}, \tag{16a}$$

where $T_4 = T/10^4$ K and $g_4(r) = g(r)/10^4\ {\rm cm\,s^{-2}}$ and $A_i = 12$.

In the same way, we can start from Eq. (10) and use $g_L = \rho_i/\rho\, g_i^{\rm rad} = v\partial_r v$ on the l.h.s. and insert the maximum value of the frictional force, i.e. $G(x) = 0.214$, on the r.h.s. After using a β-velocity field, with $x = R_*/r$ and $v_\infty = V$ one gets

$$\beta(1-x)^{3\beta-1} = 53.5 \frac{\dot{M}_{-9} Y_{-3} Z_i^2}{T_4 V_8^3 R_*/R_\odot}, \tag{16b}$$

an equation we will use later. Here $\dot{M}_{-9}$ denotes $\dot{M}/(10^{-9} M_\odot\ {\rm yr^{-1}})$.

If one does not want to employ a β-velocity field, which is not in general a good fit to the real velocity field, a precise model has to be used to get the correct radiation pressure at each point. Thus, to investigate the circumstances which favour the runaway effect, we look at existing models of τ Sco and ζ Pup (see Paper III).

From Eq. (13) it is obvious that we have to concentrate on stars with low mass loss rate and high terminal velocity. In fact if we insert the values for the O4 $I(n)$-f star ζ Puppis, which has a large mass loss rate:

$M_*/M_\odot = 42,\quad R_*/R_\odot = 19,\quad T_{\rm eff} = 42\,000$ K,

$\dot{M}/(10^{-6} M_\odot\ {\rm yr^{-1}}) = 4.6,\quad v_\infty = 2200\ {\rm km\,s^{-1}}$.

With $Y_i = 1.5\,10^{-3}$ we get

$\Gamma_B \approx 2800 Z_i^2,\quad \Gamma_L \approx 11.5\beta$,

and we see that the one-component description is absolutely sufficient, since β and Z_i are both of order unity.

However, if we take the values for the O9.5V star τ Scorpii, which has a low mass loss rate:

$M_*/M_\odot = 19.6,\quad R_*/R_\odot = 5.5,\quad T_{\rm eff} = 33\,000$ K,

$\dot{M}/(10^{-6} M_\odot\ {\rm yr^{-1}}) = 5\,10^{-3},\quad v_\infty = 3750\ {\rm km\,s^{-1}}$

we get

$\Gamma_L \approx 20\beta,\quad \Gamma_B \approx 15 Z_i^2$.

This is closer to the condition of breakthrough but still not close enough. Employing Eq. (16b) one gets for $\beta = 1$

$$(1 - x^2) = 0.42 Z_i^2 (T_{\rm eff}/T), \tag{16c}$$

which shows very clearly that a runaway will not occur for $Z_i^2 \approx 10$ *unless the temperature is increased significantly above* $T_{\rm eff}$.

However, the frictional force is accompanied by frictional heating (see Sect. 4), and since Γ_B depends on the temperature as $1/T$ [Eq. (13)], Γ_B is lowered by this heating effect (see also Fig. 2). This makes the ion runaway not only likely but even certain in this case.

Thus, a one-component description is not always sufficient to treat the wind in a satisfactory way. Consequently, due to the wide range of stellar and wind parameters, the assumption that winds are accelerated in a bulk form (shown to be valid for dense winds like that of ζ Puppis) cannot be taken for granted for all hot stars. In the following we examine the possible effects and consequences which can arise for late O-/early B-type stars such as τ Scorpii.

Whereas the inclusion of frictional heating makes the ion runaway certain, the question of whether or not in the isothermal model a break-through of the ionic component occurs depends sensitively on the parameters Y_i and Z_i. For the isothermal wind models of Paper III the dependence of Γ_L and Γ_B on the radial

