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Multi-level non-LTE calculations for very optically thick winds and photospheres under extreme NLTE conditions

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Summary. A new technique to calculate multi-level non-LTE continuum formation in the UV of stellar winds and plane-parallel, static stellar atmospheres is developed using a local Approximate Lambda Operator similar to those presented by Werner and Husfeld (1985) and Hamann (1986). In our formulation the iteration of the transfer equation solution is driven by the relative deviation of the source functions of successive iteration steps, which stabilizes the convergence. At the same time, linear equations for the statistical equilibrium are obtained, which diminishes the computational requirements.

The technique has been applied to photospheric and stellar wind models, where convergence is reached even under extreme conditions (very large optical thickness and negligible collisional ionization) for which the usual "Accelerated Lambda Iteration" fails. Test calculations are presented for radiative driven winds of central stars of planetary nebulae close to the Eddington limit, including extensive model atoms of H, He, C, N, O, Ne, Mg, Si and S (see Pauldrach, 1987) and for H and He II in the case of plane-parallel static photospheres as described by Herrero (1987a, b)

Key words: early-type stars – non-LTE – mass loss – winds

1. Introduction

The rapid development of our knowledge of O stars in the last few years is due, apart from the great improvements in observational techniques, to very detailed calculations using non-LTE physics. The main difficulty here is the simultaneous solution of the radiative transfer and statistical equilibrium equations (for the structure problem radiative and hydrostatic equilibrium must be included as well). The standard technique to overcome this is the complete linearization method of Auer and Mihalas (1969). However, since this is basically a multidimensional Newton-Raphson method, the description of the model atoms used is necessarily limited so that the search for new techniques has continued.

One of these techniques is the operator perturbation method of Cannon (1973). Following Cannon, Scharmer (1981) developed a new method that could be applied to multi-level NLTE calculations (Scharmer and Carlsson, 1985), also by means of a linearization. Their equations, however, still contain non-local terms from upper spatial layers (a consequence of the approximate lambda operator chosen). The importance of having only local approximate lambda operators was recognized by Werner and Husfeld (1985), who adapted the method of Scharmer to plane-parallel multi-level NLTE line formation calculations in hot stars. Their method, called the "Accelerated Lambda Iteration (ALI)" has been shown to be very powerful. It has been applied by Herrero (1987a, b) in performing detailed NLTE calculations for H and He II in plane-parallel, static photospheres, allowing for Stark broadening in the statistical equilibrium equations.

A similar technique was developed by Hamann (1986) for the case of spherical expanding envelopes which he also applied to He II. In both cases the method was better conditioned for line-transitions than for continuous transitions. In particular, UV-continuum transitions (which are the most important) converged only with great difficulty. The recent calculations by Pauldrach (1987) for self-consistent radiative driven stellar winds of hot stars, solving the statistical equilibrium for all atoms between hydrogen and zinc, have shown the importance of the UV-continuum formation on both, dynamics and wind line formation. He used a simple but powerful technique described by Wagner et al. (1988, in preparation) for the line formation problem. This method, which can be applied when the Sobolev approximation is valid, requires only an iteration of the escape probabilities (β, β_C – see Castor, 1970), since the line radiation transfer can be solved simultaneously with the equations of the statistical equilibrium. However, Pauldrach used a lambda iteration for the continuum which in many cases proved to be very slow. This would prevent the application of the method to objects with extremely optically thick continua (e.g. Wolf-Rayet-Stars).

The continuum formation in UV continua is therefore an urgent problem for photospherical and stellar wind models of large optical thickness under extreme NLTE conditions.

In this paper, the failure of the ALI method is analyzed (Sect. 2) and a modified perturbation technique is presented in Sect. 3, which stabilizes the convergence behaviour by means of well-conditioned equations of statistical equilibrium.

In Sect. 4 we apply the technique to a stellar wind model (including H, He and metals) and to photospheric calculations of a CPN-star close to the Eddigton limit and discuss the convergence properties with respect to the former solution method.