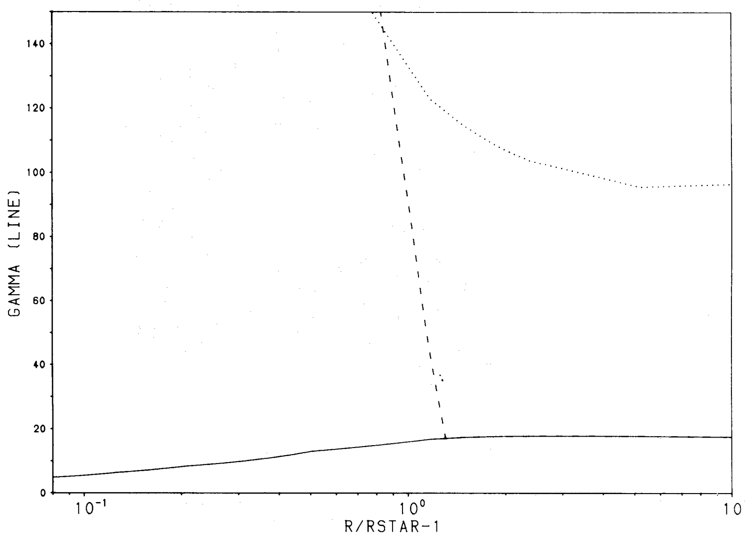

Fig. 2. The run of various radiative accelerations (normalised to local gravity) against the radial coordinate r. The full line shows the line force Γ_L for a model of τ Scorpii. The dotted line shows Γ_B for an isothermal model ($T=T_{\rm eff}$), whereas the dashed curve gives Γ_B for a consistent temperature stratification as computed from the energy equation with inclusion of the frictional heating rate (see Fig. 3). Note the intersection of the full and dashed line: at this point the runaway occurs. Here, we used the ionic abundance $Y_i=1.58\,10^{-3}$, the relative atomic mass $A_i=12$ and the ionic charge $Z_i=4$

coordinate r is shown for a realistic choice of these parameters (see below) in Fig. 2: The full line gives Γ_L and the dotted line Γ_B for this model. Since the lines do not intersect and Γ_B is always larger than Γ_L, a break-through into the runaway regime does not occur in this case. However, taking into account the aforementioned heating effect in solving the energy equation (Sect. 4), the runaway does occur, since the collisional coupling and hence Γ_B are continuously reduced. The new dependence of Γ_B on r is then given by the dashed curve in Fig. 2. The intersection of this curve with the full line (Γ_L) marks the point of decoupling. Note that the individual ion species which we lumped together in one mean ion component may have individual decoupling points depending on the ratio of abundance to individual line acceleration. In Sect. 4 we make an attempt to quantify the impact of this effect in the context of our former one-component models.

4. The contribution of frictional heating to the energy balance

On the basis of the results of the previous subsection we can investigate whether the heating resulting from the frictional force is large enough to disturb radiative equilibrium (see Sect. 1), at least in winds which are better described by a two-component flow (e.g. objects with thin winds). To that end we use Eq. (5),

$$\mathrm{div}_r(uv)=3/2\,nvk\partial_r T=-p\,\mathrm{div}_r\,v+Q^{\rm rad}+Q^{\rm fric},$$

where the first term on the right-hand side gives the adiabatic cooling rate, the second contains all radiative heating and cooling rates and the third is the specific heating rate arising from Coulomb collisions between the different species. The non-indexed variables u, n, v, and p give the mean conditions for a single-component fluid. The frictional heating rate is given by $Q^{\rm fric}=-R_{\rm ip}(v_i-v_p)$, taking into account only the interactions between the ions and the passive plasma (the drift speed between these two species v_i-v_p will henceforth be called w).

To actually solve Eq. (13) one would have to include all line-contributions to the radiative rates, which is beyond the scope of this work. However, since we need only an estimate of the influence of the frictional rate in the case of thin winds, we can simply compare it with the dominating photoionisation heating rate resulting from the nebular approximation (Osterbrock 1989):

$$G_p=3/2\,T_{\rm eff}\,n_e n_H[\alpha_A(H^0,T)+Y_{He}\alpha_A(He^+,T)], \tag{17}$$

where α_A is the total recombination coefficient and He is assumed to be completely ionised. In scaled form the two heating rates read:

$$Q^{\rm fric}/n_H^2=6\;10^{-20}(Y_i/10^{-3})(Z_i^2\ln\Lambda/100)\times(T/10^4\,{\rm K})^{-1/2}G(w/a_p)\,w/a_p\;{\rm erg\,cm^3\,s^{-1}} \tag{18}$$

and

$$G_p/n_H^2=2.5\;10^{-25}(T_{\rm eff}/10^4\,{\rm K})\times[(\alpha_A(H^0)+0.1\alpha_A(He^+)]/10^{-13}\;{\rm erg\,cm^3\,s^{-1}}. \tag{19}$$

From the above equations it follows directly that a drift speed of just 1% of the proton thermal velocity a_p already yields a frictional heating rate comparable to the photoionisation heating rate. Since in the linear regime ($w \ll a_p$) the drift speed w as well as $Q_{\rm fric}$ are proportional to $T^{3/2}$ [Eq. (10)], $Q^{\rm fric}$ cannot be cancelled by cooling rates which vary only as T, such as recombination cooling (bound–free processes). The temperature dependence of the most important cooling mechanism, i.e. cooling by collisionally excited lines, is more complicated and is discussed separately below.

Now the value of the collisional coupling coefficient $k_{\rm ip}$, which is proportional to $1/T$, is significantly lowered by the additional heating rate, so that the drift speed rises with temperature. We can give a lower bound for the dependence of the drift speed on temperature by the following argument: According to Eq. (10), the argument of the Chandrasekhar function w/a_p will grow as T in the regime where the frictional force behaves as $G \sim w/a_p$. From Fig. 1 we see that G depends actually sublinear on w/a_p, which means that w/a_p *rises at least as* T.

The behaviour described above is illustrated in Figs. 2 and 3 for a wind model of τ Scorpii. The run of Γ_L an Γ_B is given in Fig. 2 and the temperature structure resulting from the integrated energy Eq. (5) in Fig. 3. The full and dashed lines in Fig. 3 give the

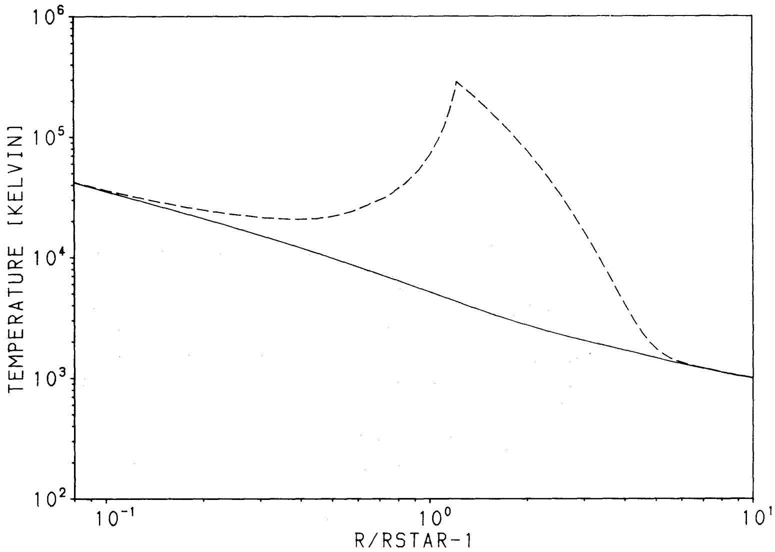

Fig. 3. For a model of τ Scorpii the run of temperature as computed from the energy equation (5) is shown. The full and dashed lines give the temperature without and with inclusion of the frictional heating rate, respectively. The cusp in the dashed line corresponds radially to the intersection of dashed and full line in Fig. 2.

run of temperature without and with inclusion of frictional heating, respectively. For the wind model we used the hydrodynamical structure of Paper III with the force multiplier parameters $k=0.029$, $\alpha=0.675$, $\delta=0.08$, yielding a mass loss rate of $5.5\,10^{-9}M_\odot\,\mathrm{yr}^{-1}$ and a terminal velocity of $3752\,\mathrm{km\,s}^{-1}$, with the ion parameters $Y_\mathrm{i}=1.58\,10^{-3}$, $A_\mathrm{i}=12$ and $Z_\mathrm{i}=4$. In the computation of the temperature structure, the rates of photoionisation, radiative recombination, free–free emission and adiabatic cooling were considered in the energy equation on the basis of the nebular approximation, whereas cooling by collisionally excited lines of heavy elements was included only in a rough way by adapting the ratio of line to recombination cooling rates from Drew's (1989) models. (Note that in the case of τ Sco Gabler (1991), Gabler et al. (1992, in preparation) confirmed Drew's result via detailed NLTE calculations including metal lines.) Since in Drew's models the two rates have the same order of magnitude for the underlying temperature structure of the wind, which does not exceed T_eff, we may ask whether our results are changed significantly by taking into account the frictional heating which enhances the temperature and thus influences the line cooling rate. To test this we assumed that the cooling rate calculated by Drew increases with $(T/T_\mathrm{eff})^2$. This corresponds to the Rosner et al. (1978) approximation to the radiative loss function of Raymond et al. (1976) in the temperature range from 40 000 to 80 000 K. We regard this procedure to be more appropriate than simply adopting the Rosner approximation directly, since the radiative loss function of Raymond is designed for gaseous nebulae with much thinner plasmas than the ones employed here. However, even in this case the generated frictional heating could not be balanced by radiative line cooling and our results remain completely unchanged. This behaviour becomes apparent in Fig. 4, where the frictional heating rate is compared to the main cooling rates.