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2. The problem of continuum formation under extreme NLTE conditions

The solution of the radiative transfer equation is usually written as

$$J_\nu = \Lambda(S_\nu). \tag{1}$$

where J_ν is the mean intensity, S_ν is the source function and Λ is the so-called Λ operator, which allows us to calculate the radiation field for a given source function. This is defined by the occupation numbers, which in non-LTE are determined by means of the statistical equilibrium equations

$$n_i(C_{ij}+R_{ij}+\sum_{j\neq i} P_{ij}) = n_j(C_{ji}+R_{ji}) + \sum_{j\neq i} n_j P_{ji} \tag{2a}$$

(P_{ij} consists of a bound-bound collisional and radiative term) through collisional and radiative processes. The latter are described by the radiative transition probabilities

$$R_{ij} = 4\pi \int \frac{\alpha_{ij}}{h\nu} J_\nu d\nu \tag{2b}$$

$$R_{ji} = 4\pi \left(\frac{n_i}{n_j}\right)^* \int \frac{\alpha_{ij}}{h\nu} \left(\frac{2h\nu^3}{c^2} + J_\nu\right) e^{-h\nu/kT} \mathrm{d}\nu, \tag{2c}$$

$(n_i/n_j)^*$ being the LTE ratio of the occupation numbers as usual; $i \leftrightarrow j$ can be a bound-bound or a bound-free transition.

The simplest way to solve the loop implicit in Eqs. (1) and (2) is the lambda iteration. Given starting values for the occupation numbers, we solve alternately for the radiation transfer and the statistical equilibrium. As is well known, this technique does not converge for optically thick cases.

In the perturbation technique of Cannon (1973) one writes Eq. (1) as

$$J_\nu^i = \Lambda^*(S_\nu^i) + (\Lambda - \Lambda^*) S_\nu^{i-1}. \tag{3}$$

That is, the radiation field in iteration i is obtained by applying an approximate operator to the source function of the current iteration, plus a "correction term", given by the inconsistency in the last iteration.

The selection of the approximate operator Λ^* is now the problem, as it is clear that its properties will determine the stability and convergence behaviour of the iteration process.

There are a variety of such operators in the literature (Scharmer, 1981; Scharmer and Carlsson, 1985; Werner and Husfeld, 1985; Hamann, 1986; Olson et al., 1986). From these, however, only local operators really offer a good chance for detailed multi-level non-LTE calculations. Here "local" means that the only unknown terms are the local ones, that is, the operator can use information from layers lying beneath the current one. The reason is that non-local operators need a linearization and the inversion of a matrix of dimension ND × NL (ND being the number of depth points and NL the number of linearized levels). Although the matrix is nearly upper diagonal, the inversion requirements may still be large. With a local operator, however, it is only necessary to invert an NL × NL matrix, ND times. Even if the matrix were full the advantages are evident. Thus, local operators can treat up to 100 NLTE levels (or perhaps more if the matrix is not full). Moreover, linearization is also possible but not essential (Werner and Husfeld, 1985; Herrero, 1987a, b).

If we restrict ourselves to local operators, so that at depth τ

$$\Lambda_\tau^*(S_\nu(\tau)) = C_\nu(\tau) S_\nu(\tau) \tag{4}$$

and assume that the source function S_ν is dominated by the transition $i \leftrightarrow j$, so that $S_\nu = S_{ij}$, we may write

$$R'_{ij} = 4\pi \int \frac{\alpha_{ij}}{h\nu} \Delta J_\nu d\nu \tag{5a}$$

$$R'_{ji} = 4\pi \left(\frac{n_i}{n_j}\right)^* \int \frac{\alpha_{ij}}{h\nu} \Delta J_\nu e^{-h\nu/kT} d\nu \tag{5b}$$
$$+ 4\pi \left(\frac{n_i}{n_j}\right)^* \int \frac{\alpha_{ij}}{h\nu} \frac{2h\nu^3}{c^2} (1 - C_\nu) e^{-h\nu/kT} d\nu$$

for the radiative rates by forming a net rate with respect to $C_\nu(\tau) S_\nu^i$.

Here we have dropped the τ index for simplicity, as the formulation is similar for all depths. The equations of statistical equilibrium now read

$$n_i(C_{ij}+R'_{ij}+\sum_{j\neq i} P_{ij}) = n_j(C_{ij}+R'_{ji}) + \sum_{j\neq i} n_j P_{ji} \tag{5c}$$

and ΔJ_ν is the correction term of Eq. (3), $\Delta J_\nu = (\Lambda - \Lambda^*) S_\nu^{i-1}$. In this formulation, no linearization is needed. The equations of statistical equilibrium are solved directly for the occupation numbers and a new radiation field is computed. The procedure is then iterated until some convergence criterium is fulfilled.

As already indicated by Werner and Husfeld (1985), the selection of Λ^* in the outer layers, where $\tau < 1$, is not so important as a Λ-iteration will converge (but note that a Λ-iteration would converge slower than any other convergent procedure). However, in this paper we are interested in the optically thick case where $\tau \gg 1$. Here usually $C_\nu = 1$ is chosen, $(1 - C_\nu)$ vanishes, and the iteration procedure is driven by ΔJ_ν-the inconsistency between the previous calculated source function and the radiation field obtained by formal solution.

In general, this solution scheme is stable and converges quickly when applied to photospherical multi-level line formation calculations (Werner and Husfeld, 1985; Herrero, 1987a, b). However, the situation may be different if the ΔJ-terms dominate the rate equations when the optical thickness is very large; under these conditions, ΔJ is a very small quantity and may even become negative during iteration, especially if J_ν and S_ν are equal to the available numerical accuracy.

This is usually not a problem in photospheres. There, the very large optical depth corresponds to layers where the stabilizing collisional rates are large enough to dominate the reduced radiative rates calculated with Eqs (5). Other radiative transition rates may also act as stabilizing terms.

But, if these stabilizing terms are not large enough, the R'_{ij} and the much smaller R'_{ji} rates exchange rôles in the statistical equilibrium when ΔJ changes its sign, and this yields obviously ill-conditioned matrices, with subsequent divergence. Even more catastrophic will be the situation if only one of the terms changes its usual sign (the other, generally the R'_{ji} term, being dominated by the stabilizing terms). In this case the statistical equilibrium equations may be only fulfilled with negative occupation numbers in (at least) one of the levels.

The situation described in the preceding paragraph appears in the photospheres only in extreme cases (e.g. objects close to the Eddington limit because of the lower densities). In fact, in our plane parallel calculations we have found this behaviour for the resonance lines of He II preferentially in models near the Eddington limit, but only with the approximate operator of Werner and Husfeld (1985). Use of the more refined operator of Olson, et al. prevents divergence. This is because at very great depths this operator is approximately equal to $(1-1/(\Delta\tau)^2)$, so that it differs significantly from 1 down to layers where the collisions may again stabilize the situation.

For stellar winds, however, the situation is far more problematic in the case of very optically thick continua (lines are optically thinner due to the velocity gradients). Because of the slow decrease in density outwards (quadratic instead of exponential as in static photospheres), very large optical thickness may be reached, whereas the local density may still not be high enough to guarantee well-conditioned matrices through collisions.

3. A modified technique for continuum formation using local approximative Λ-operators

Here we present a modified technique for the continuum formation which is adequate to solve the difficult cases described in the preceding section. This technique combines the stabilizing properties of the normal Λ-iteration with the convergence advantages of the ALI method.

We start by writing Eq. (3) as

$$J_\nu^i = \Lambda(S_\nu^{i-1}) + \Lambda^*(S_\nu^i - S_\nu^{i-1}) \tag{6}$$

and construct a net rate in a similar way as before, but with respect to $\Lambda^*(S_\nu^i - S_\nu^{i-1})$. Instead of Eqs. (5) we now find:

$$R_{ij} = 4\pi \int \frac{\alpha_{ij}}{h\nu} \Lambda(S_\nu^{i-1})\, d\nu \tag{7a}$$

$$R_{ji} = 4\pi \left(\frac{n_i}{n_j}\right)^* \int \frac{\alpha_{ij}}{h\nu}\left(\frac{2h\nu^3}{c^2} + \Lambda(S_\nu^{i-1})\right) e^{-h\nu/kT}\, d\nu \tag{7b}$$

$$R_{ji}^* = 4\pi \left(\frac{n_i}{n_j}\right)^* \int \frac{\alpha_{ij}}{h\nu} \frac{2h\nu^3}{c^2} C_\nu \left(\frac{S_{ij}^{i-1}(\nu) - S_{ij}^i(\nu)}{S_{ij}^i(\nu)}\right) e^{-h\nu/kT}\, d\nu \tag{7c}$$

$$n_i\Big(C_{ij} + R_{ij} + \sum_{j\neq i} P_{ij}\Big) = n_j\big(C_{ji} + R_{ji} + R_{ji}^*\big) + \sum_{j\neq i} n_j P_{ji} \tag{7d}$$

Equations (7a) and (7b) represent the usual Λ-iteration and Eq. (7c) an accelerating term. Here we use the total source function in (7a) and (7b), but have used the source function in the $i \leftrightarrow j$ transition to build up the "perturbation rate" R_{ji}^*.