Thus, in the case of τ Scorpii, where drift speeds of the order of the thermal speed a_p are reached, the breakthrough into the runaway regime will definitely occur, while the remaining parameters Y_i, A_i and Z_i can only influence the temperature at which this breakthrough takes place. The remaining important parameter is the density, or the corresponding value of the mass loss rate. Here, we found that with a mass loss rate of $\dot{M}_{-9}=5.5$ τ Sco

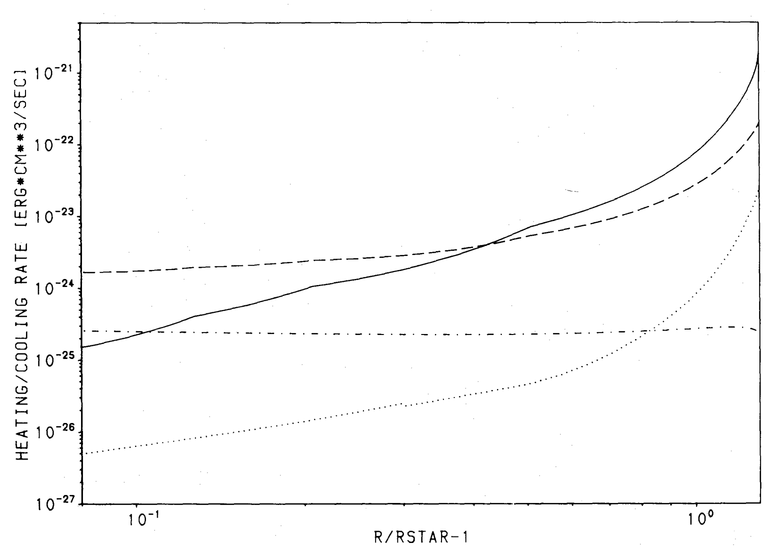

Fig. 4. The frictional heating rate (full line) and the main competing cooling rates are. shown for our τ Sco model (dashed: adiabatic cooling rate, dash–dot: energy loss by recombination, dotted: line cooling with an assumed T^2-dependence). At the right end of the figure runaway occurs

at the border of the runaway condition since even a 20% increase destroys the effect. Observational results give, however, a lower mass loss rate of $\dot{M}_{-9}=2$ (Hamann 1981).

The most important result of our calculations is that the decoupling of the radiatively driven ions from the wind reduces the terminal wind velocity from 3850 km s^{-1} (see Paper III) to 2400 km s^{-1} along with a temperature which may exceed 100 000 K.

Although the ions alone are accelerated solely beyond the breakthrough point, which results in huge velocity gradients and enormous ion velocities, such large velocities cannot be observed, since the ionic densities decrease steeply by virtue of the continuity equation. This reduces the Sobolev optical depths drastically, so that the ionic material is no longer observable, and the observed wind speed is that of the "passive plasma", which is not further increased after the decoupling occurs. Concerning the value of the reduced terminal velocity our new result is really encouraging, since it is consistent with those inferred from spectral observations (Lamers & Rogerson 1978; Hamann 1981).

A further comment is to explain the cusp in the run of temperature with radius. This arises from the unphysical assumption that immediately after break-through the frictional heating drops to zero. Since the decoupling will occur in a very short time we do not think that it is necessary for the moment to include the effect of a continuous transition, for which one would have to solve the dynamical equations of motion for the different species. For this kind of calculation one would also have to take into account the distortion of the Maxwellian velocity distribution which arises from the fact that ions from the tail of the distribution will have been decoupled before the hydrodynamical decoupling occurs.

Since our result reproduces the observed terminal velocity of τ Sco in surprising detail, it has to be discussed whether this result would be significantly changed if instead of a mean ion the real situation of different subspecies is considered, each with its abundance Y_k, charge number Z_k and contribution to the total force $M_k(t)/M(t)$ [$M(t)$ is the total force multiplier – see Paper III]. The condition of Eq. (15) has then to be replaced by

$$\Gamma_L > (Y_k/Y_i)[M(t)/M_k(t)](Z_k/Z_i)^2\Gamma_B, \qquad (20)$$

where subscript k denotes the ionic subspecies and subscript i the mean ion species. Table 1 gives the contribution of individual ion species to the total line force – the values have been taken from the calculations of Paper III. From Table 1 we infer that S and Ar decouple at a lower velocity than 2400 km s^{-1}, but since the abundance of these elements is small (1.5%), this has a negligible effect on the heating rate. For the remaining elements the individual decoupling points are close to the point found for the mean species. Due to the steep run of Γ_B these points differ only by a few hundred km s^{-1}. However, for these considerations we ignored the influence of the considerably increased temperature on the ionisation fractions, which will again reduce the decoupling points by several hundred km s^{-1} and also limit the increase of temperature.