The inconvenience of nonlinear equations for the statistical equilibrium is apparently recovered in Eqs. (7). However, as the perturbation rate depends only on the relative deviations of the current and previously calculated source function, it is only the inconsistency between two succesively evaluated source functions which drives the iteration. Thus, the unknown term $(S_{ji}^{i-1} - S_{ij}^i)/S_{ij}^i$ may be replaced by the known quantity $(S_{ij}^{i-2} - S_{ij}^{i-1})/S_{ij}^{i-1} \equiv \Delta S_{ij}$, and thus linear equations are again obtained for the statistical equilibrium.

Now, when during the iteration process $\Delta S_{ij} \to 0$, the matrices remain well conditioned due to R_{ij} and R_{ji}, even if no other stabilizing rates (mainly collisions) are present, as long as the perturbation rate is not subjected to catastrophically amplified corrections. If necessary, such corrections can be avoided by comparing the values of R_{ji}^* and R_{ji} and damping the corrections, which is equivalent to choosing a smaller value for C_ν (but of course, not so small that they become meaningless). Another reason to choose values of C_ν slightly below 1 at the beginning of the iteration is to avoid oscillations in the convergence behaviour which may appear during the iteration process (Suppose, e.g., that after i iterations we obtain the correct source function S_{ij}^i. Thus, the radiation field calculated for iteration $(i+1)$, $\Lambda(S_{ij}^i)$, would also be correct, but we would still be correcting with $\Delta S_{ij} = (S_{ij}^{i-1} - S_{ij}^i)/S_{ij}^i$. Clearly, the advantage of very fast convergence could be compensated by the induced oscillations, so that a compromise has to be found).
Following Hamann (1986) in the case of stellar winds we have found that a good choice for Λ^* is:

$$\Lambda^*(S_\nu) = C_\nu S_\nu = (1 - e^{-\tau_\nu(r)/\gamma\tau_\nu^0(r_s)}) S_\nu \tag{8}$$

where $\tau_\nu(r)$, the local optical thickness in the current iteration, is scaled by $\tau_\nu^0(r_s)$, the optical thickness of the He II ground state edge at the sonic point obtained in our first iteration assuming radiative detailed balance, and γ is a free parameter.

We now proceed as follows: we set γ to 10^6 (thus performing a Λ-iteration) for two iterations to obtain starting values for ΔS_{ij}. Then we reduce systematically the value of γ shown in Table 1 (it should be mentioned that for cases in which line transitions are neglected γ has to be reduced moderately, since the damping influence of the radiative b-b rates is missing).

In this way values of C_ν equal to 1.0 are used in the problematic layers of the wind in the last iterations. In the upper layers these values are smaller but this does not matter as the Λ-iteration becomes effective (see Sect. 4).

Finally, it is important to mention that the source function in the $i \leftrightarrow j$ transition used to build up the "perturbation rate" is not a restriction to dominating transitions, since the described iteration process is superimposed by a normal Λ-iteration. Thus, this technique can be and is also used for metals.

4. Results

4.1. Stellar winds

In order to illustrate the situation, we have chosen an extreme case of continuum formation for stellar winds allowing for a differential comparison of the results of the ALI method and our modified technique. These calculations are based on the self-consistent treatment of radiatively driven stellar winds of hot stars (Pauldrach, 1987), which comprises detailed multi-level statistical equilibrium calculations for 133 ionization stages of 26 elements, including 10,000 radiative bound-bound transitions, electron collisions and the correct continuous radiation field obtained from consideration of the ground levels of the elements H, He, C, N, O, Ne, Mg, Si, and S as contributors to the bound-free opacity, self-consistently with hydrodynamics.

Table 1

$N_{\rm It}$	0–1	2–6	7–11	11–$N_{\max}$
γ	10^6	1	$\begin{bmatrix} 10^{-2} \\ 4/\tau_\nu^0(r_s) \end{bmatrix}_{\max}$	$\begin{bmatrix} 10^{-4} \\ 4/\tau_\nu^0(r_s) \end{bmatrix}_{\max}$

Before the methods outlined above were applied to a stellar wind model of crucial optical thickness in the ionization continuum shortward of the He II edge, as a first step we recalculated the wind models of ζ Pup using the same parameters and assumptions as Pauldrach (1987), in order to investigate the properties of the continuum formation techniques under non-critical conditions. All three methods – ALI, our technique and the lambda iteration – yielded identical results for both dynamics and wind line formation without difficulties.

The choice of NGC 2392, the central star of planetary nebulae close to the Eddington limit as a test case seems to be appropriate. This object shows on the one hand extreme emission features (see Pauldrach et al., 1987) that indicate enormous wind densities and on the other it has a well determined position in the $\log g$, $\log T_{\rm eff}$ diagram found by detailed NLTE analysis (see Mendez et al., 1988). Hence, our results of the continuum formation calculations are discussed in detail for a model of NGC 2392 with the following stellar parameters (see Mendez et al., 1988)

$T_{\rm eff} = 45000$ K

$\log g = 3.6$

$R/R_\odot = 2.76$

$N({\rm He})/N({\rm H}) = 0.1$

From self-consistent detailed wind calculations – for which our modified technique has already been incorporated – Pauldrach et al. (1987) obtained a terminal velocity of

$v_\infty = 420\ {\rm km\,s^{-1}}$

for this object and a mass loss rate

$\dot{M} = 1.4\ 10^{-6}\ M_\odot/{\rm yr}$

which supports the suggestion that this CPN star has an extremely dense wind and consequently an ionization continuum of large optical thickness. Figure 1 shows that for the He II ground state edge this really is the case.

This large optical thickness is caused by recombination of He^{++} in nearly the whole wind region. Hence, it is not surprising that it is the He II ground state that causes the greatest convergence problems. As a function of proton density, the population numbers of the He II ground state relative to the total He number are given in Fig. 2 for our modified technique (converged exact solution), the ALI method just before divergence (see discussion below), the lambda iteration after 40 and 200 iterations and the starting values.

First, we see that in the range of $11 < \log n_p < 13$ the lambda iteration is not able to yield convergence, since the method produces solutions that stabilize far from the exact solution and this demonstrates the necessity of the more elaborate techniques.

Although the attempt to obtain an iterative solution with the ALI method seems to be successful after 5 iterations (note that the values of the iteration go quickly in the right direction compared with the values of the lambda iteration), it has to be emphasized that the next iteration diverges catastrophically in the wind region given above. The reasons for this behaviour have already been described in Sect. 2, and can be summarized as follows. Due to the large optical thickness of the UV continuum the radiative rates are dominated by the very small quantity ΔJ (see Eq. (5)) which can become negative during iteration. This yields ill-conditioned matrices, since the compensating influence of the collisional rates is missing as they are too small in this wind region. As a consequence, the statistical equilibrium equations can be fulfilled only with negative occupation numbers and the method diverges. This behaviour is expressed numerically in Table 2, where the perturbation rates (R^*_{ji} – see Eq. (17)) of our modified technique are also given.

The order of magnitude of these perturbation rates ($|R^*_{ji}| \le R_{ji}$ and $|R^*_{ji}| \ll R_{ij}$) confirm our conclusion in Sect. 3 that even under extreme conditions the normal lambda iteration radiative rates act as stabilizing terms, so that the matrices remain well conditioned even if no stabilizing collisional rates are present. Hence,

Table 2

$N_{\rm It}$	0	2	6
R'_{ij}/R_{ij}	1.0	0.67	−0.05
R'_{ji}/R_{ji}	1.0	0.82	0.02
R^*_{ji}/R_{ji}	0.0	0.03	0.39
R_{ji}/R_{ij}	$3.0\ 10^{-3}$	$3.7\ 10^{-3}$	$9.7\ 10^{-3}$
C_{ij}/R_{ij}	$1.6\ 10^{-5}$	$2.0\ 10^{-5}$	$5.3\ 10^{-5}$
C_{ji}/R_{ji}	$1.2\ 10^{-6}$	$1.2\ 10^{-6}$	$1.2\ 10^{-6}$