Table 1. Abundances Y_k and the individual contribution $M_k(t)$ to the total line force of the ion components relative to the mean ion component Y_i. For a detailed discussion see text

	Element					
	C	N	O	Ne	S	Ar
Y_k/Y_i (%)	30.7	6.4	54.2	3.5	1.05	0.44
$M_k(t)/M(t)$ (%)	8	29	36	4	18	5
$\dfrac{Y_k/Y_i}{M_k/M}$	3.84	0.22	1.51	0.875	0.058	0.088

Since all of these approximations act in the same direction, namely reducing the terminal velocity, our main conclusions are not affected. There might, however, exist the possibility that ion-wave-coupling prevents the drift velocity from exceeding the thermal velocity by very large values. The two-stream instability which arises in a two-component plasma with a drift that exceeds the thermal velocity may be seen as a source for such waves. In this case the wind will continue to heat up to coronal temperatues and, hence, terminate the acceleration due to drastic changes of the ionisation balance (very high ionisation stages will appear). The line force is cut off by these changes, and the terminal velocity will consequently be that at the decoupling point.

5. Conclusions

The main aim of this work was to investigate whether the frictional heating rate resulting from collisional coupling of radiatively accelerated metal ions to bulk matter (H, He) influences the temperature structure in the wind. The means to deal with this problem were provided by a multicomponent hydrodynamical description, from which we derived a nonlinear diffusion equation for the drift velocity between metal-ions and H and He nuclei, and an equation for the energy balance which includes the frictional heating rate.

On the basis of this procedure we found the drift speed is too small to noticeably affect the wind properties of normal O-stars which exhibit dense winds (e.g. ζ Pup). Since for these objects also the ratio of the mean scale length of the cooling layers, which are connected to hot gas produced by periodic shocks ploughing through the wind (Lucy 1982), to the mean distance between shocks (Lucy & White 1980; Chen & White 1991) is small, we conclude in contradiction to Paper IX that at the present stage of understanding dense winds of O-stars can indeed assumed to be in radiative equilibrium.

On the other hand, this is definitely not the case for hot stars which exhibit thin winds (e.g. τ Scorpii). For these objects we found a multicomponent description to be indispensable for two reasons: (1) The frictional heating rate dominates the energy balance in the wind and thus *destroys the assumption of radiative equilibrium*. As a consequence, the temperature increases to values which may exceed 100 000 K. (2) The one-fluid terminal velocity cannot be reached, since after passing a point of ionic decoupling *a breakthrough into the runaway regime* occurs. This reduces the predicted terminal velocity of τ Scorpii to 2400 km s^{-1}. Although our present models are not yet fully consistent, this value is found to be in excellent agreement with observations.

Lucy & White (1980) also proposed a lower terminal velocity and an extremely hot wind for τ Sco resulting from a simple shock-model. However, our results do not compete with those, since our approach has a fundamental character – not the whole material but only the metal ions are directly accelerated. Since the part of the wind from which Lucy and White deduced their results is strongly related to our runaway regime, a correct shock description of τ Sco should obviously also be based on our approach. A second reason for a multicomponent shock description follows from the inherent damping behaviour which accompanies the momentum transfer from metal ions to bulk matter. This effect, which has been neglected up to now, might stabilize the wind by smoothing out otherwise undamped instabilities.

The severe reduction for the terminal velocity that we found for τ Scorpii (almost 40%) has also important consequences for wind diagnostics, affecting most masses and distances determined using the theory of radiatively driven winds (Paper X). Since these stellar parameters are derived in that framework by comparing observed and predicted mass loss rates and terminal velocities, the reduction of the latter quantity is of course decisive. Apart from late O V–III stars this effect is also important for early main sequence B-stars as α Vir, β CMa, β Ceu, for a group of subdwarf O-stars which exhibit winds (e.g. HD 49798, HD 128220B) and for some central stars of planetary nebulae [see Pauldrach et al. (1988; hereafter Paper V)].

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