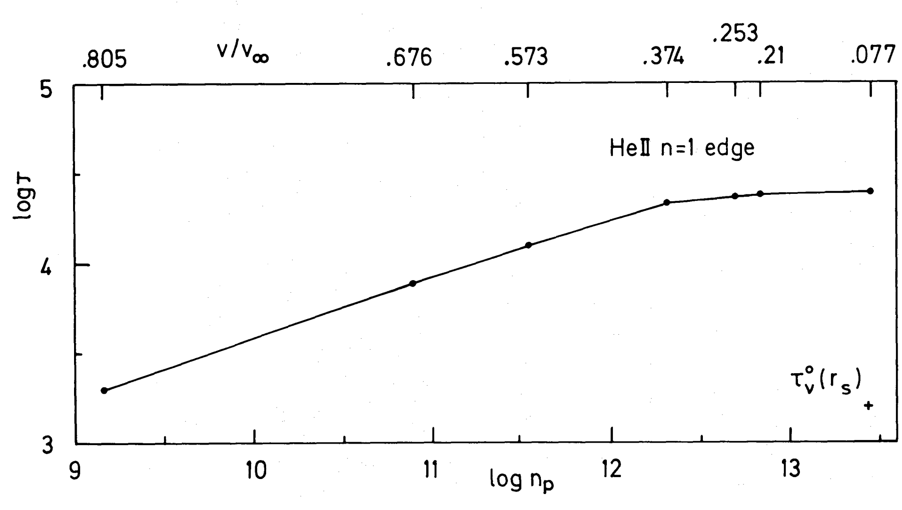

Fig. 1. Illustration of the continuous optical depth at 227.84 Å (He II ground state edge) as a function of the proton density

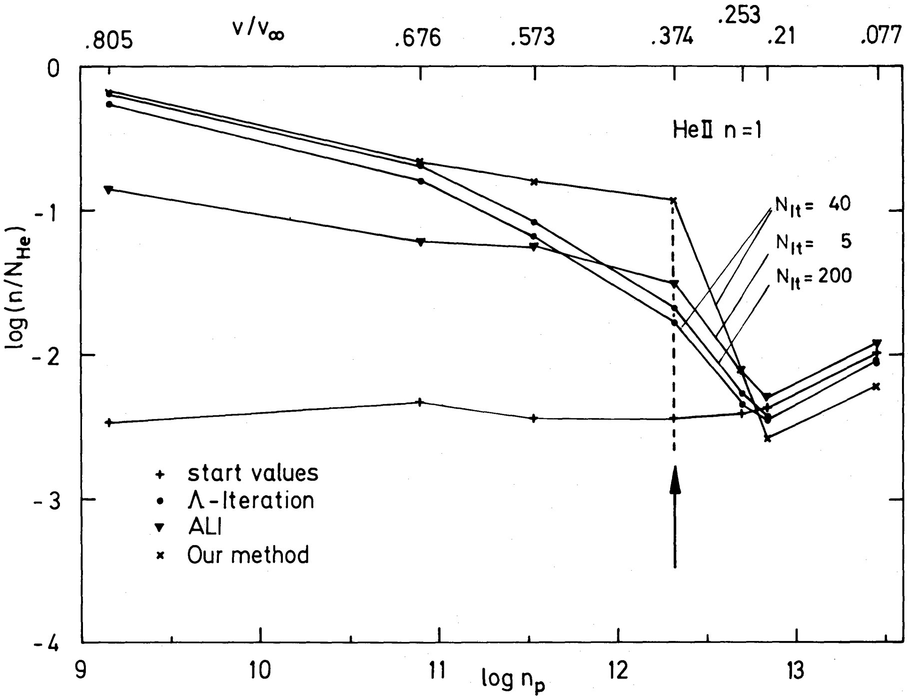

Fig. 2. Run of the population numbers of the He II ground state relative to the total He number as a function of the proton density for our modified technique (×), the ALI method (▼) and the lambda iteration after 40 and 200 iterations (●). The starting values (common to all three methods) are also given (+). The critical continuum forming layer is marked by the arrow (see Fig. 3)

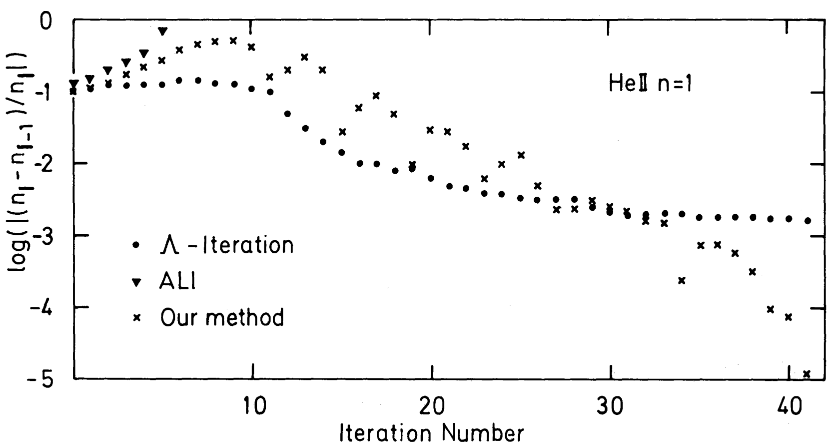

Fig. 3. The decrease of the relative corrections of the He II ground state at a depth of $\log n_p = 12.32$ (marked by the arrow in Fig. 2) is shown versus the iteration number for the different iteration processes (lambda iteration (●), ALI technique (▼), our method (×)). Note the divergence of the ALI-method after five iterations

our technique for the continuum formation is adequate to overcome the problematic situation described above, since it combines the stabilizing properties of the Λ-iteration with the fast convergence of the ALI method. To demonstrate this in more detail the decrease of the relative corrections of the He II ground state are shown for the different iterations processes at a depth of long $n_p = 12.32$ in Fig. 3. Here we see, that for ten iterations our technique yields nearly the same large corrections as the ALI iteration, although the latter diverges after 5 iterations (see above). The relative corrections of our iteration cycle decrease monotonically after 40 iterations, whereas the usual lambda iteration gives constant corrections confirming the failure of this method. The already mentioned oscillations (see Sect. 3) of our techniques between the 10th and 30th iterations are related to the perturbation rate, which favours ionization and recombination alternately (see Fig. 4) due to the remaining inconsistency of the last two successively evaluated source functions (see Eq. (7)). In addition, Fig. 3 and Fig. 4 together demonstrate that our method still converges when the accelerating term is very small compared with the radiative recombination lambda term.

4.2. *Hydrostatic photospheres*

As it may be necessary in some cases to solve for the photospheric layers simultaneously with the envelope of the wind (e.g. in the case of spherically extended NLTE model atmosphere calculations for a gas in radiative equilibrium), we have to test our continuum formation method for its applicability in the photosphere layers so that we can ensure consistency from innermost layers through to the outermost.

Again, even if the conditions described in the previous sections hardly occur in the photosphere, it is essential to investigate

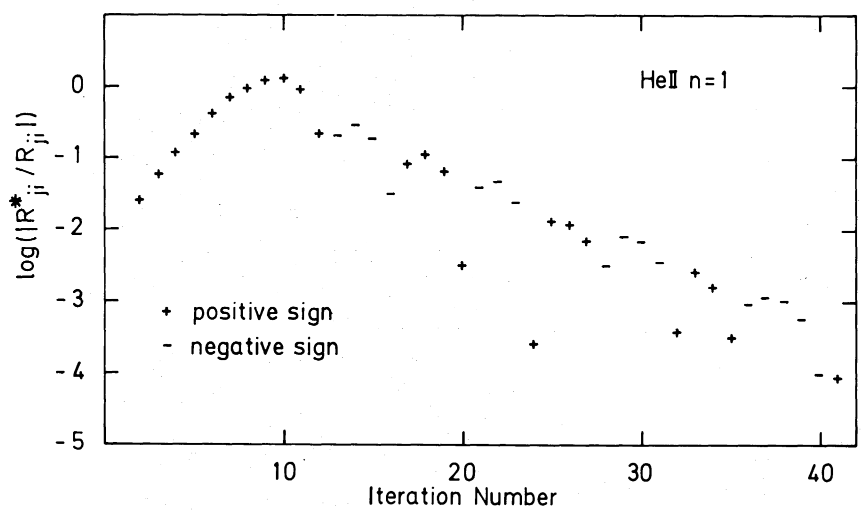

Fig. 4. The logarithm of the perturbation rate (R^*_{ji}) relative to the lambda recombination rate versus the iteration number for the He II ground state (our method). The different signs indicate how ionization (−) and recombination (+) are alternately favoured by the perturbation rate (see Eq. (7))

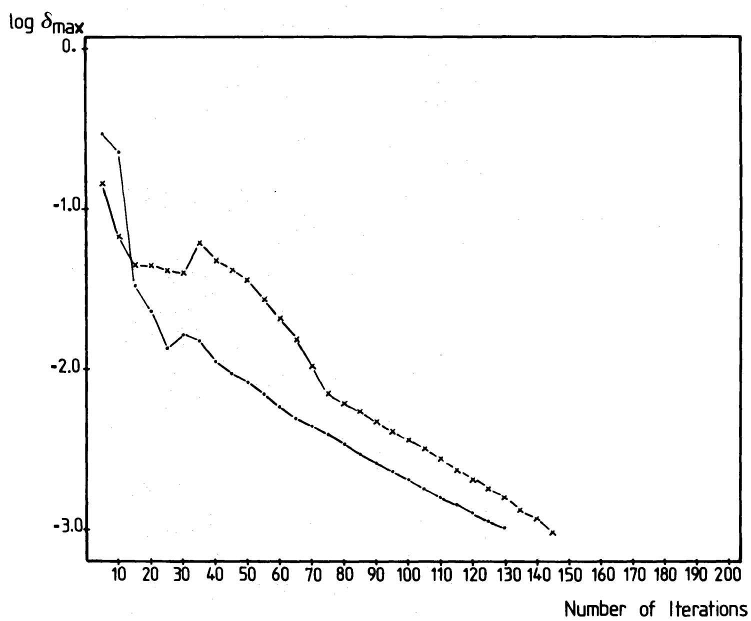

Fig. 5. Greatest relative corrections ($\delta_{\max}=\max|(n_i-n_{i-1})/n_i|$) of the He II ground state versus the iteration number for the ALI method (×) and our technique (•) in the case of hydrostatic photospheres

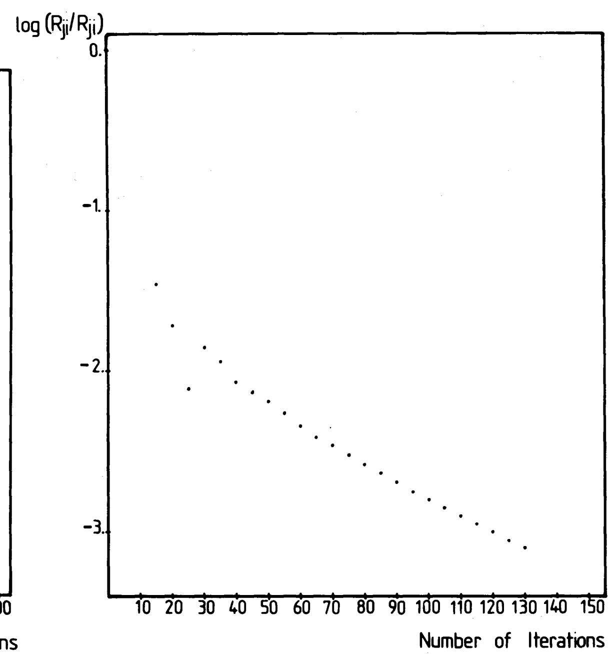

Fig. 6. For the hydrostatic photospherical model the logarithm of the perturbation rate relative to the lambda recombination rate is shown for our technique, as a function of the iteration number

the properties of the modified method in these layers, since it is desirable to have a new method to deal with the known convergence problems for continuum formation.

This test has been performed on the basis of NLTE multilevel line formation calculations in plane-parallel, static stellar atmospheres, including a He II model which consists of 14 NLTE levels and 92 transitions (see Herrero, 1987b).

The calculations have been carried out for a model with $T_{\rm eff}=40\,000$ K, $\log g=4.0$ and normal helium abundance (N(He)/N(H)$=0.1$). Again, the decrease of the largest relative corrections of the He II ground state occupation numbers is plotted versus the iteration number in Fig. 5 for the ALI method and our technique. In addition, Fig. 6 shows the behaviour of the perturbation rate during the iteration process. It can be seen from the decrease in the corrections that both methods are of equal merit. But for objects very close to the Eddington limit one should be careful with the ALI technique, since large optical thickness can also be reached in such cases together with local densities that are not high enough to guarantee the stabilizing influence of collisions.

Finally, we wish to point out that in contrast to the ALI method our continuum formation technique has also been successfully applied to metals (C, N, O, Mg, Si, S) in self-consistent radiatively driven wind calculations. This is of great importance, since Pauldrach (1987) has shown that metals must in some cases be regarded as main contributors to the continuum opacity, which indirectly influences the dynamics and of course the line formation.

5. Conclusions

Together with analytical considerations we have demonstrated through our calculations that the accelerated lambda iteration fails for the continuum formation of stellar winds under extreme NLTE conditions. Moreover, we suggest that in the case of NLTE multi-level continuum formation calculations in plane parallel, static stellar atmospheres the same problem may occur for objects very close to the Eddington limit.

Hence, we have presented a modified application of the perturbation technique, also based on entirely local approximate lambda operators, which stabilizes the convergence behaviour by means of well-conditioned equations of statistical equilibrium, so that convergence is obtained for the continuum formation of stellar winds and photospheres even under extreme conditions. In addition, Gabler et al. (1988, submitted) confirm the results of our investigation using a completely independent code and have shown that our modified perturbation technique is also appropriate for spherically extended NLTE model atmosphere calculations, in which the constraint of radiative equilibrium and the density structure of radiation driven winds are included.

Thus the investigation described in this paper brings us a decisive step nearer to our aim of calculating completely self-consistent wind models for stars close to the Eddington limit.

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